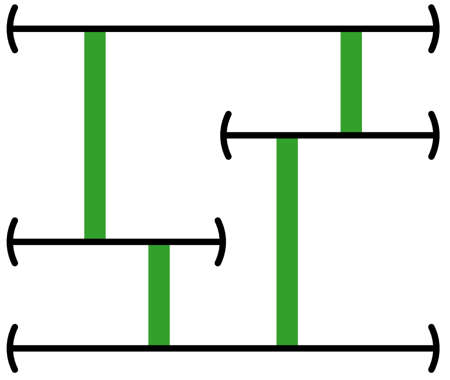
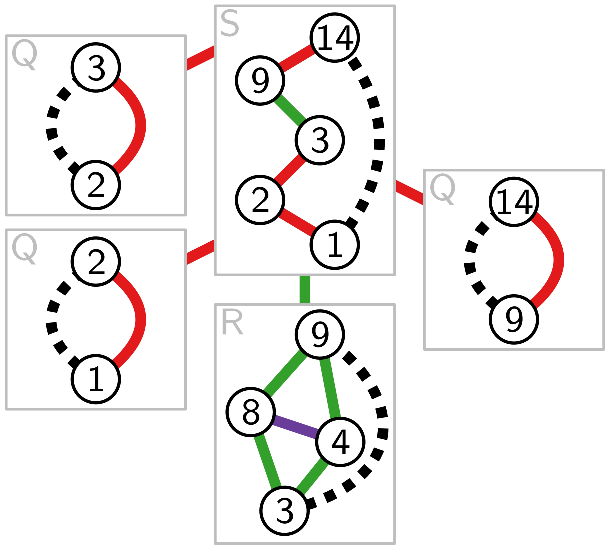
Visualization of graphs

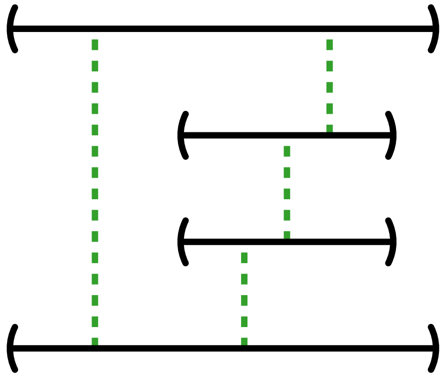
Partial visibility representation extension

With SPQR-trees

Jonathan Klawitter · Summer semester 2020







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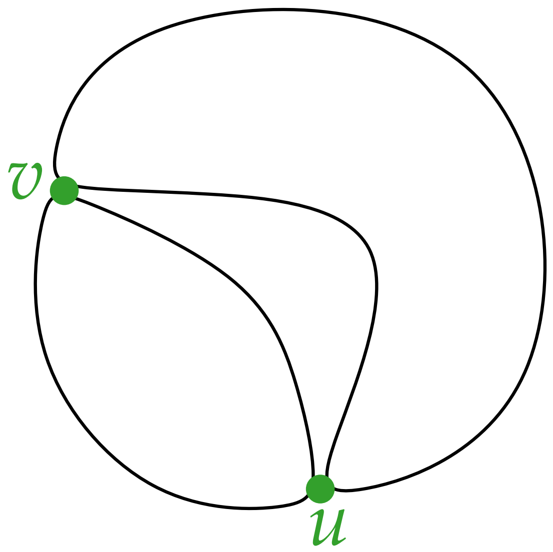

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- An **SPQR-tree** T is a decomposition of a planar graph G by **separation pairs**.
- The nodes of T are of four types:
  - **S** nodes represent a series composition
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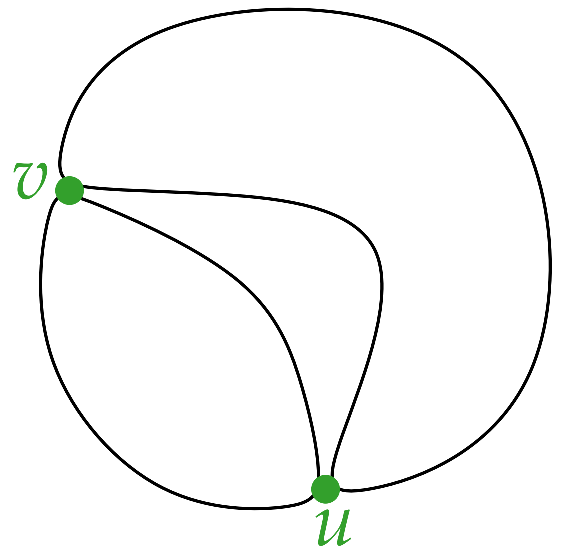
  - **Q** nodes represent a single edge
  - **R** nodes represent 3-connected (*rigid*) subgraphs



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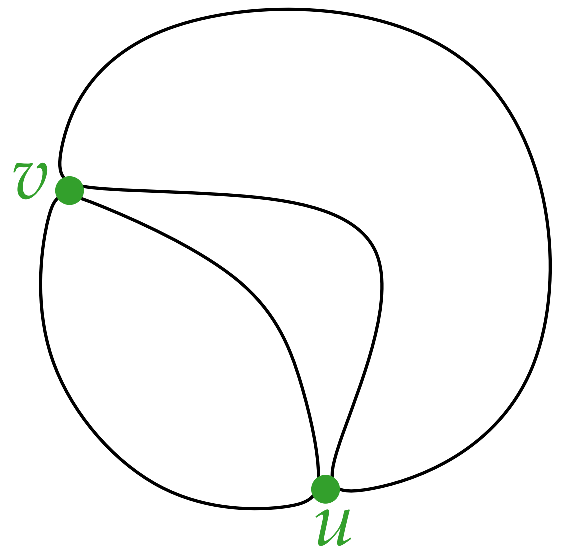

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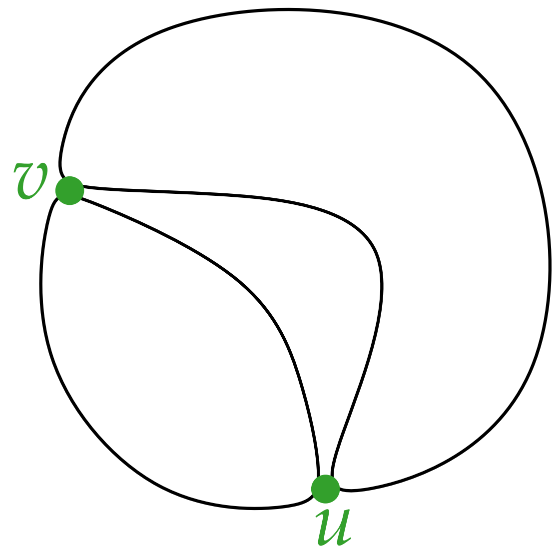

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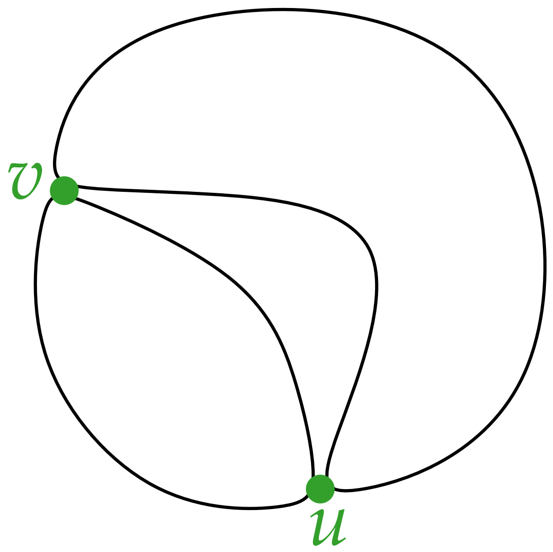
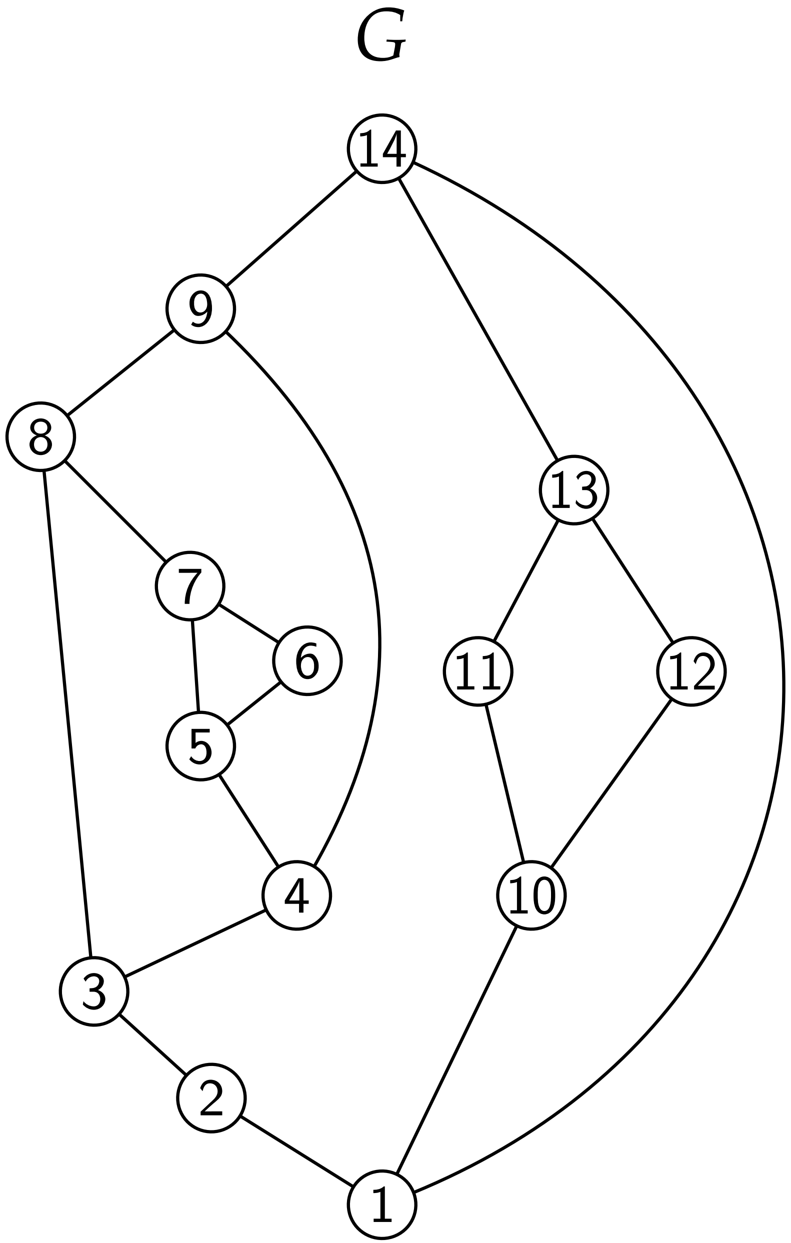
- T represents all planar embeddings of G.
- T can be computed in $\mathcal{O}(n)$ time. [Gutwenger, Mutzel '01]



SPQR-tree example



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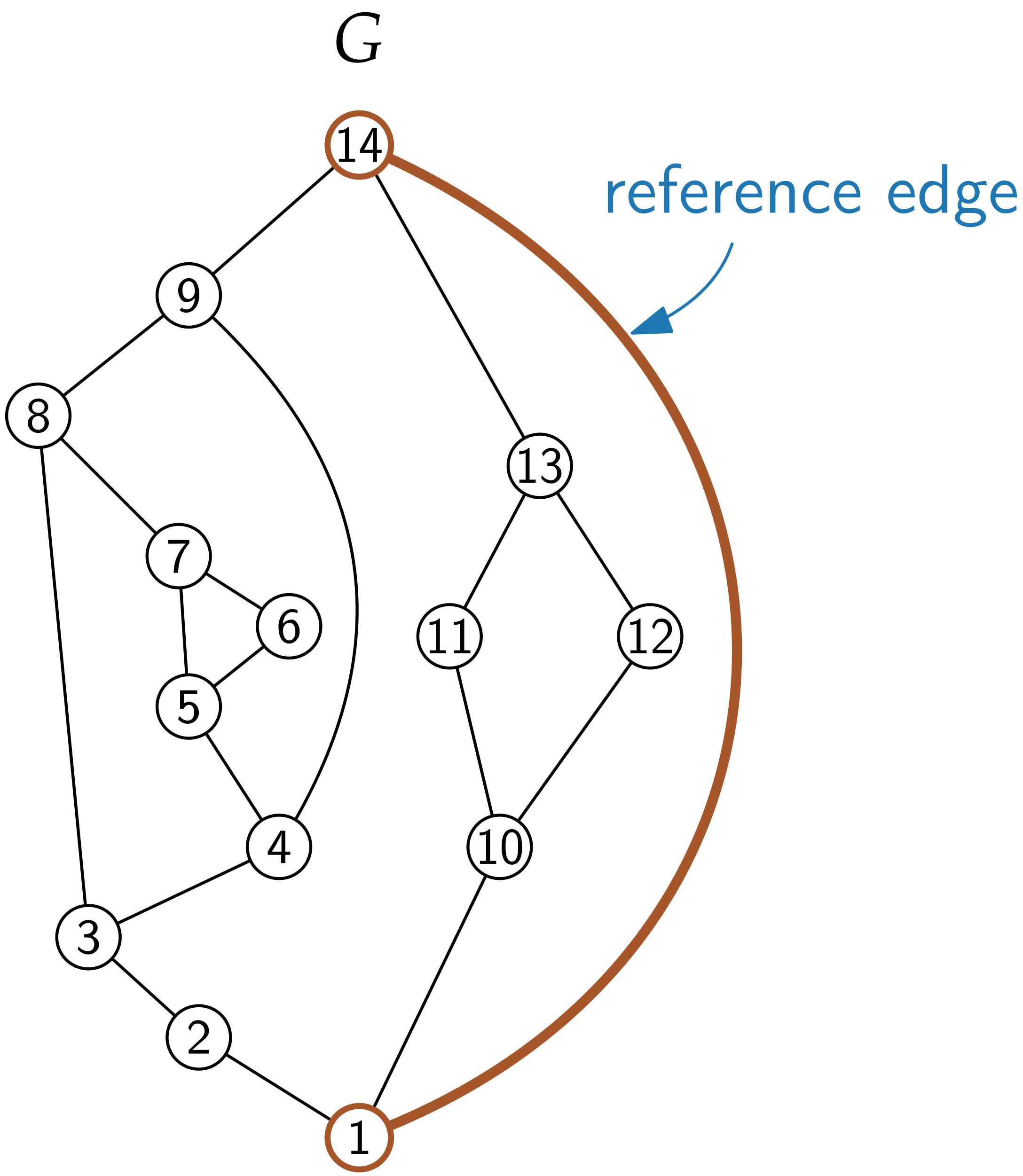

