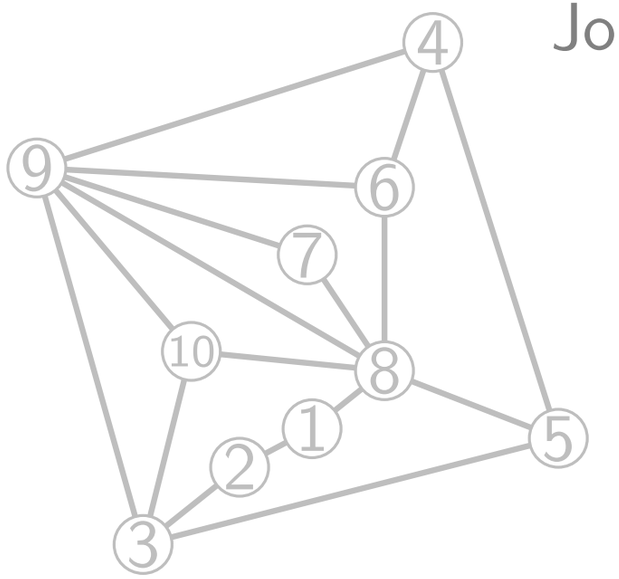
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

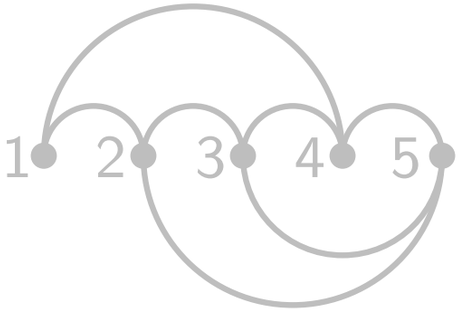
Visualisation of graphs

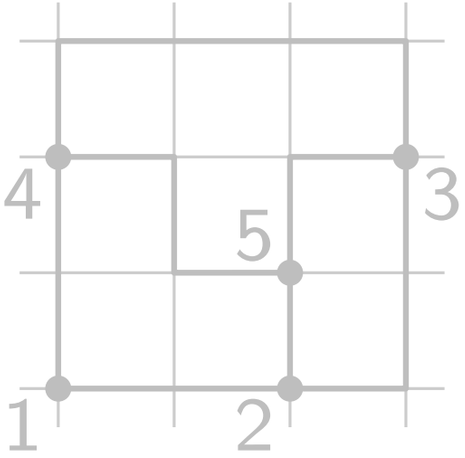
Introduction

The graph visualisation problem

Jonathan Klawitter · Summer semester 2020







Graphs and their representations

What is a graph?

- graph $G = (V, E)$
- vertices $V = \{v_1, v_2, \ldots, v_n\}$
- edge $E = \{e_1, e_2, \ldots, e_m\}$

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- Adjacency matrix

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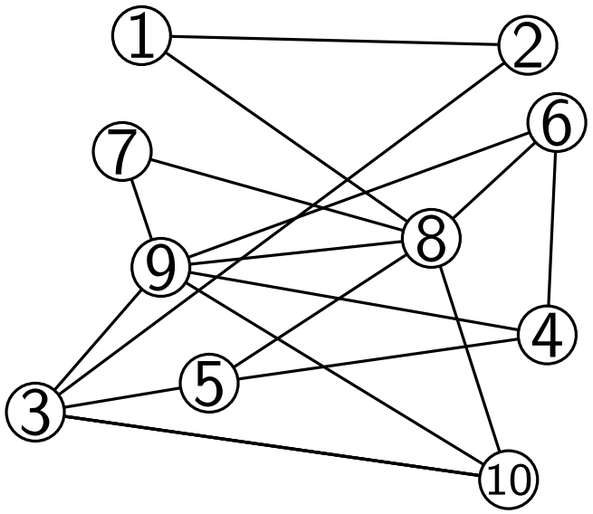
- Drawing



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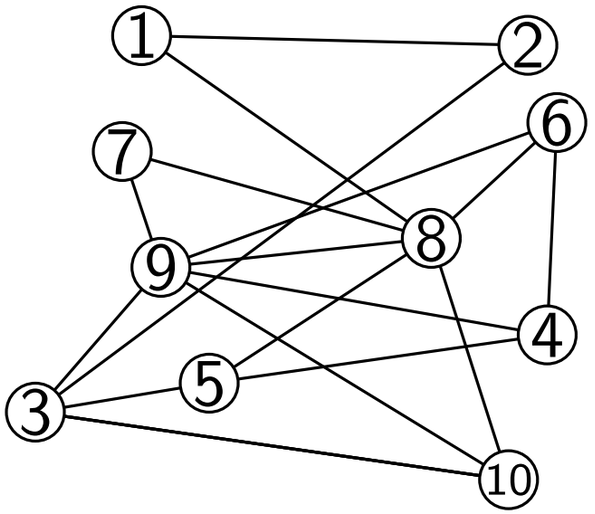

