Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, 3 Mai 2024

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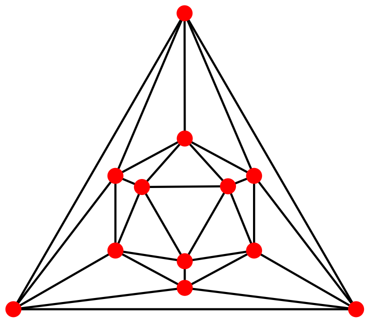
Exercise Sheet #3

Graph Visualization (SS 2024)

Exercise 1 – Canonical order and shift method for the icosahedron

Let G be the 1-skeleton of the icosahedron, i.e., the graph shown below.



a) Find a canonical order of G. **3 Points**

b) Draw G using the shift algorithm from the lecture. Show the intermediate drawings step by step. **5 Points**

Exercise 2 – Canonical orders for outerplanar graphs

A graph is *outerplanar* if it has a planar embedding such that all vertices are on the same face, usually the outer face. It is a *maximal outerplanar graph* if it is biconnected and internally triangulated.

Describe a special canonical order built precisely for maximal outerplanar graphs.

a) Reformulate the conditions (C1)–(C3) for maximal outerplanar graphs. Can we enforce a bound on the degree of v_{k+1} in G_{k+1}? **2 Points**

b) How can we adjust the algorithm CanonicalOrder for maximal planar graphs to obtain a canonical order for maximal outerplanar graphs? **4 Points**

Exercise 3 – An alternative shift algorithm

We want to examine an alternative algorithm for drawing a plane triangulation G:

- Let $(v_1, v_2, \ldots, v_n)$ be a canonical order of the vertices of G.
- Draw v_1 at $(0, 0)$, v_2 at $(2, 0)$, and v_3 at $(1, 1)$.
- Incrementally draw the graph $G_k = G[\{v_1, \ldots, v_k\}]$ for $k \in \{4, 5, \ldots, n\}$:

  Let $w_1, \ldots, w_p, \ldots, w_q, \ldots, w_t$ be the vertices on the boundary of the outer face of G_{k-1} (in this order), where $w_1 = v_1$, $w_t = v_2$, and $w_p, \ldots, w_q$ are the neighbors of v_k in G_{k-1}. As the x-coordinate of v_k, choose an integer value $x(v_k)$ with $x(w_p) < x(v_k) < x(w_q)$. If no such value exists, first shift the right part of the drawing to the right by 1; i.e., for $q \leq i \leq t$ move each $L(w_i)$ to the right by 1. Now choose the smallest positive integer y-coordinate for which the drawing stays planar and v_k lies on the outer face.

a) Argue why this algorithm always yields a planar drawing. In particular, in step 3, why does a suitable y-coordinate exist? **3 Points**

b) Find a good lower bound for the maximum area requirement of the resulting drawing: find an infinite family of graphs where making bad choices for the x-coordinate in step 3 gives exponentially large y-coordinates. **3 Points**

This assignment is due at the beginning of the next lecture, that is, on May 10 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on May 8 at 16:00 and the solutions will be discussed one week after that on May 15.