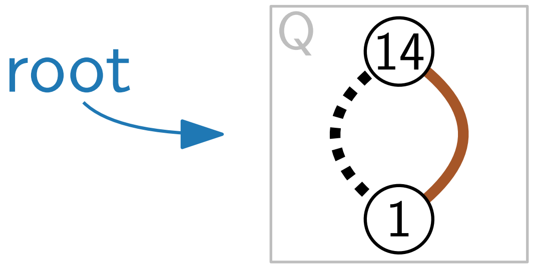

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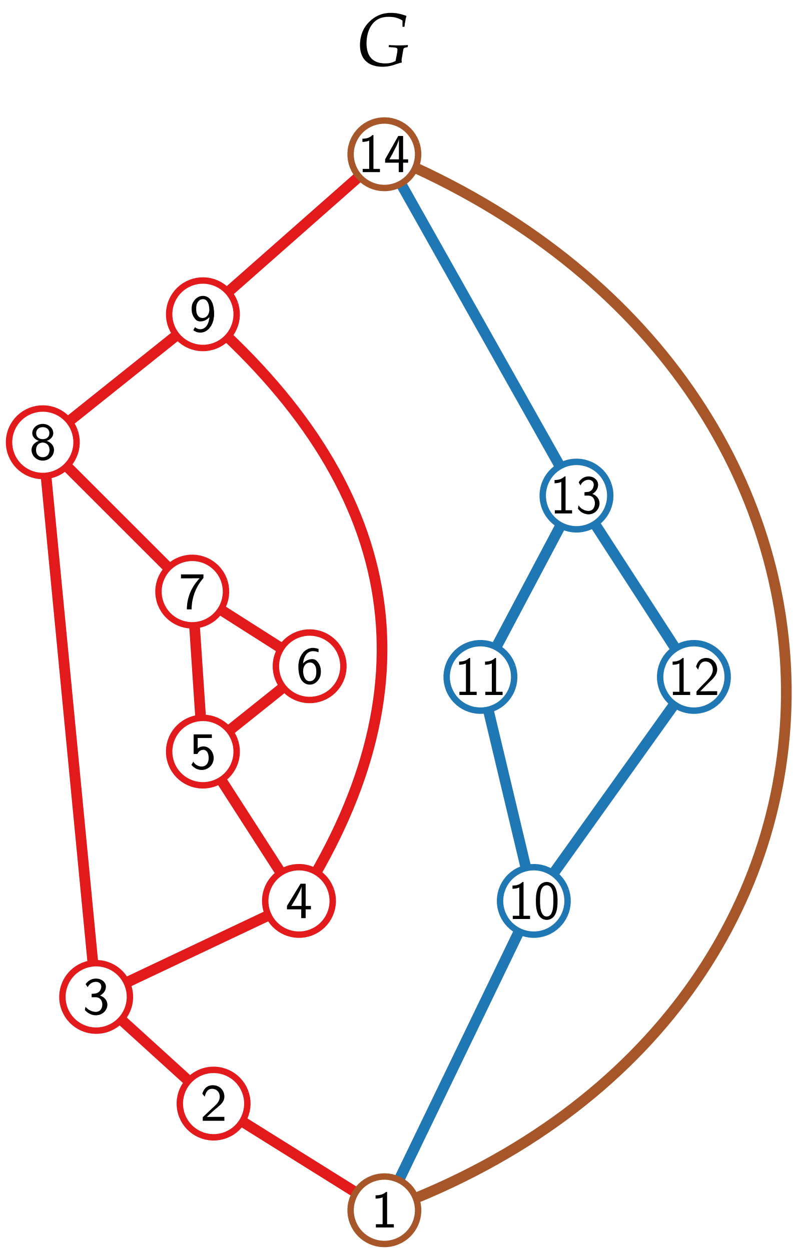

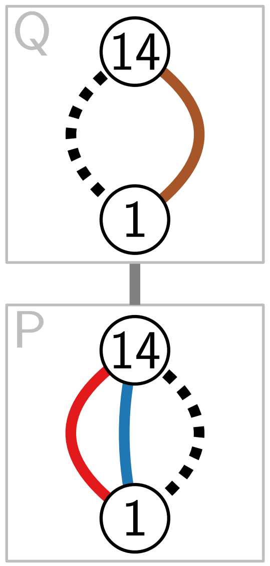

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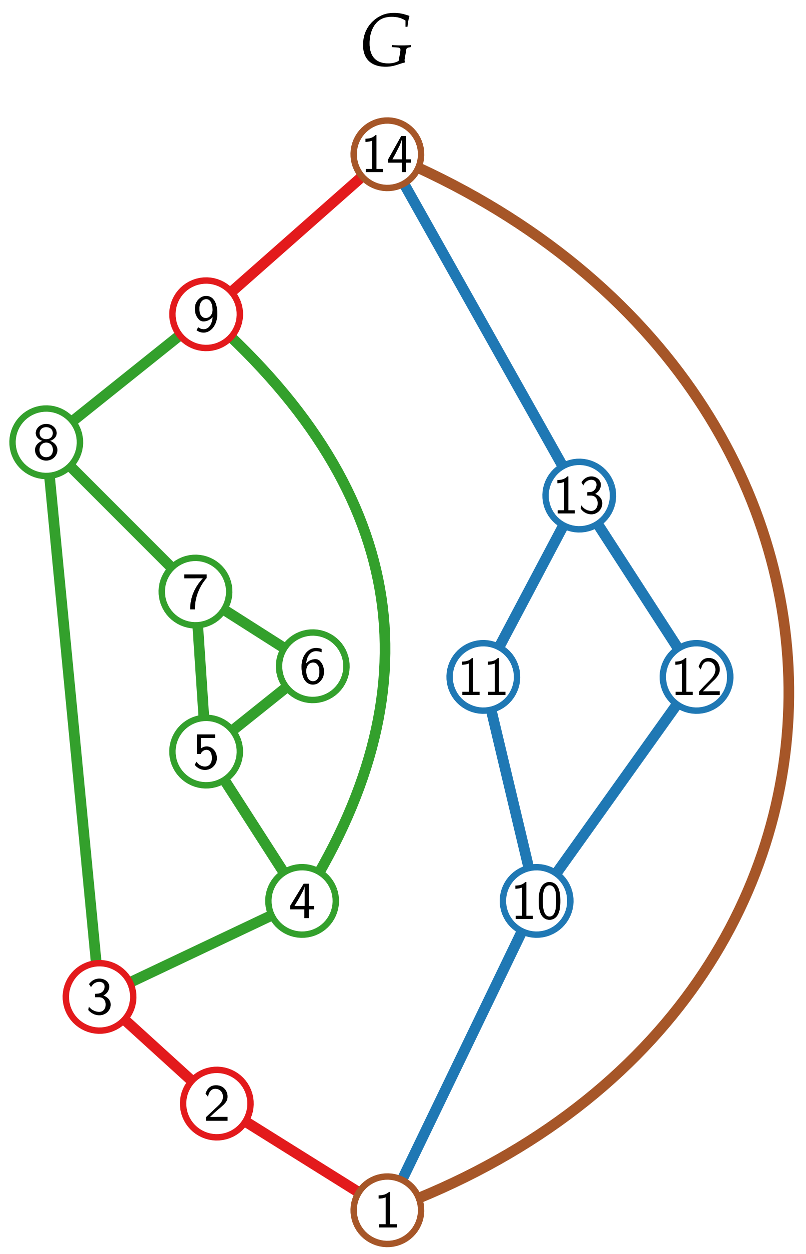

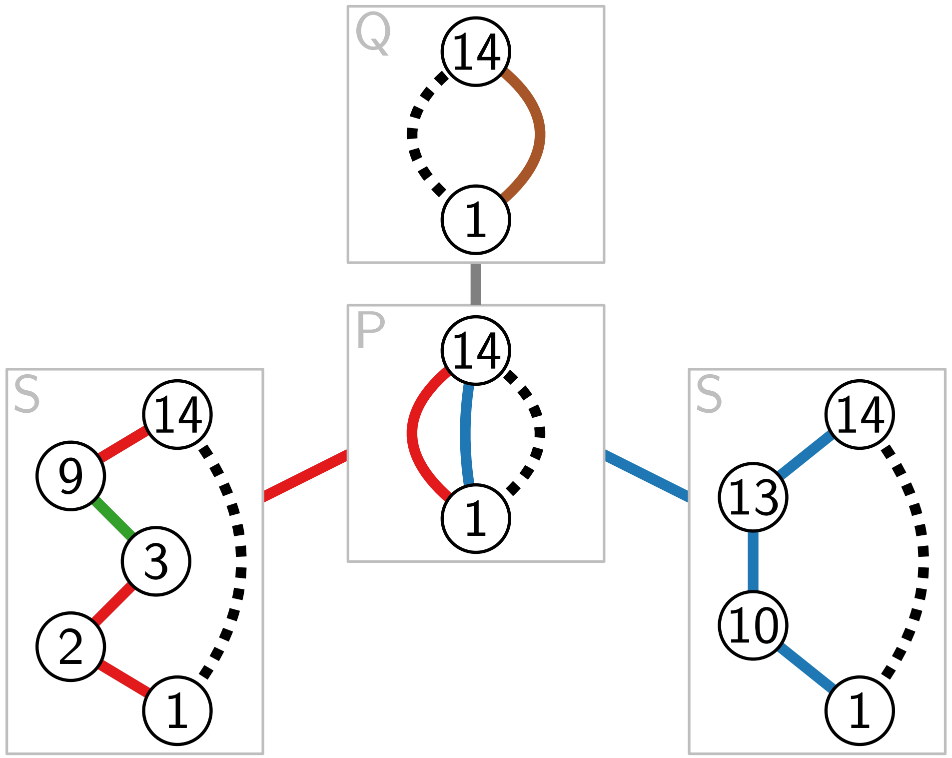

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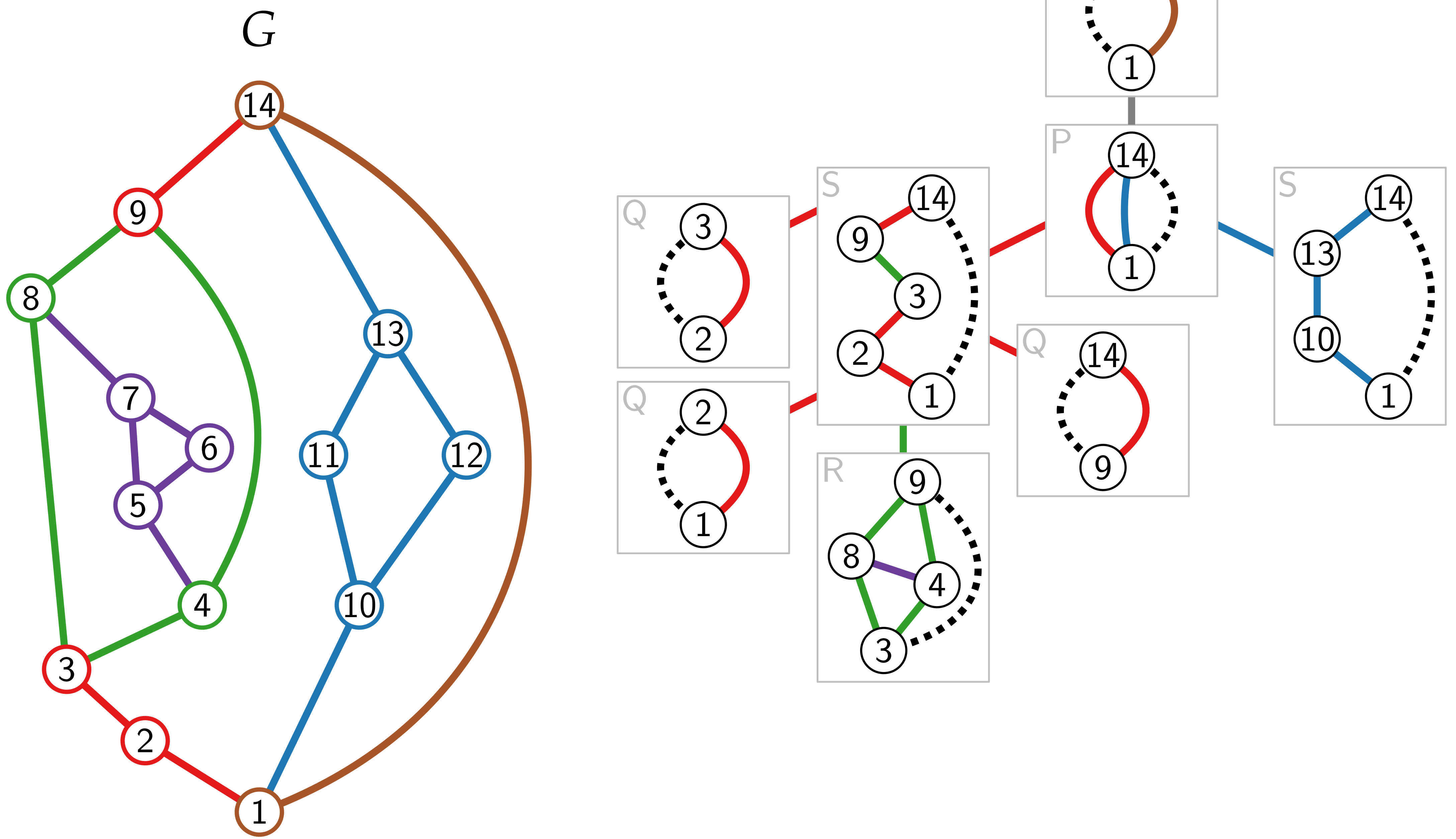