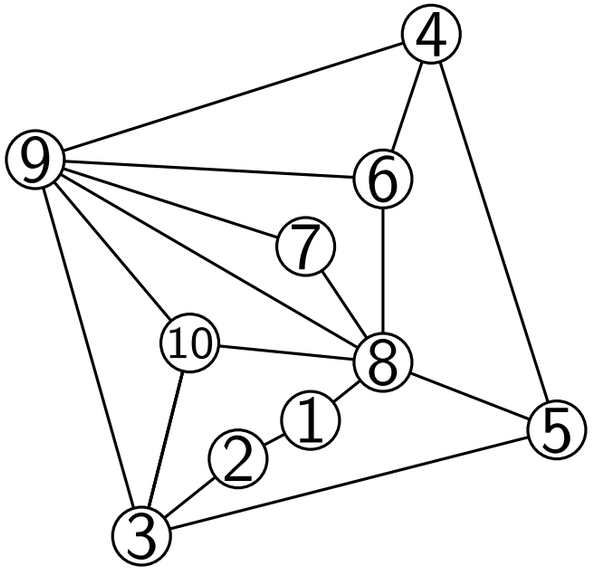

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Abstract networks

- Social networks
- Communication networks
- Phylogenetic networks
- Metabolic networks
- Class/Object Relation Digraphs (UML)
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Physical networks

- Metro systems
- Road networks
- Power grids
- Telecommunication networks
- Integrated circuits
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We need algorithms that draw graphs automatically to make networks more accessible to humans.

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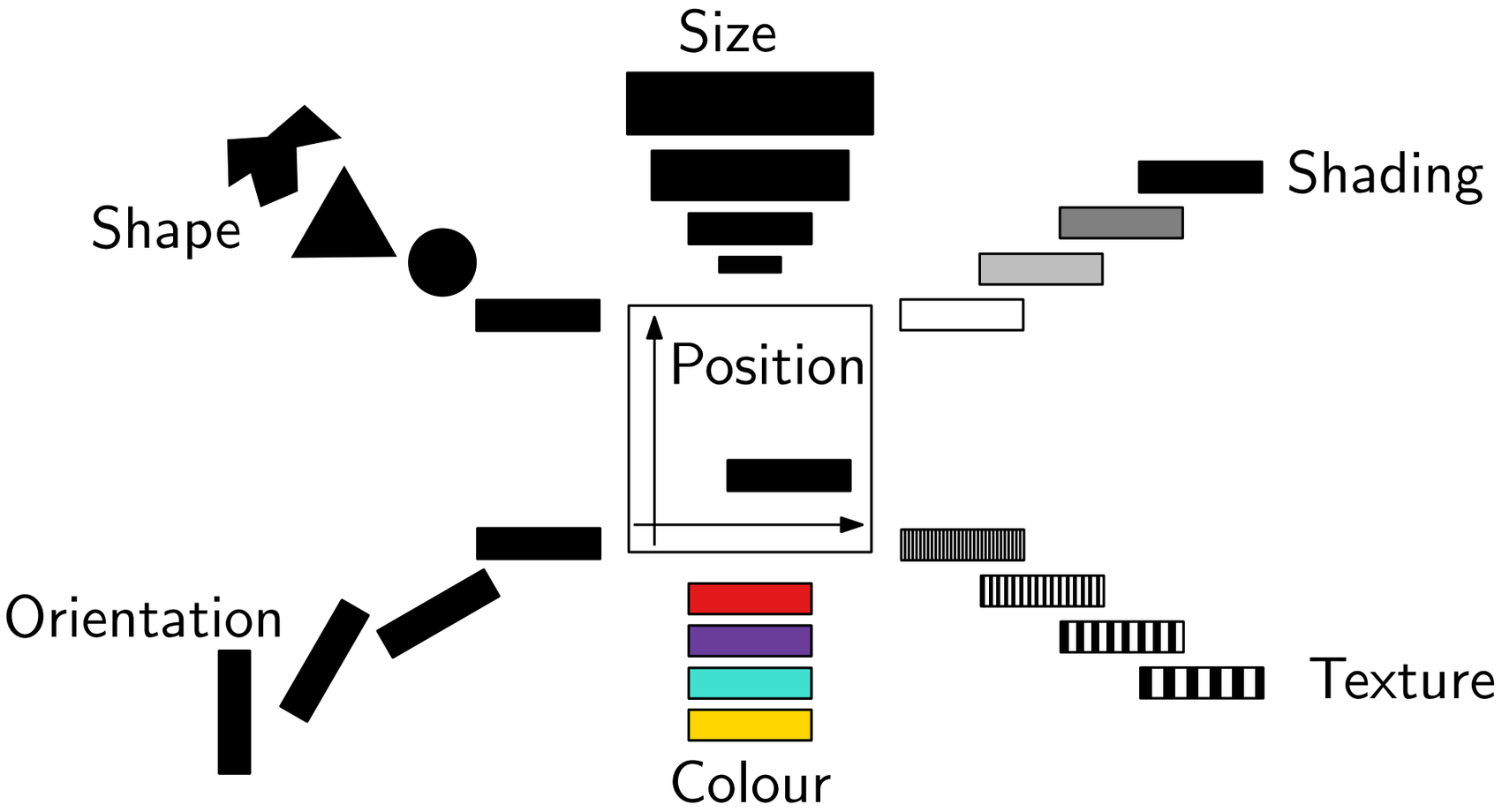