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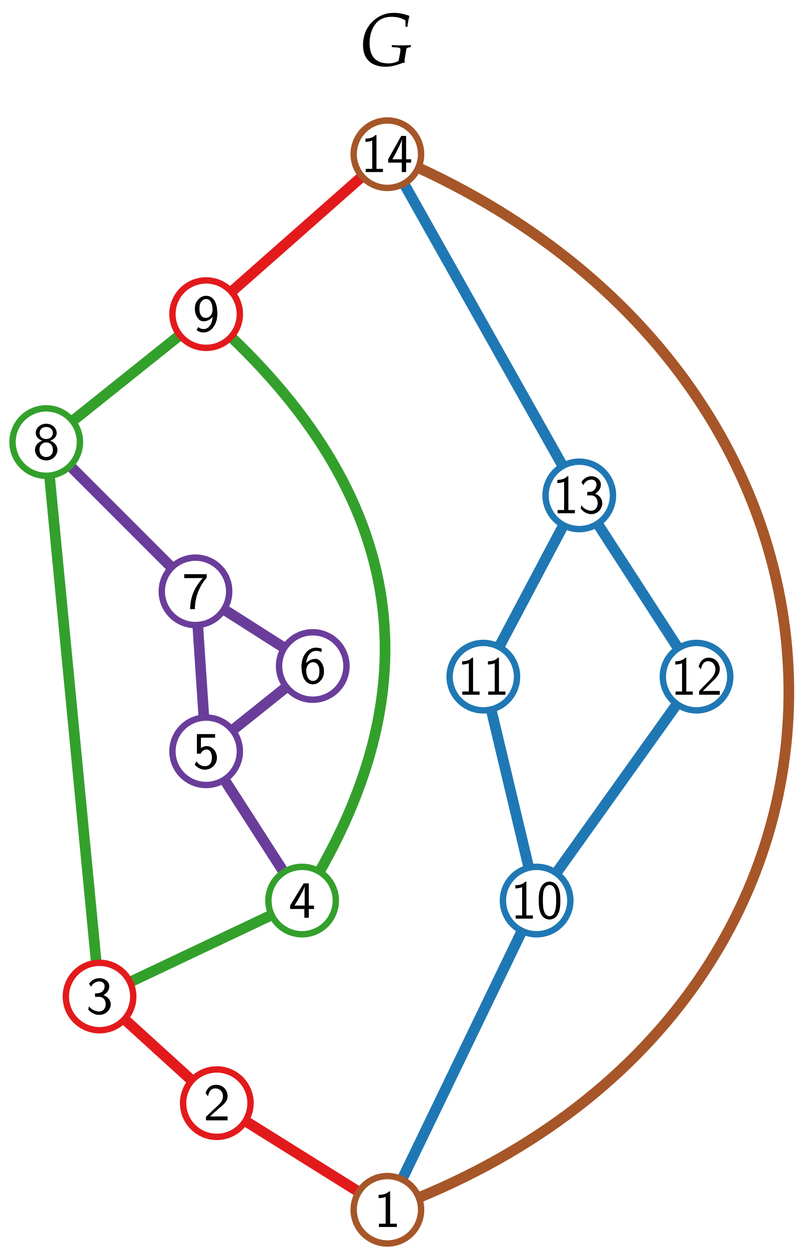

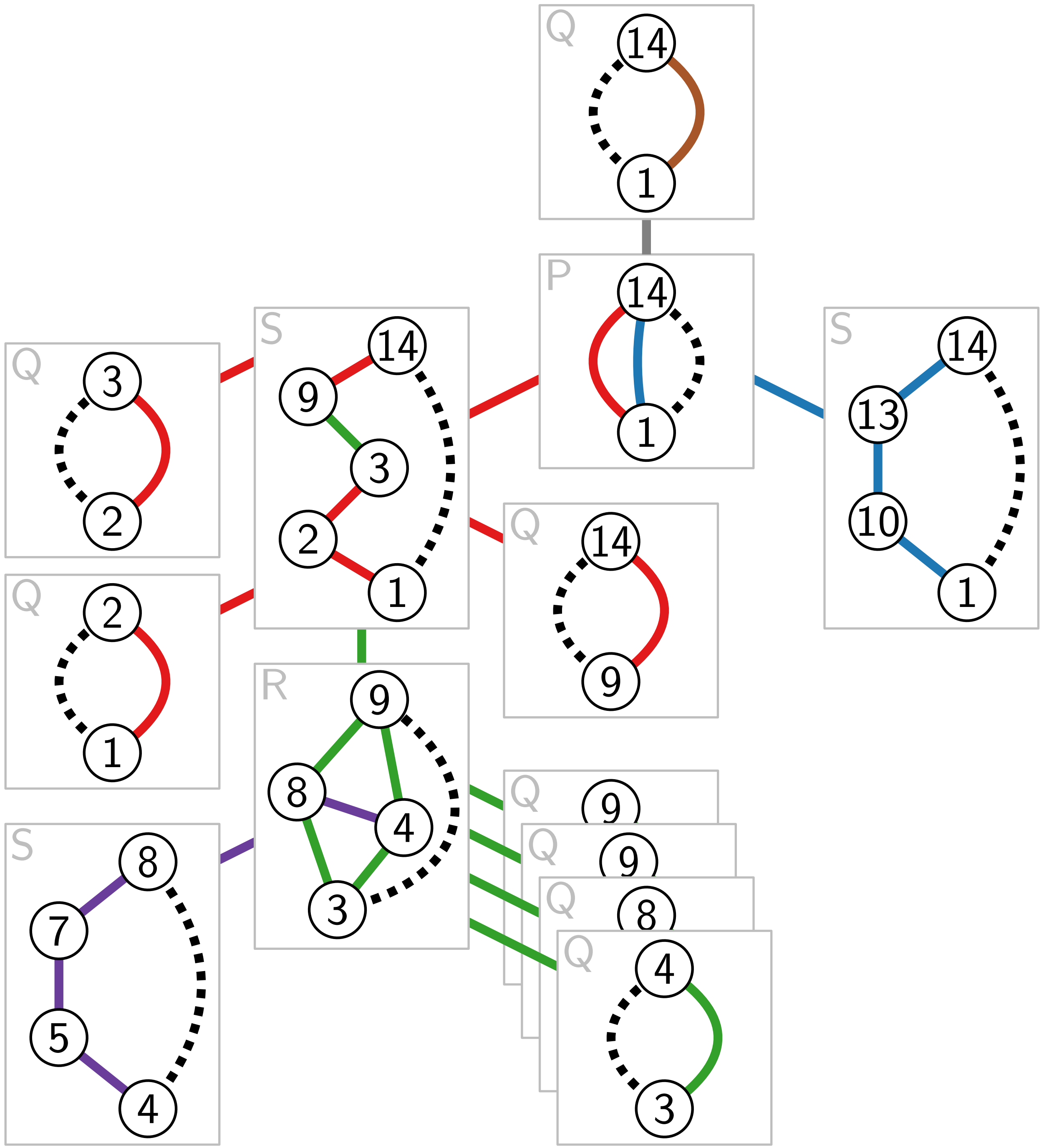

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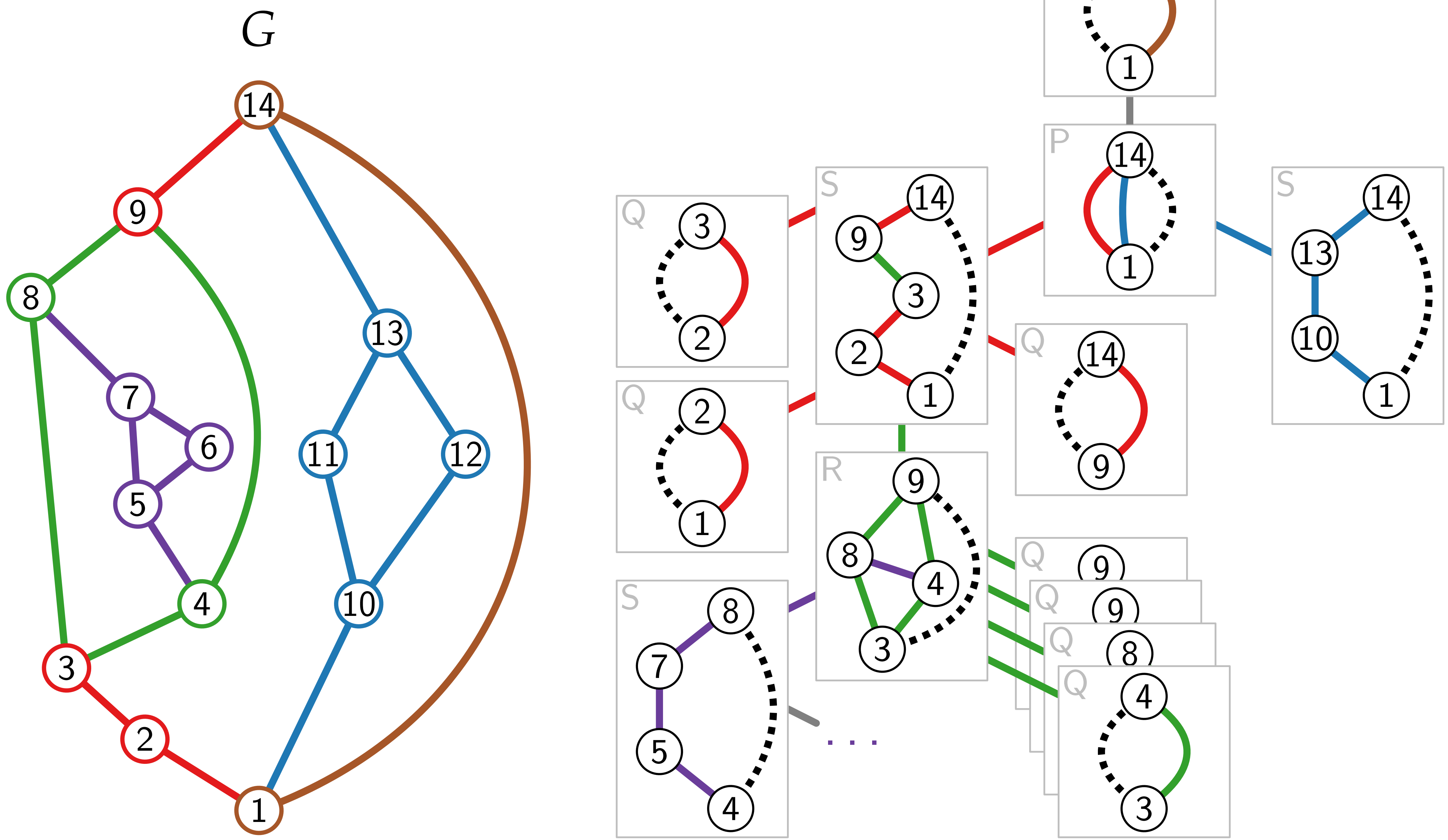

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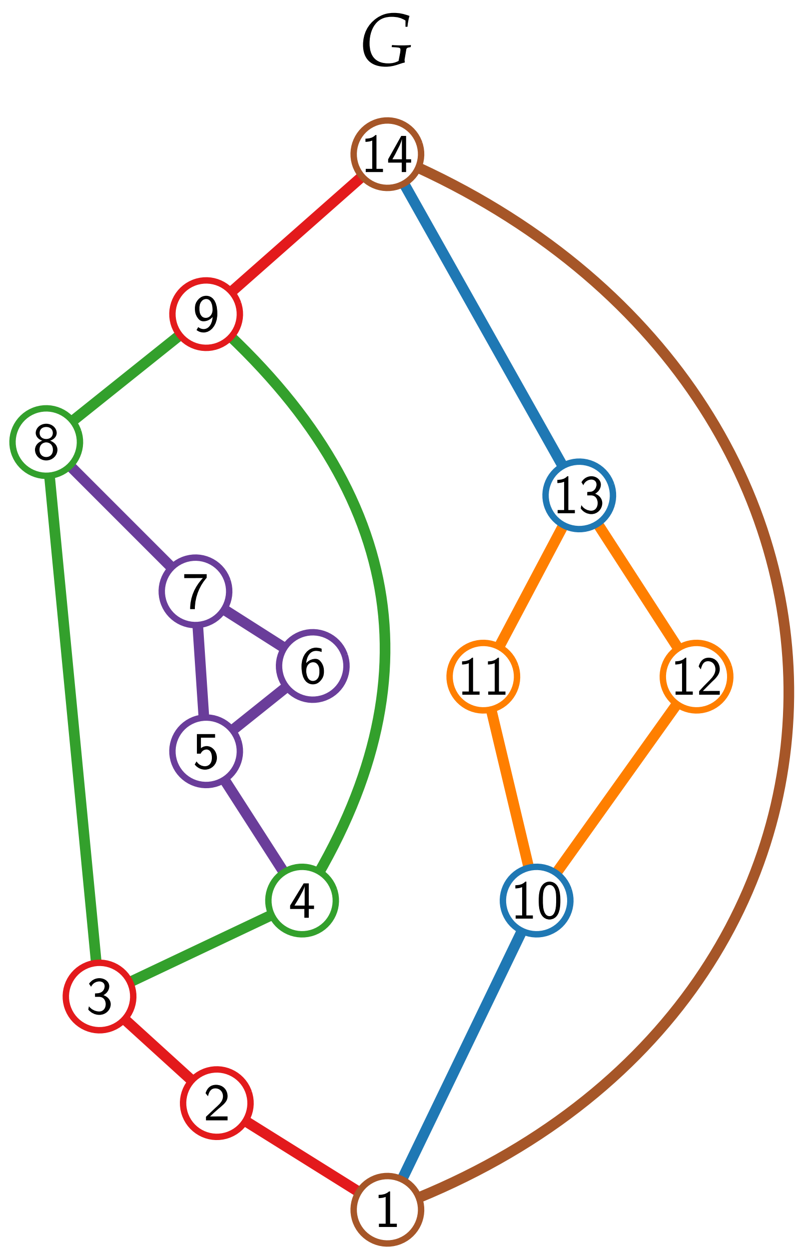

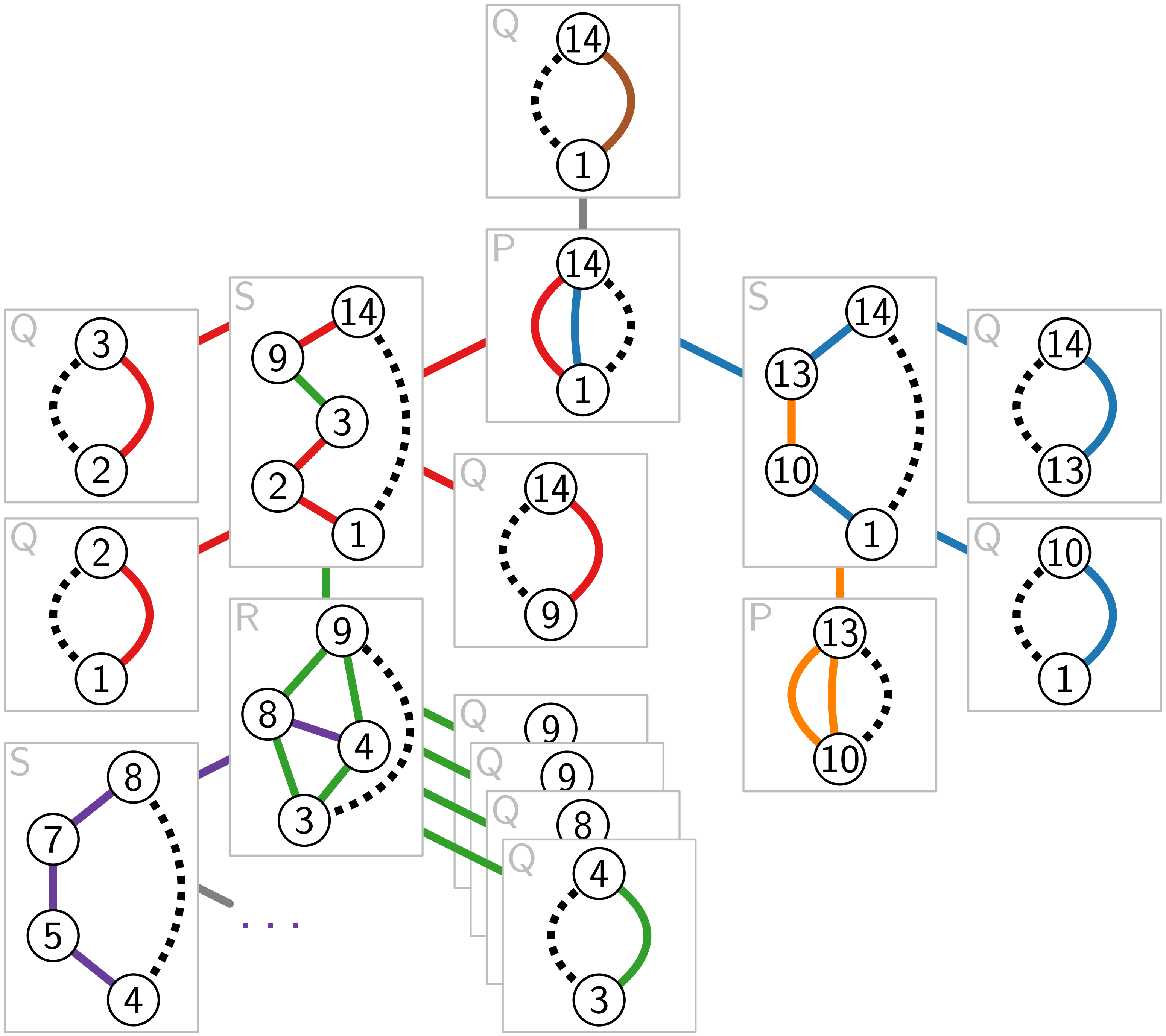

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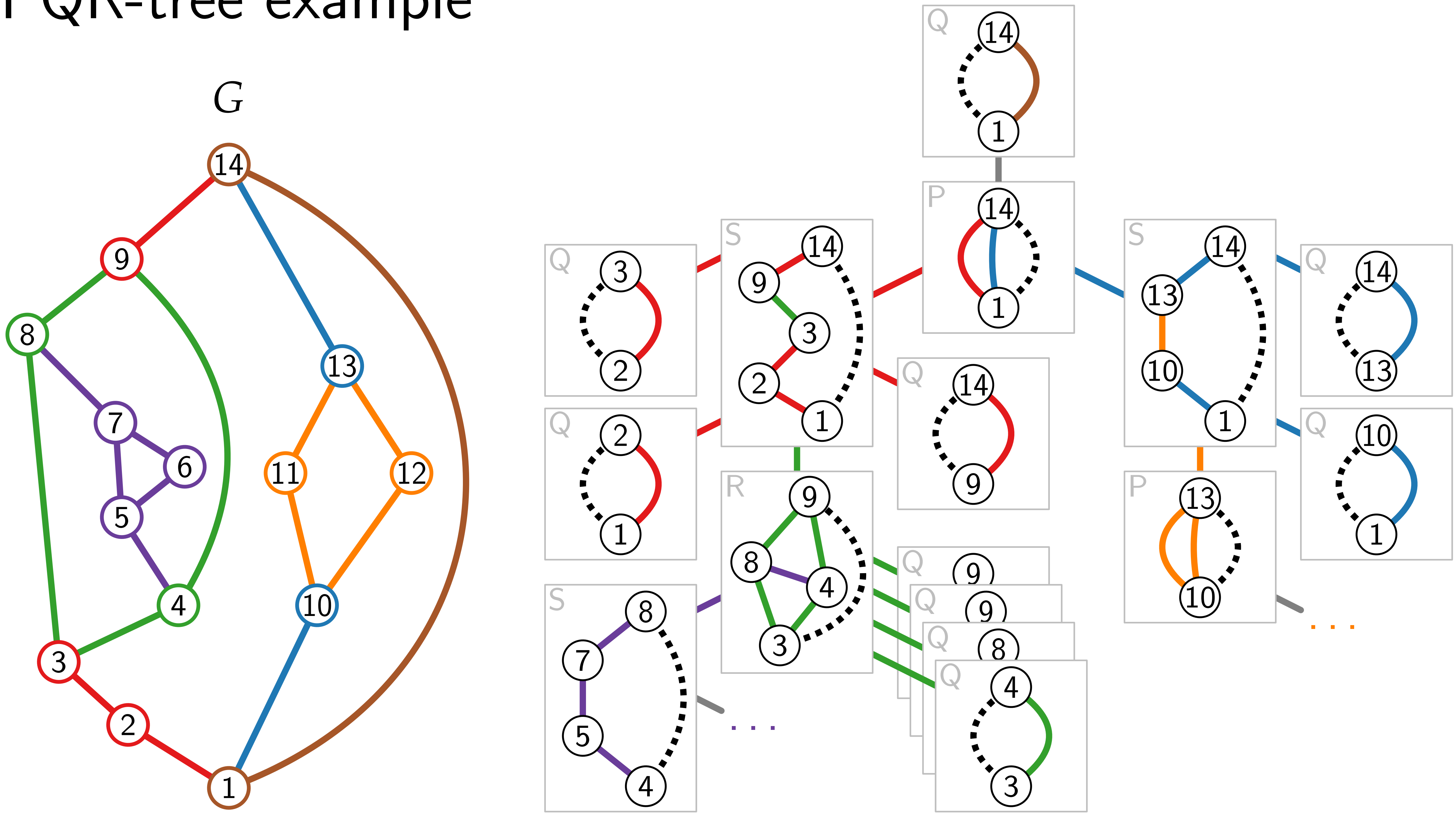
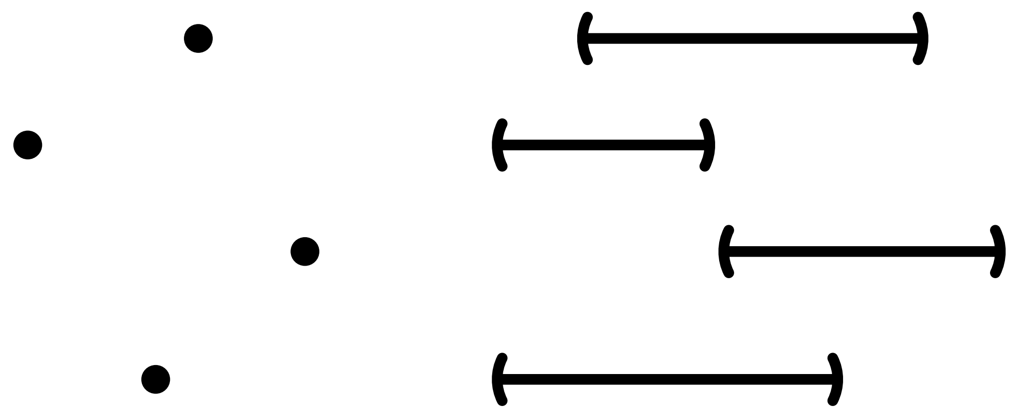


Bar visibility representation

- Vertices correspond to horizontal open line segments called **bars**



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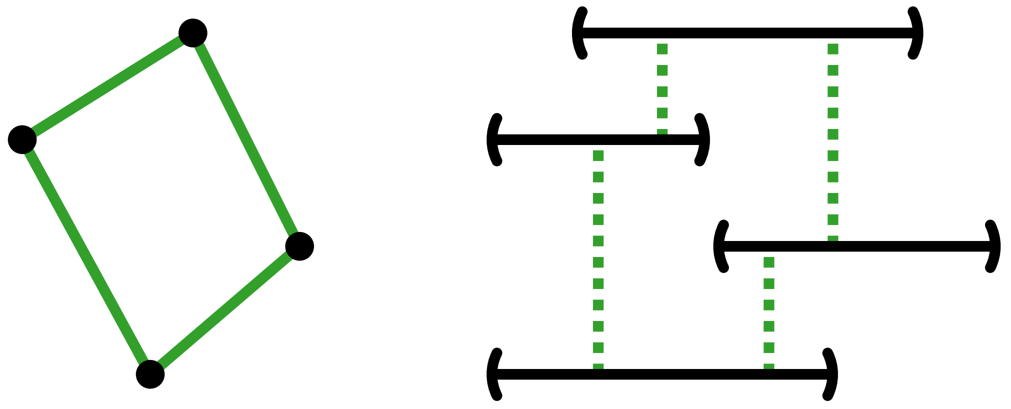

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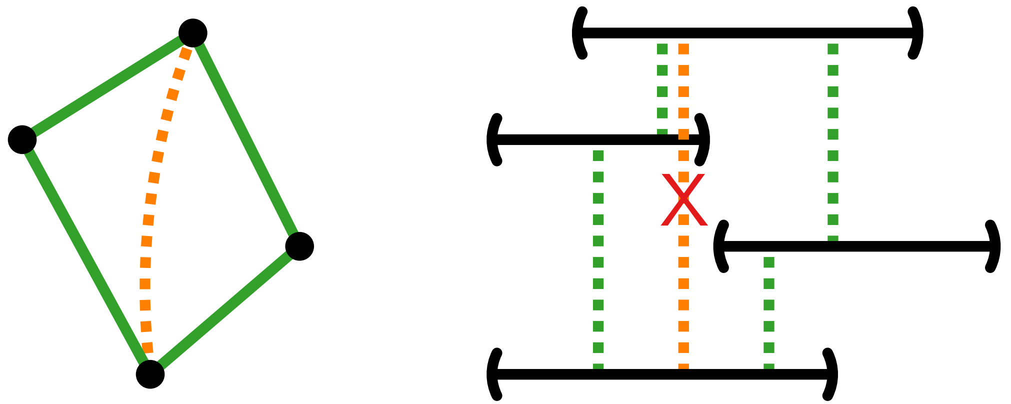