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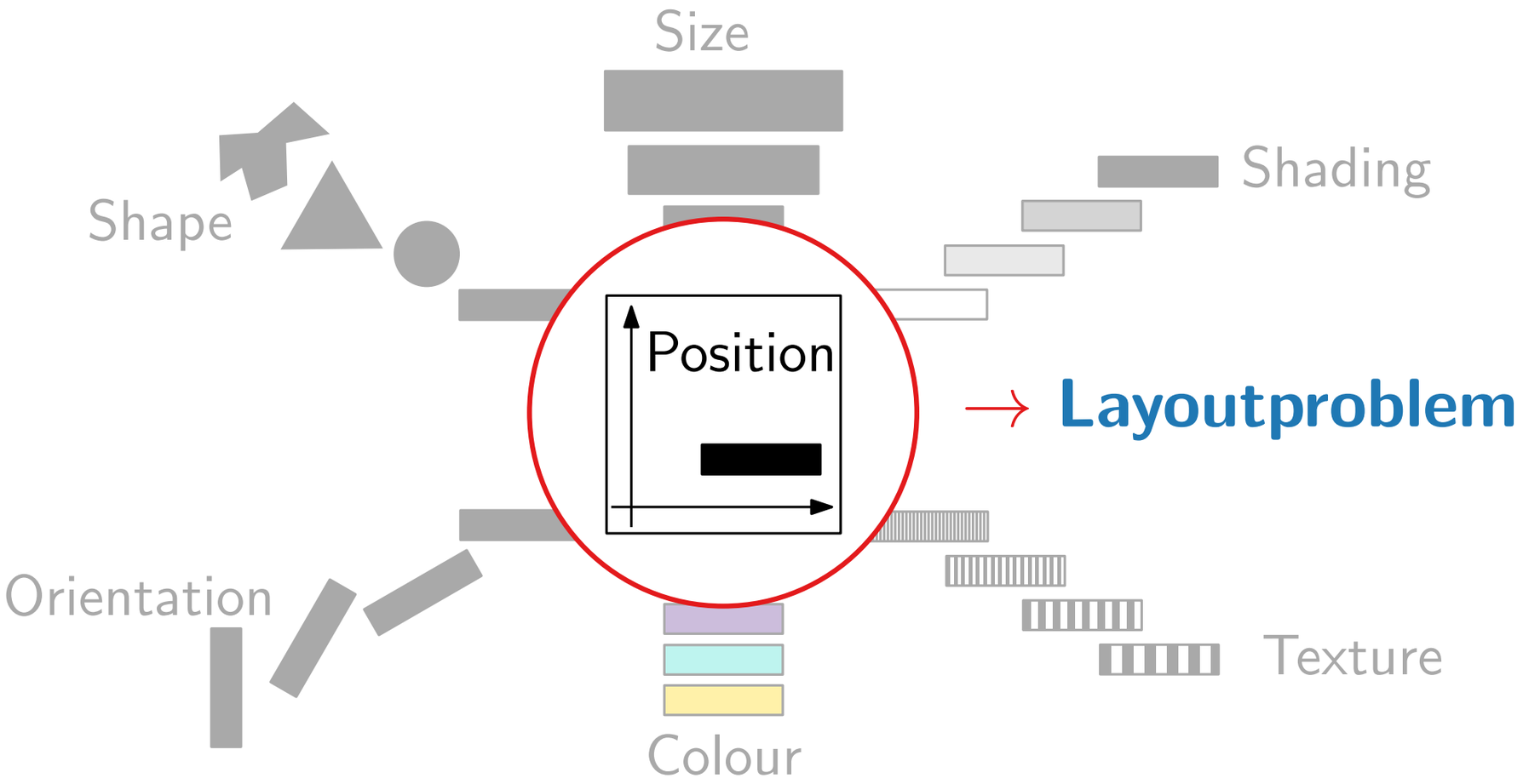

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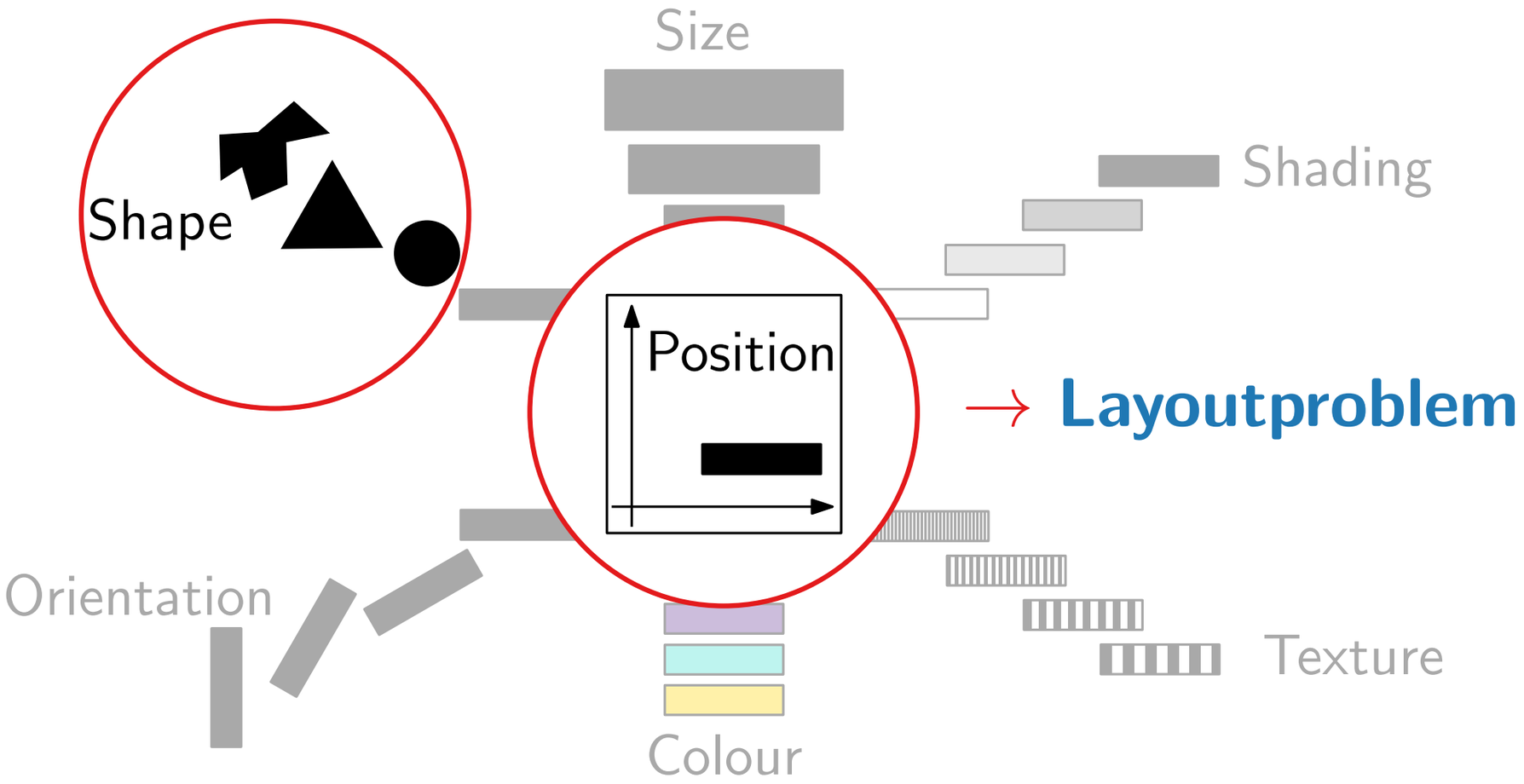
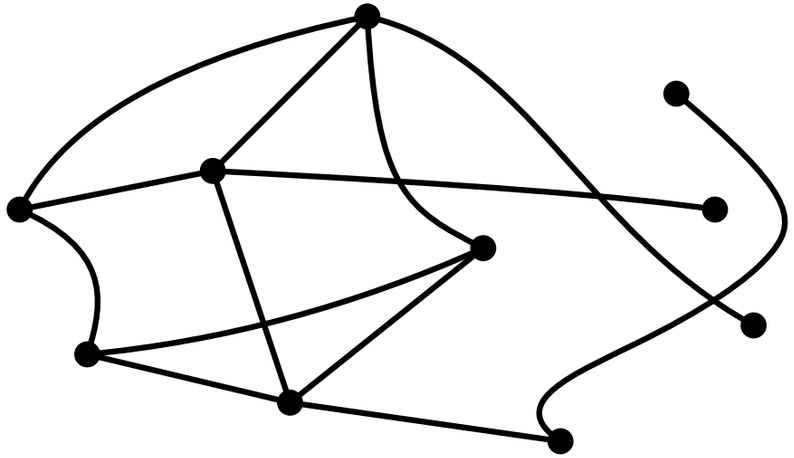


The layout problem

- *Here* restricted to the **standard representation**, so-called node-link diagrams.



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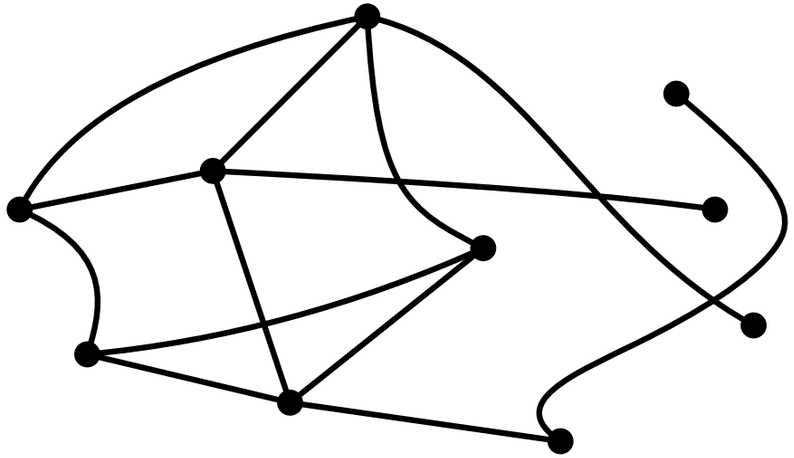