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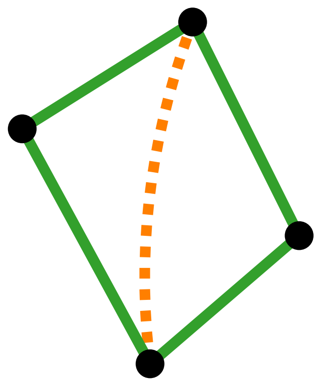

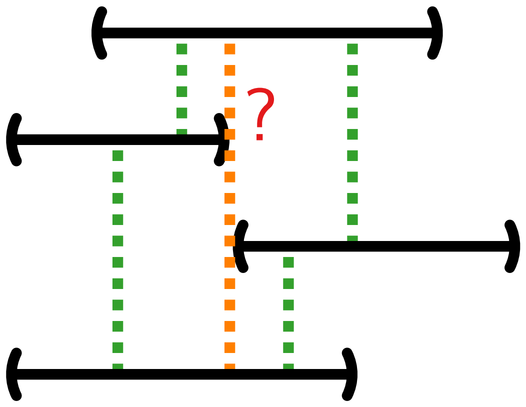

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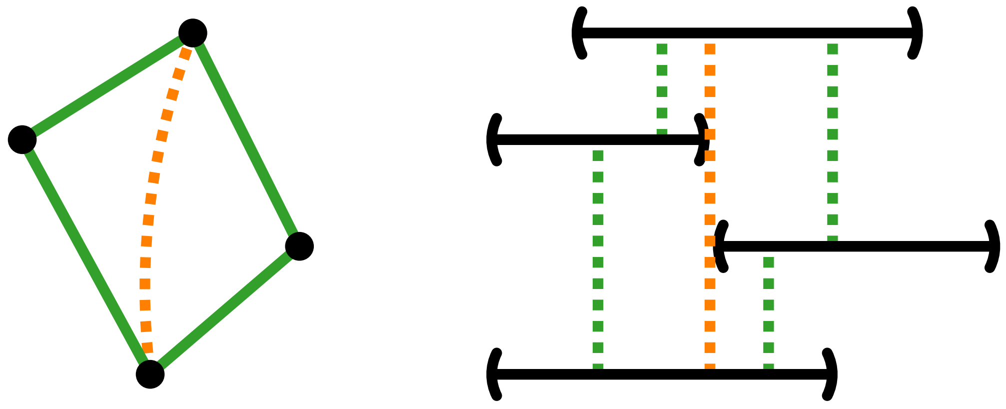

Models.

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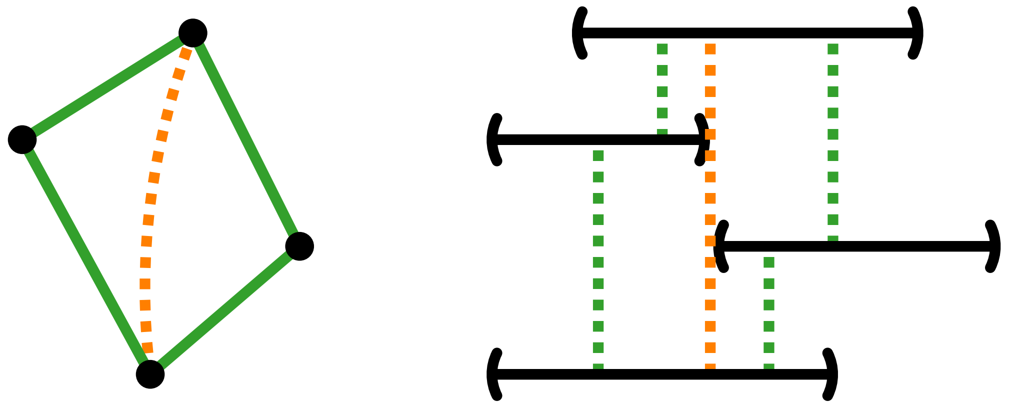

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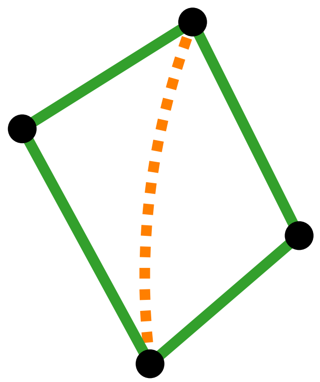

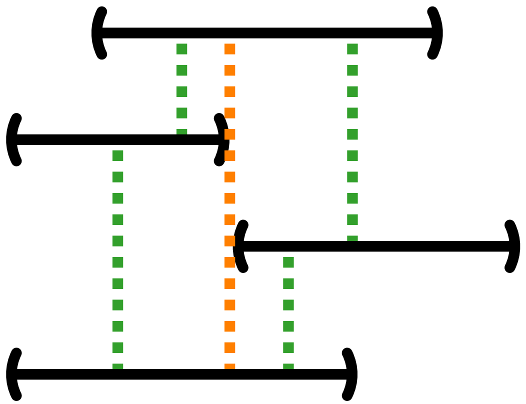

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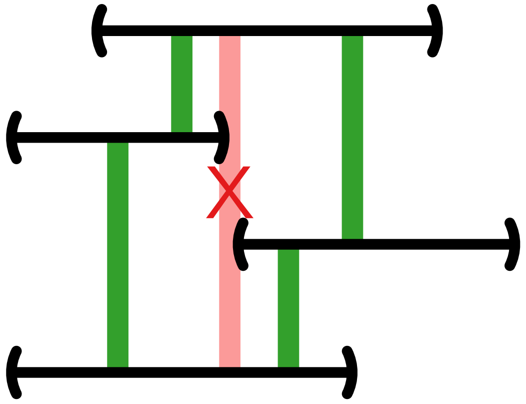

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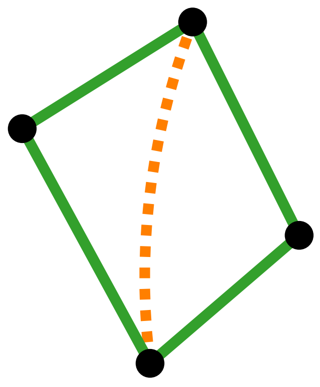

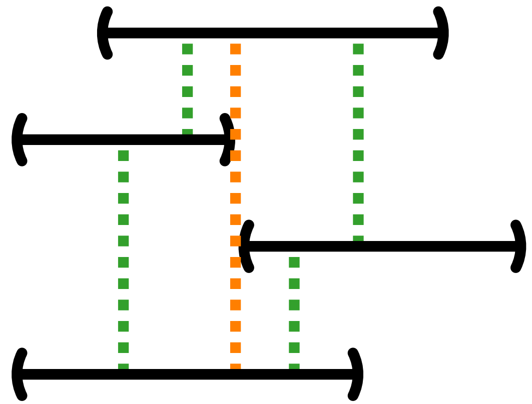

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- **Strong:** Edge $uv \Leftrightarrow$ unobstructed **0-width** vertical sightlines
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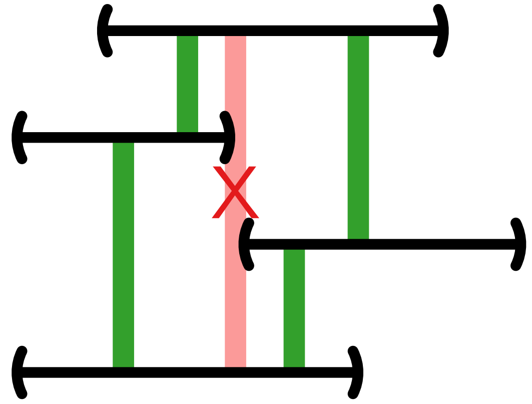
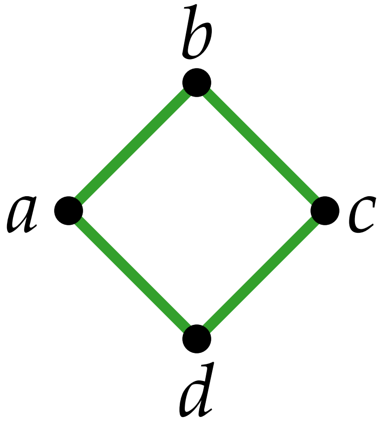
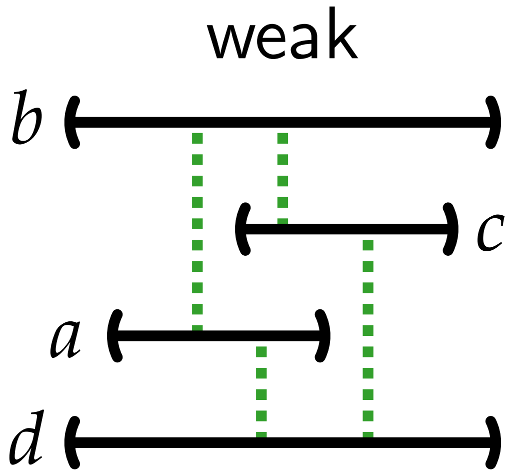
- **Weak:** Edge $uv \Rightarrow$ unobstructed vertical sightlines exists, i. e., any subset of *visible* pairs



Problems





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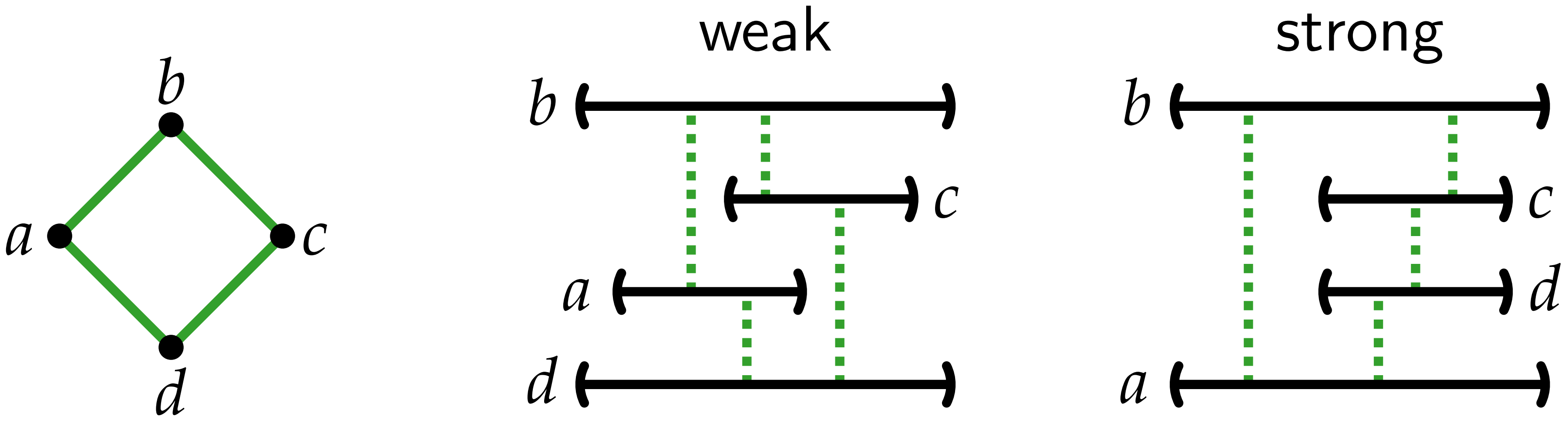

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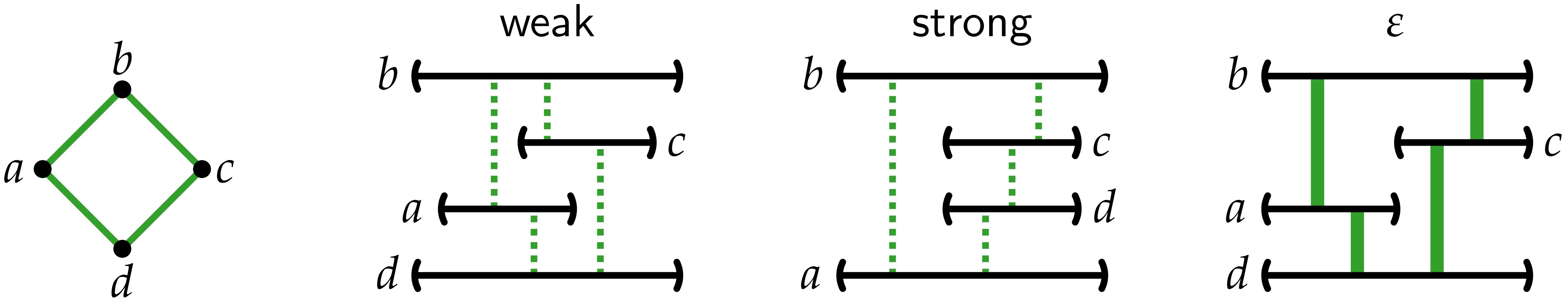