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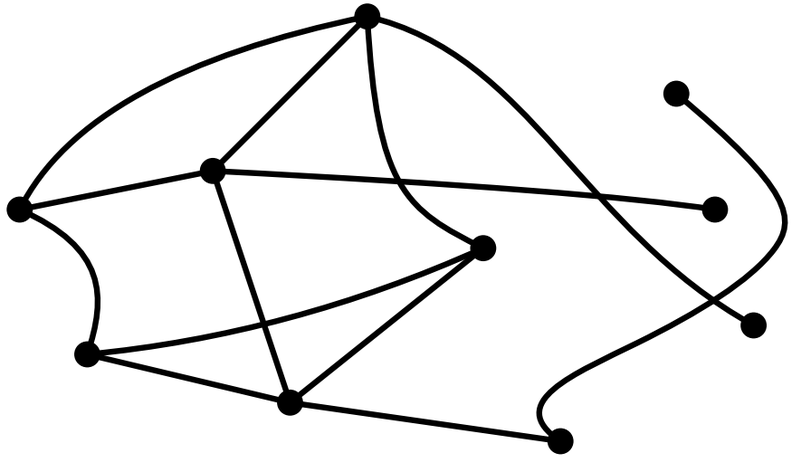

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- $\Gamma\colon V \to \mathbb{R}^2$, vertex $v \mapsto$ point $\Gamma(v)$
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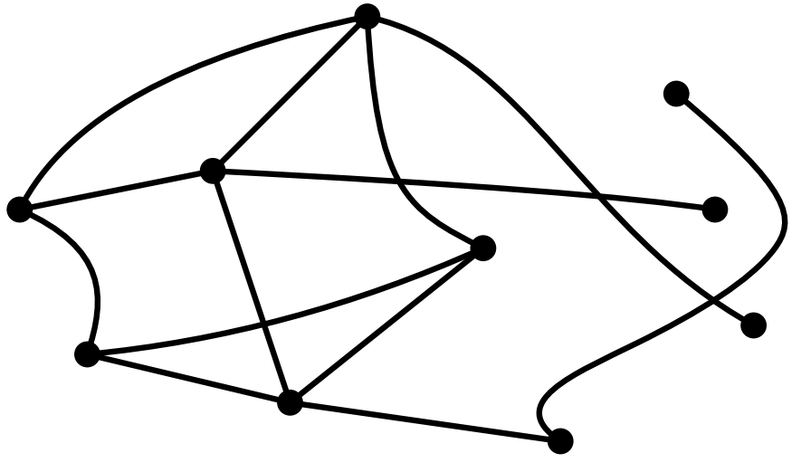

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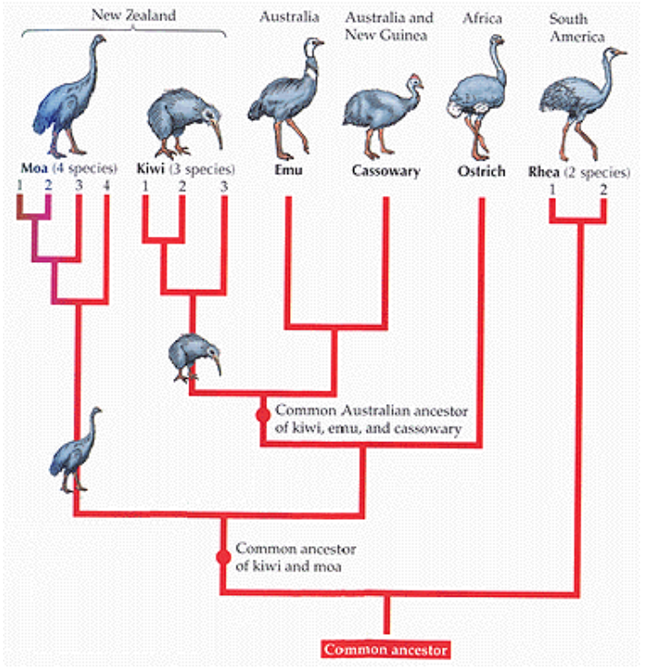
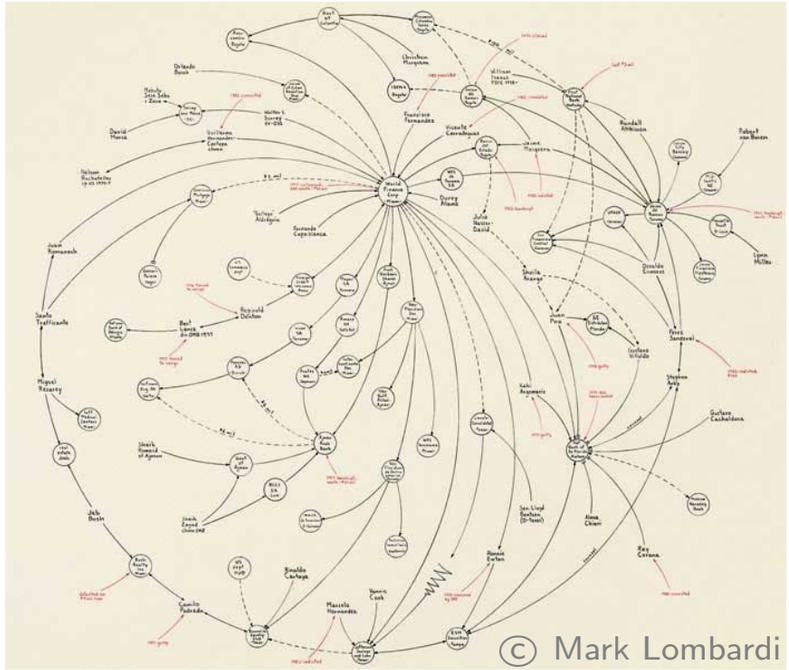
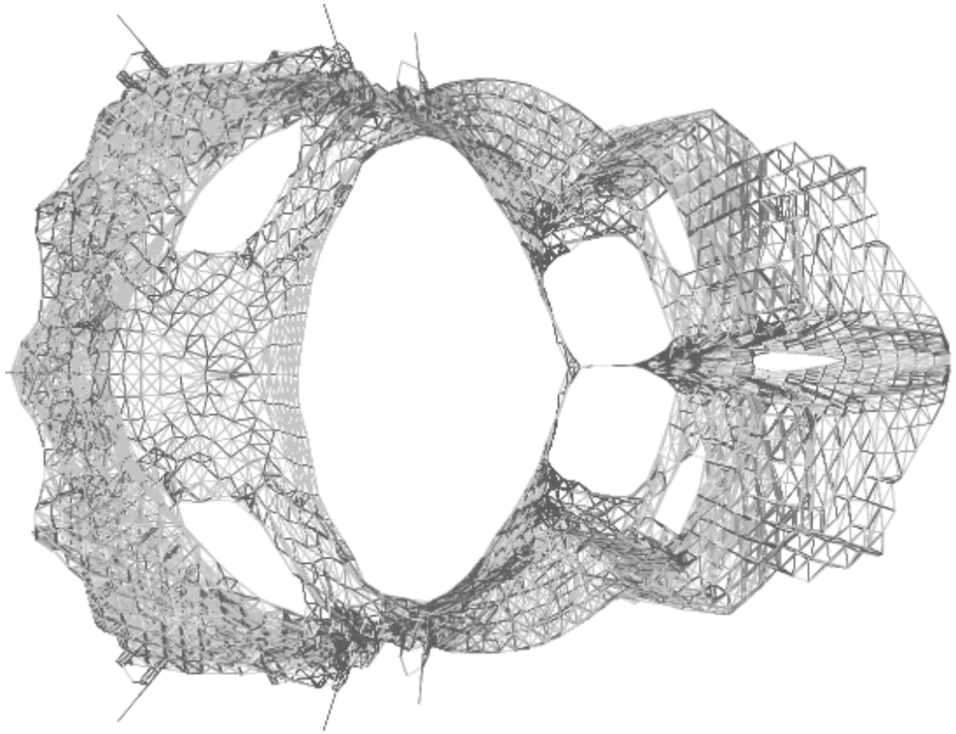
But what is a **nice** drawing?

Examples







- See slides (and video) with more examples.

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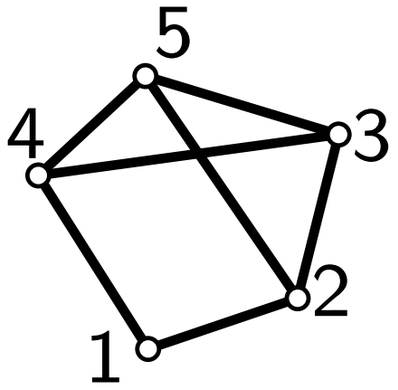

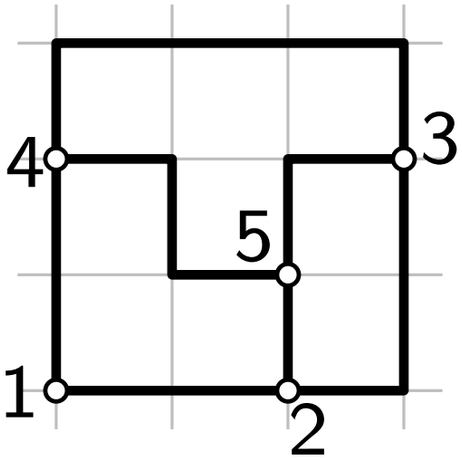

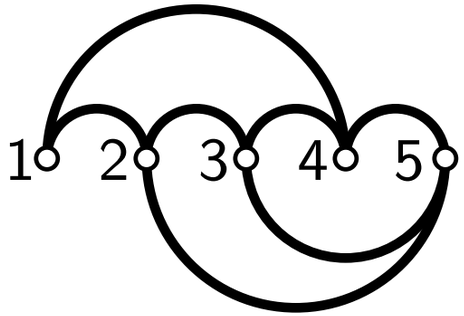

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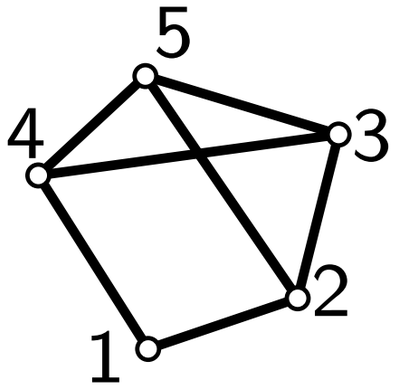