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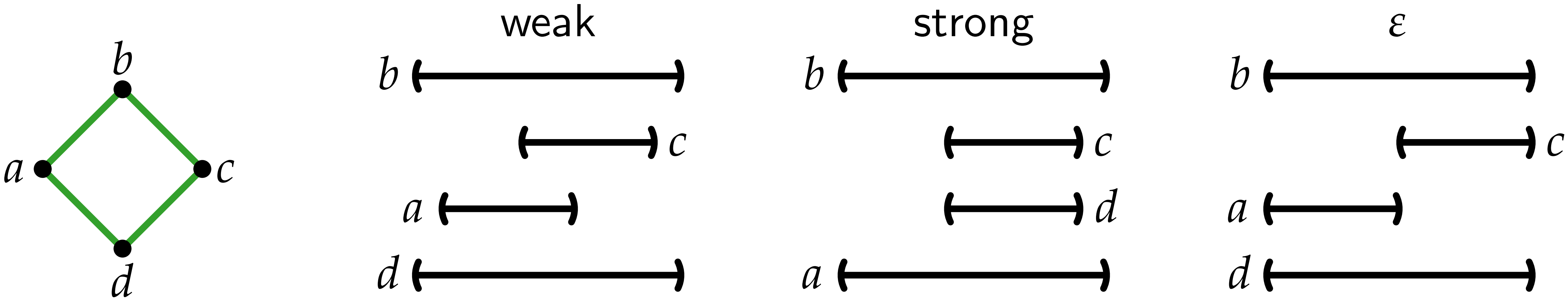

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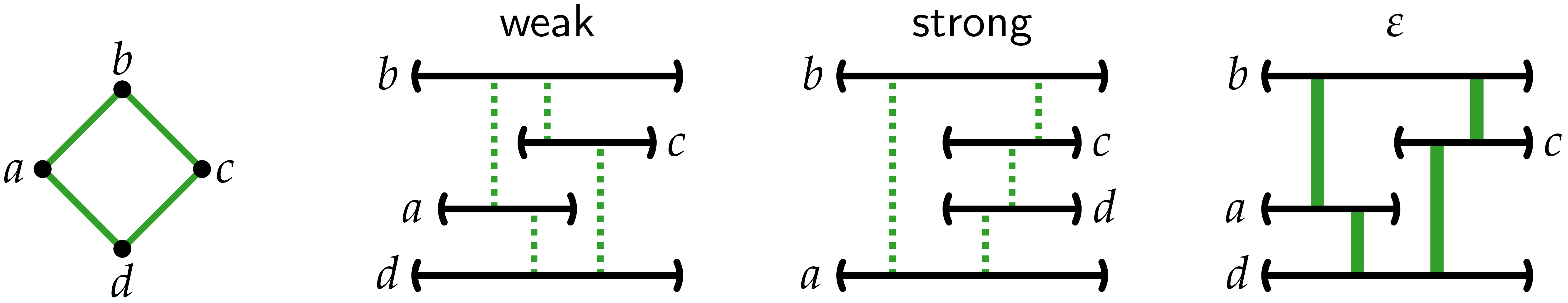

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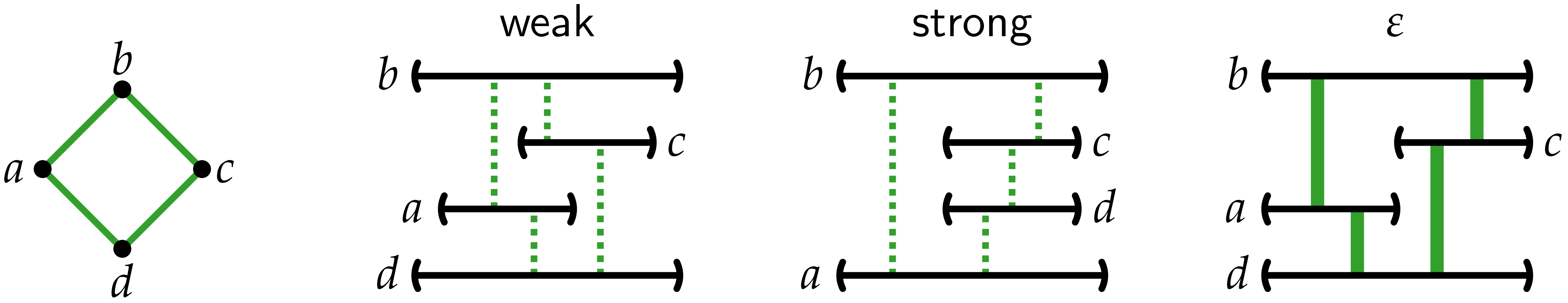

Recognition problem.

Given a graph G, **decide** if there exists a weak/strong/ε bar visibility representation ψ of G.

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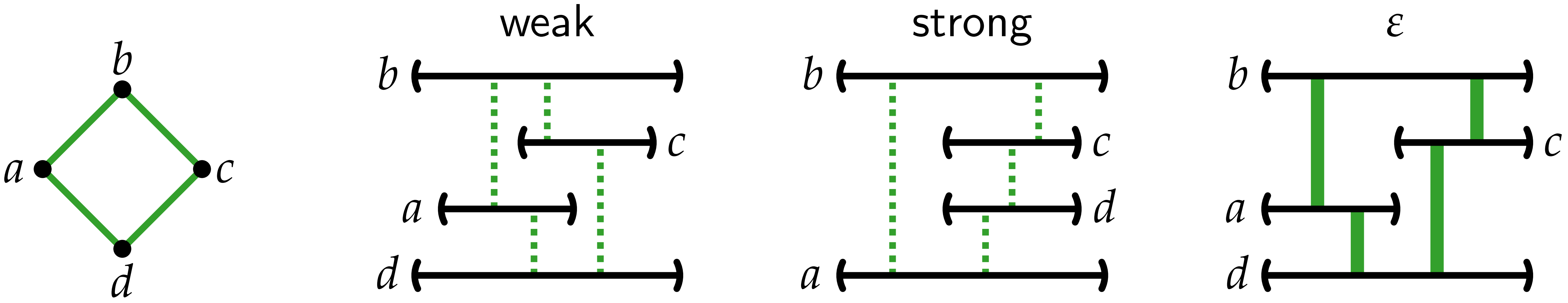

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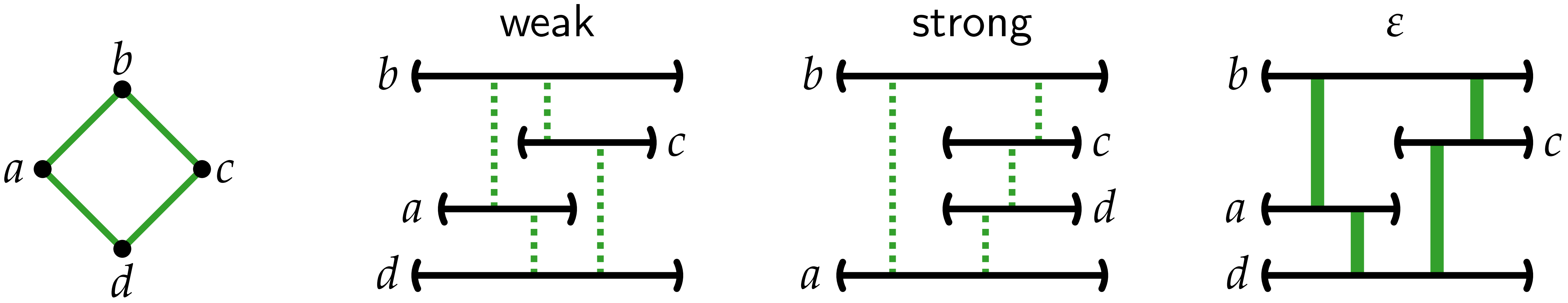

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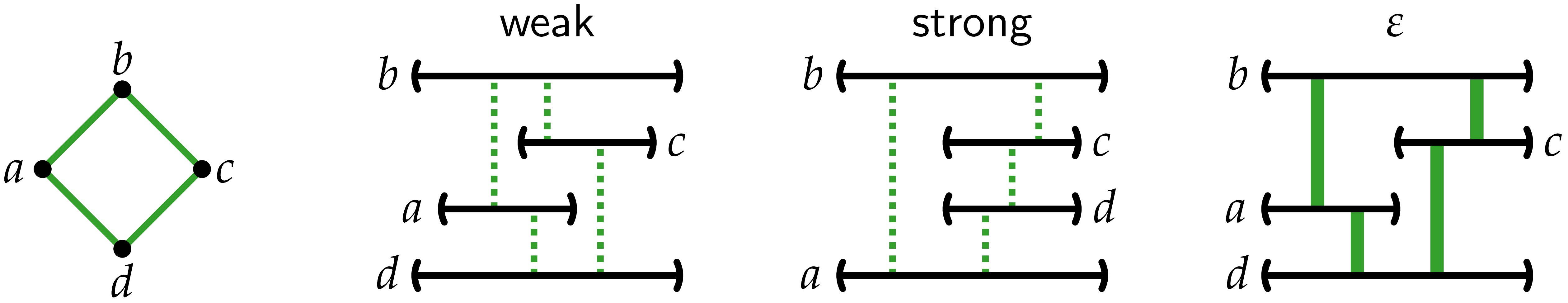
Partial Representation Extension (& Construction) problem.
Given a graph G and a **set of bars ψ' of $V' \subset V(G)$**, **decide** if there exists a weak/strong/ε bar visibility representation ψ of G **where $\psi|_{V'} = \psi'$** (and **construct** ψ when it exists).

Background



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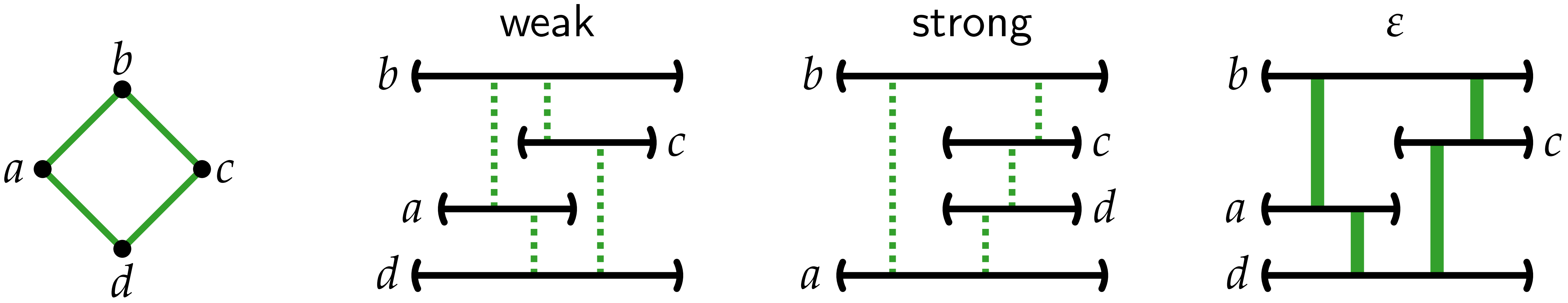

Weak Bar Visibility.

- All planar graphs. [Tamassia & Tollis 1986; Wismath 1985]
- Linear time recognition and construction [T&T '86]
- Representation Extension is NP-complete [Chaplick et al. '14]

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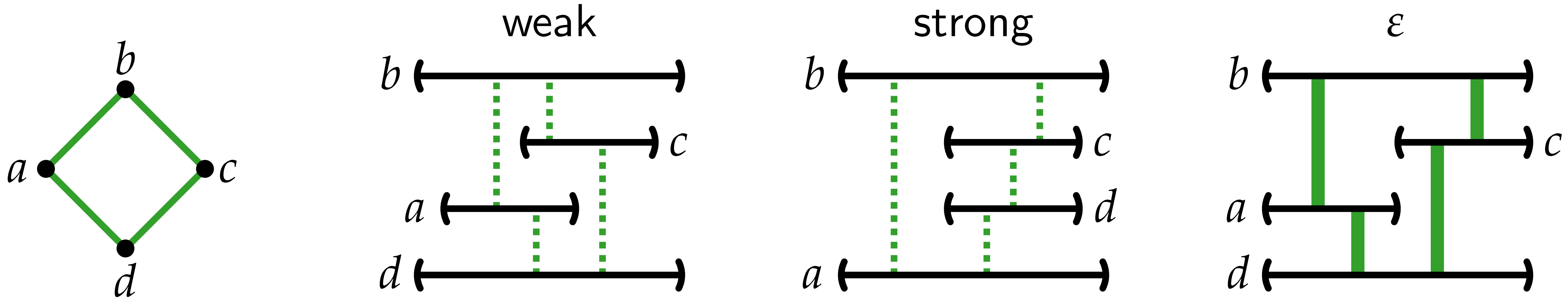

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Strong Bar Visibility.

- NP-complete to recognize [Andreae '92]

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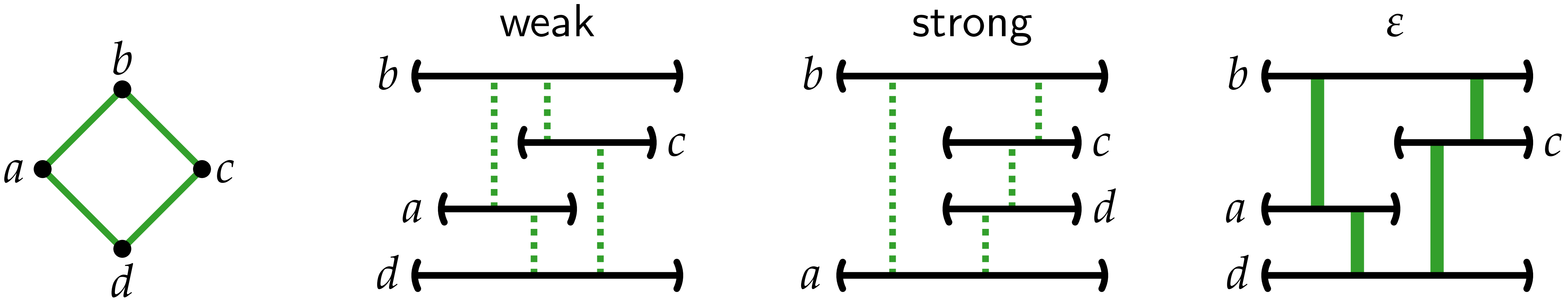

ε-Bar Visibility.