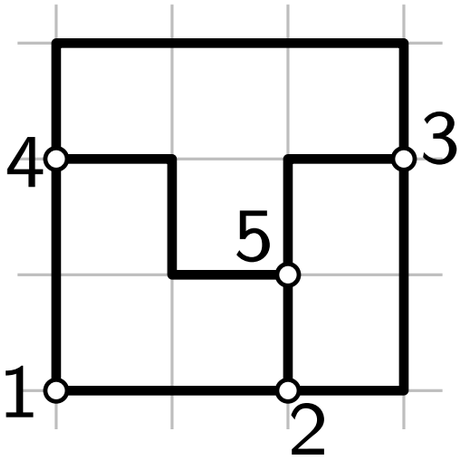

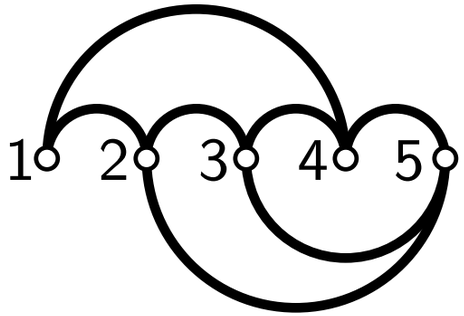

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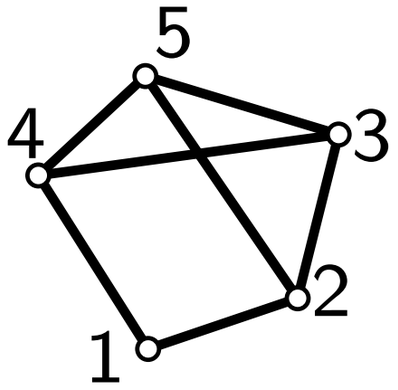

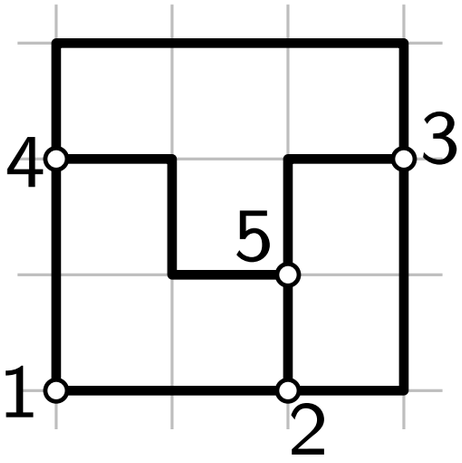

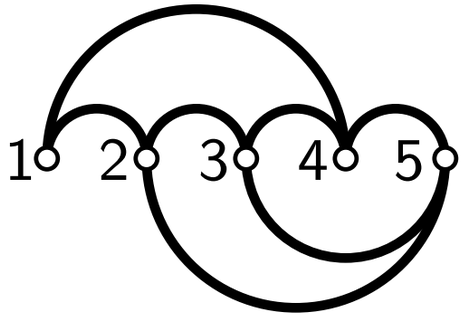

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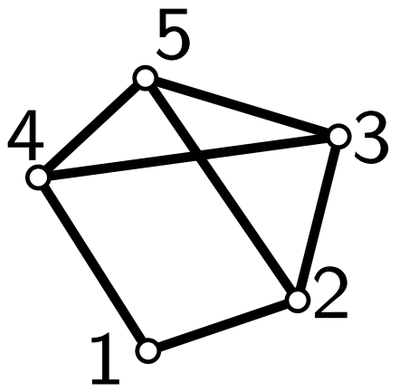

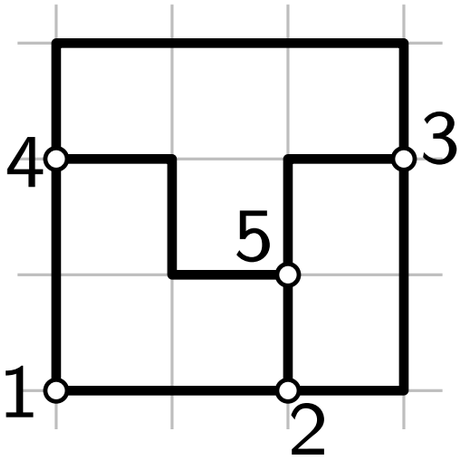

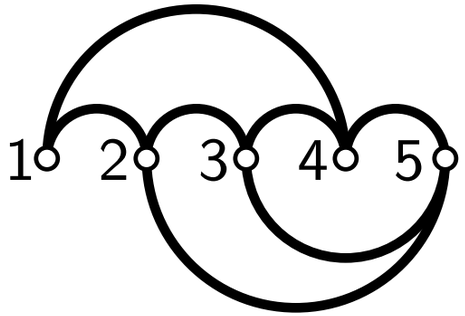

$\rightarrow$ lead to NP-hard optimization problems

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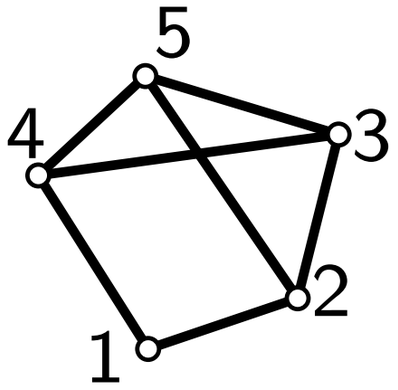
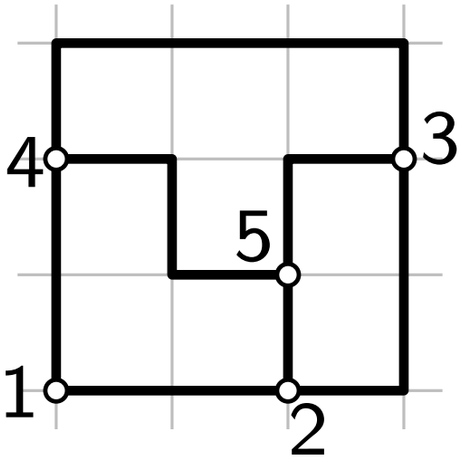
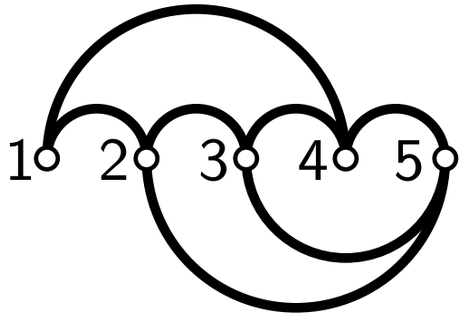
   - straight edges with $\Gamma(uv) = \overline{\Gamma(u)\Gamma(v)}$
   - orthogonal edges (i.e. with bends)
   - grid drawings
   - without crossing
2. Aesthetics to be optimised, e.g.
   - crossing/bend minimisation
   - edge length uniformity
   - minimising total edge length/drawing area
   - angular resolution
   - symmetry/structure







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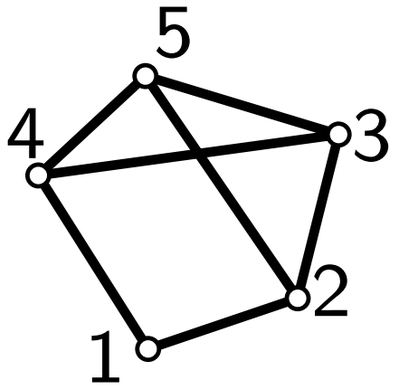

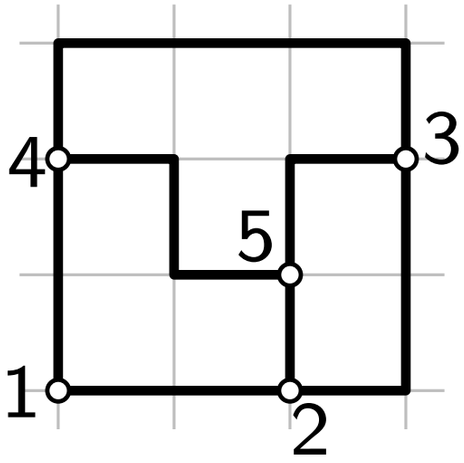

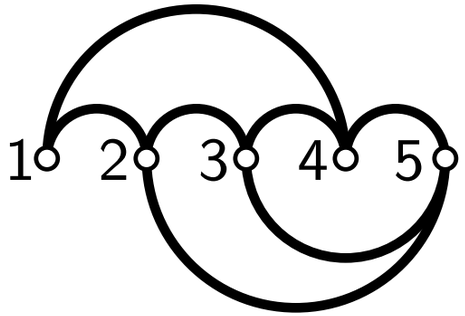

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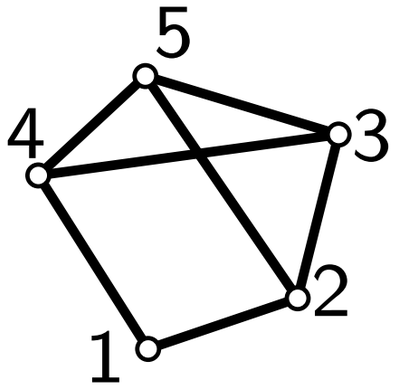

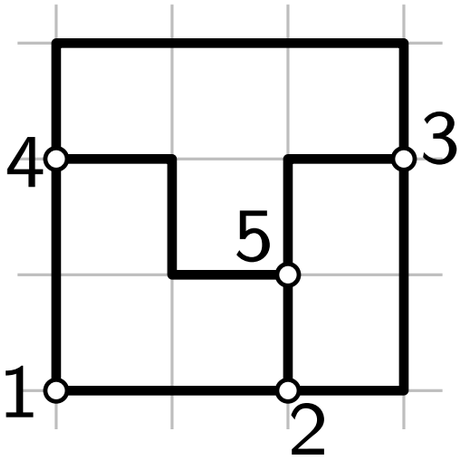

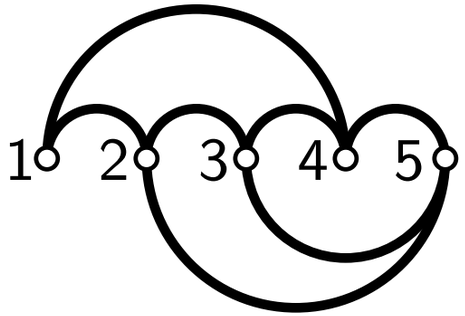

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3. Local Constraints, e.g.
   - restrictions on neighbouring vertices (e.g., “upward”).
   - restrictions on groups of vertices/edges (e.g., “clustered”).

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in: Graph $G = (V, E)$

out: Drawing Γ of G such that

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- Many algorithmically interesting questions arise.
- Rendering problem downstream is ignored.