- Planar graphs that can be embedded with all **cut vertices** on the outerface. [T&T 1986, Wismath '85]
- Linear time recognition and construction [T&T '86]
- What about Representation Extension?

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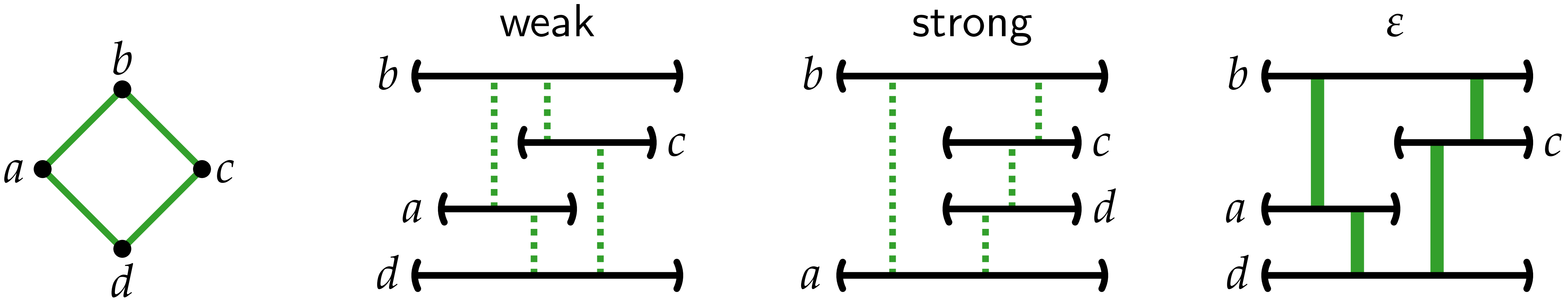

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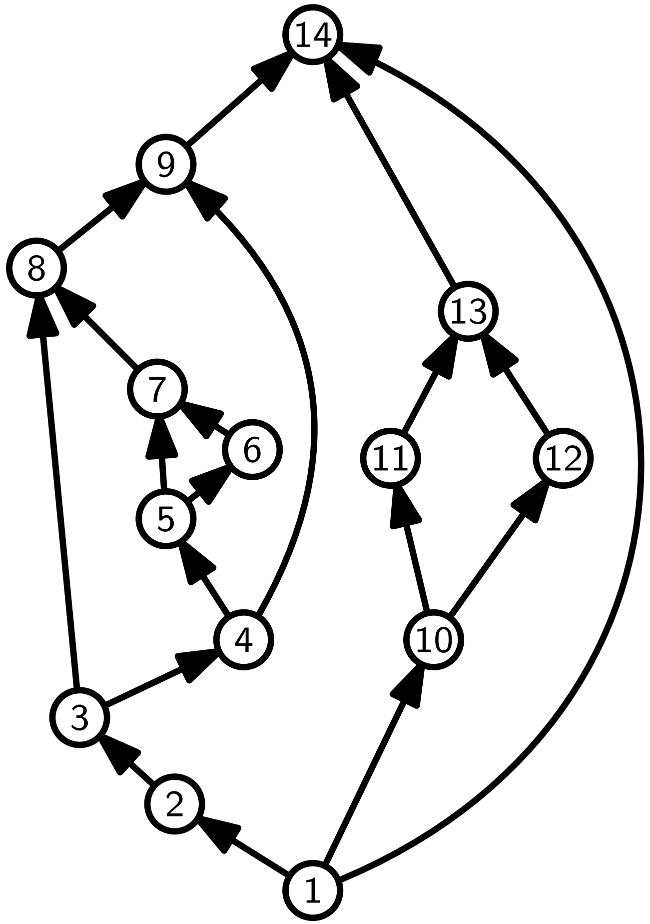
Let's see!

ε-bar visiblity and st-graphs

Recall that an **st-graph** is a planar digraph G with exactly one soure s and one sink t where s and t occur on the outer face of an embedding of G.



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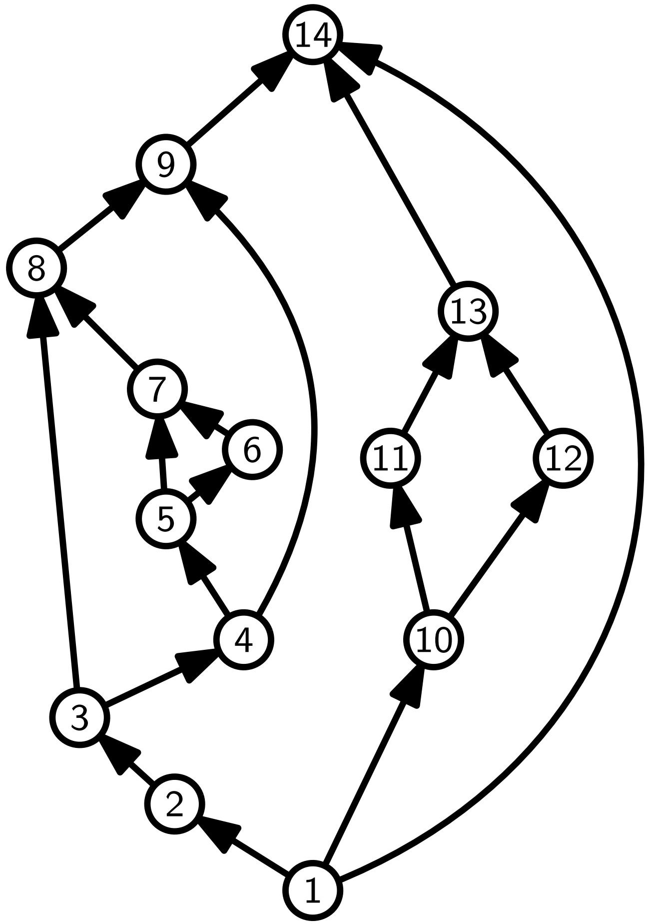
Observation.
st-orientations correspond to ε-bar visibility representations.



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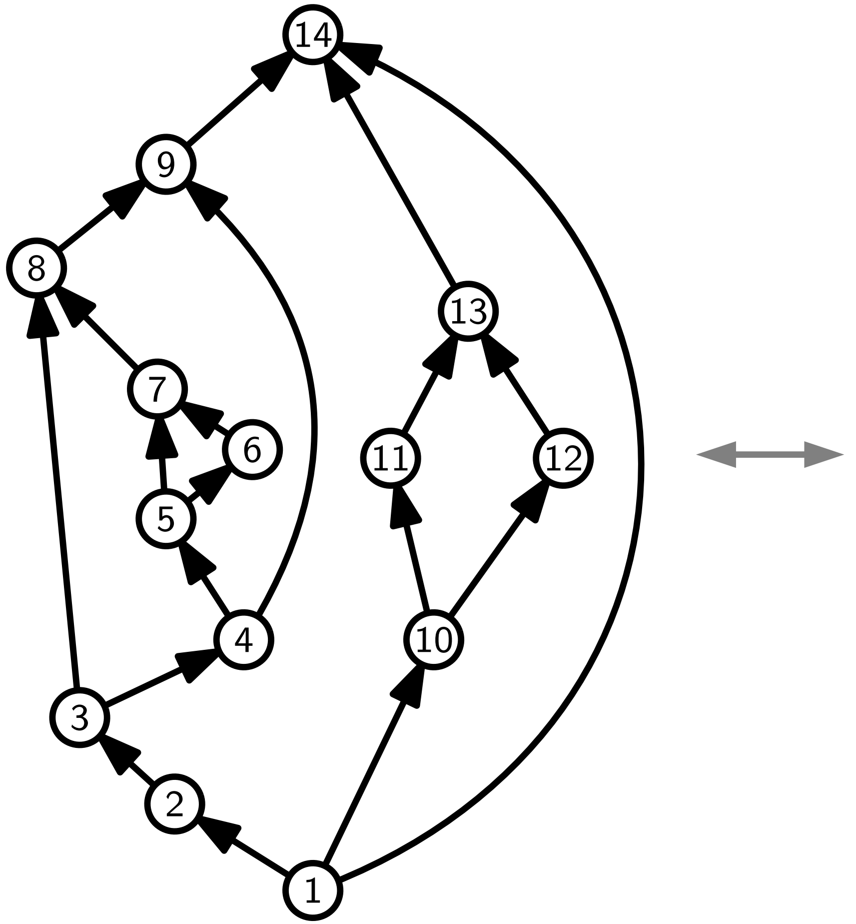

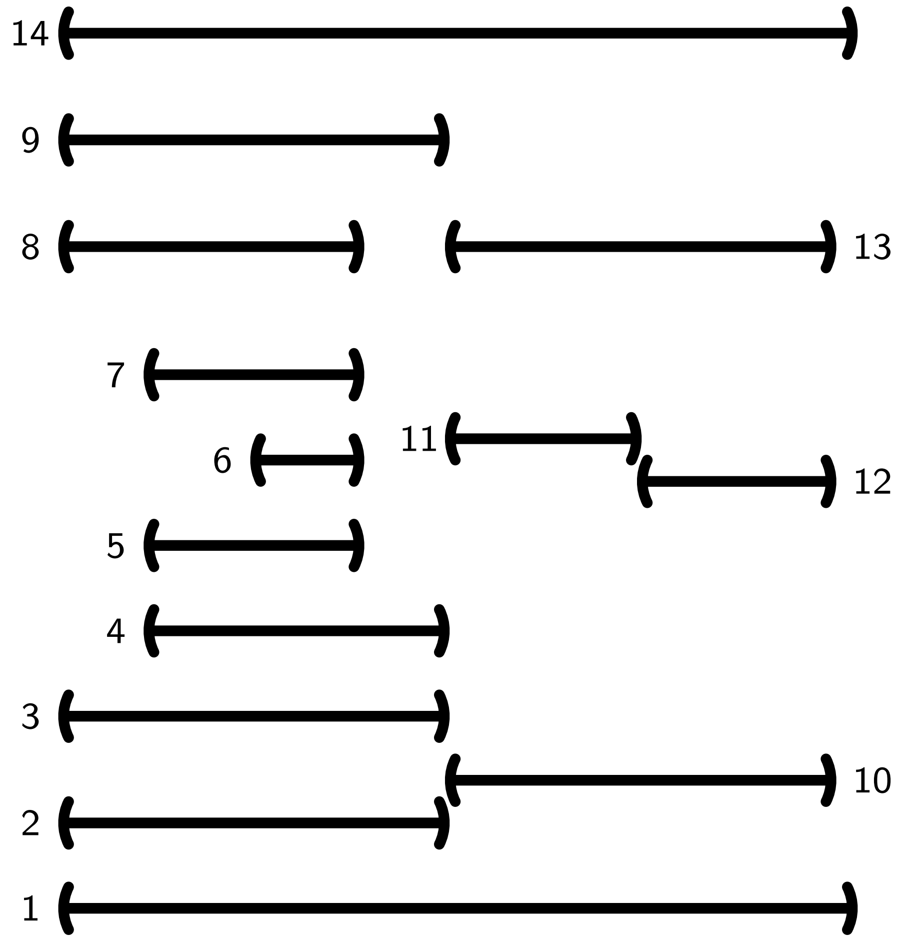

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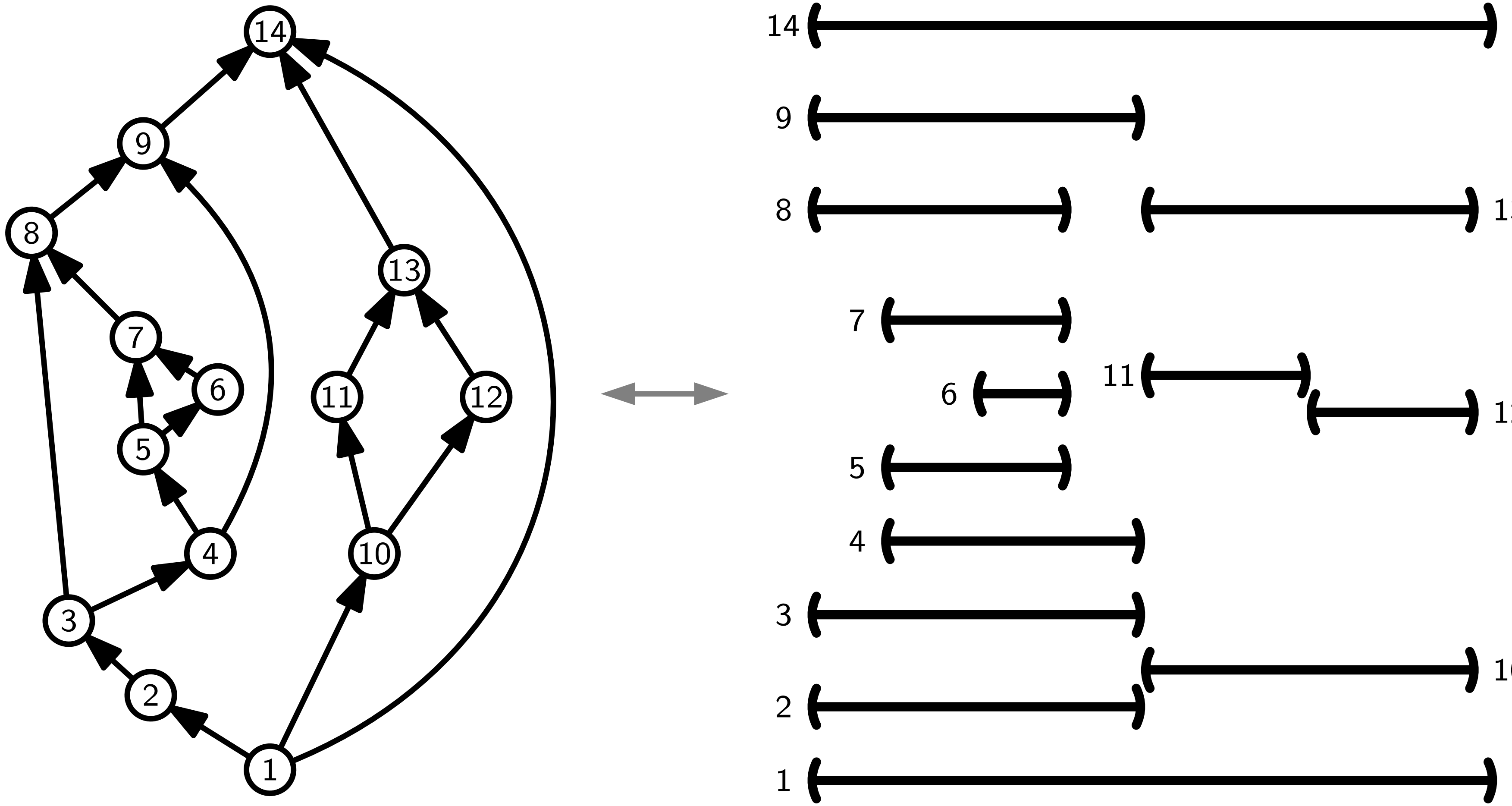
Note that testing whether an acyclic planar **di**graph has a weak bar visibility representation is NP-complete.



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- This is upward planarity testing! [Garg & Tamassia '01]

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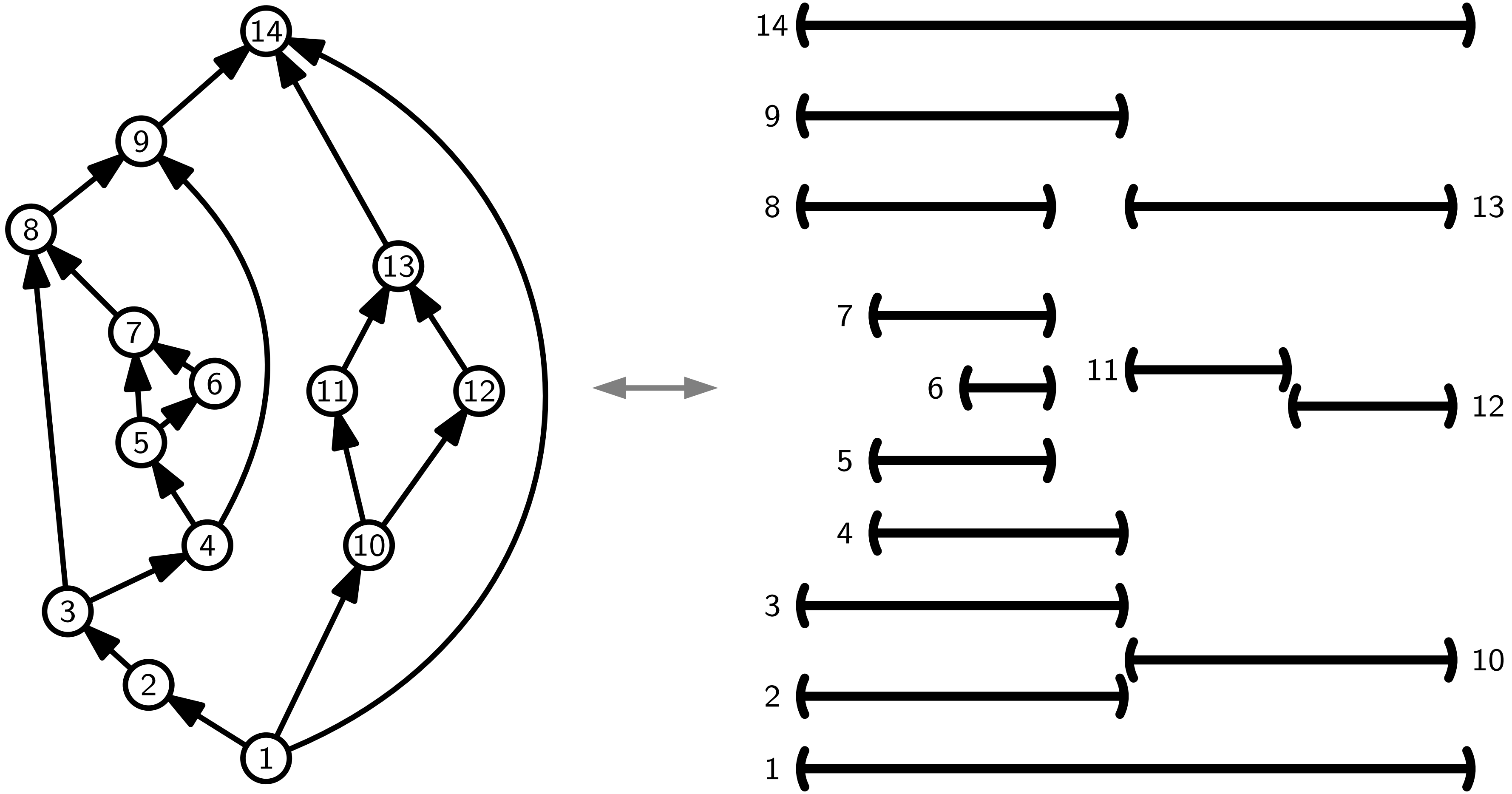

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- ε-bar visability testing is easily done via st-graph recognition.

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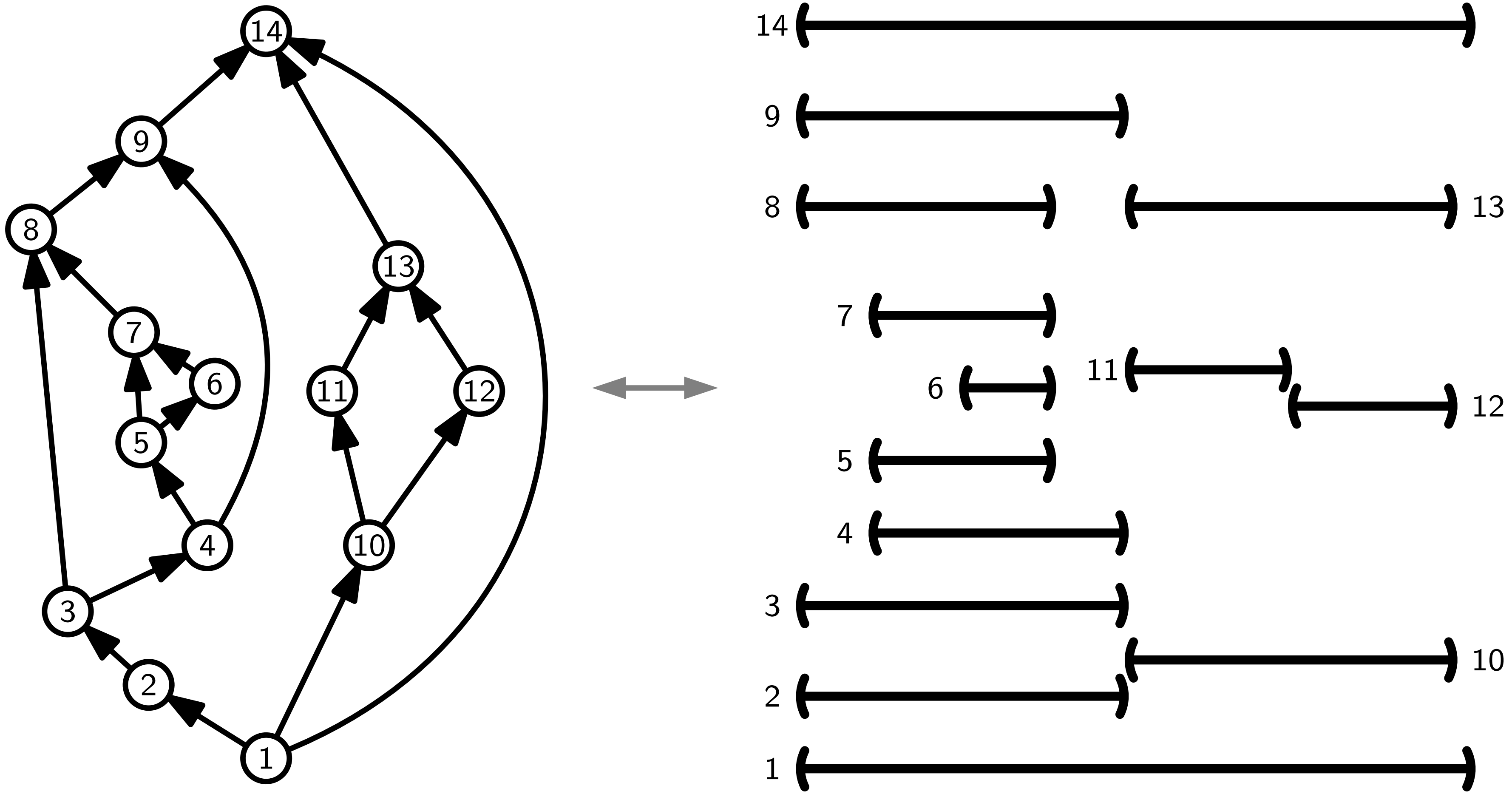

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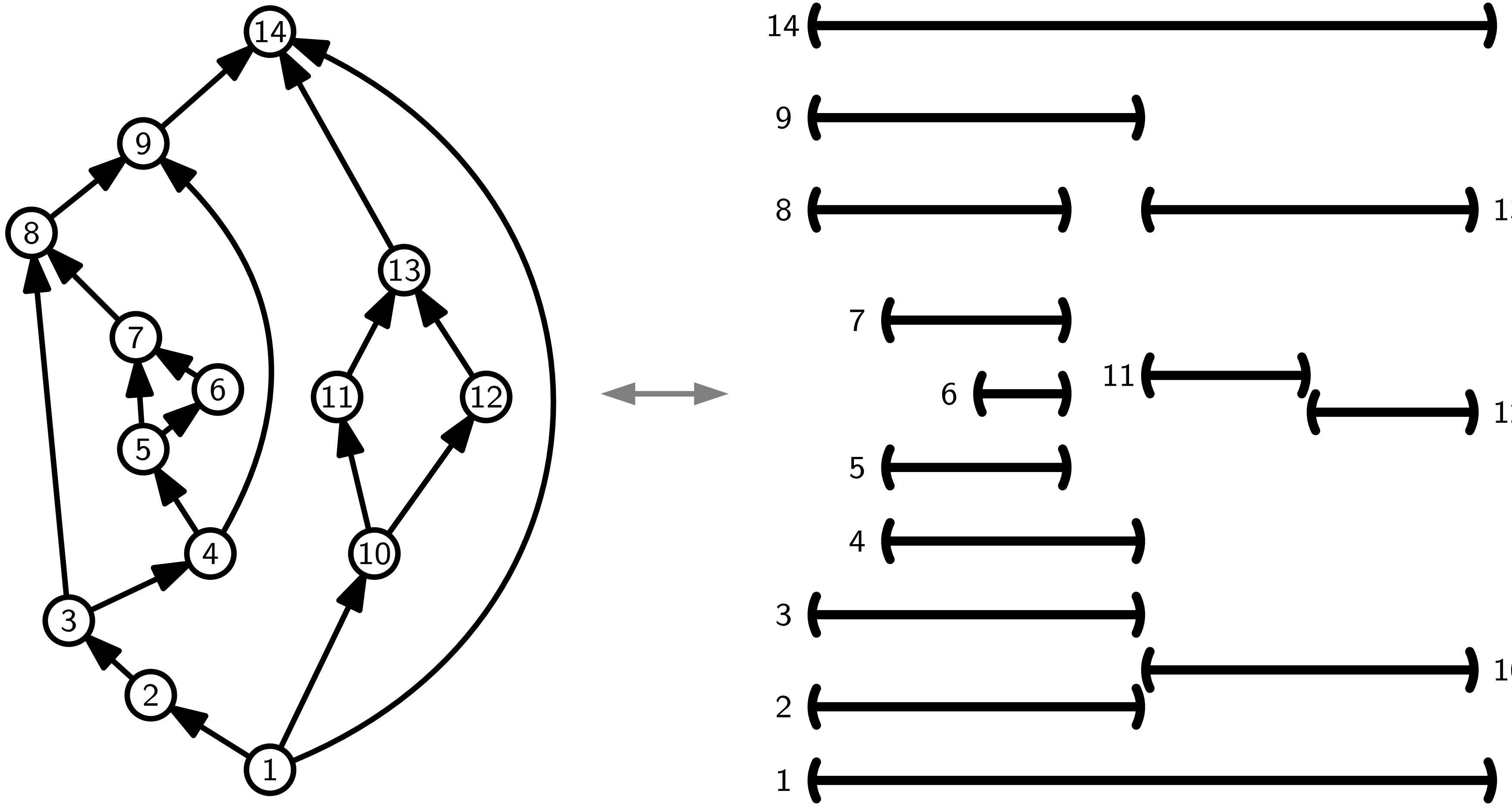
Observation.
st-orientations correspond to ε-bar visibility representations.

- ε-bar visability testing is easily done via st-graph recognition.
- Strong bar visability recognition. . . open?



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- In a **rectangular** bar visability representation $\psi(s)$ and $\psi(t)$ span an enclosing rectangle.

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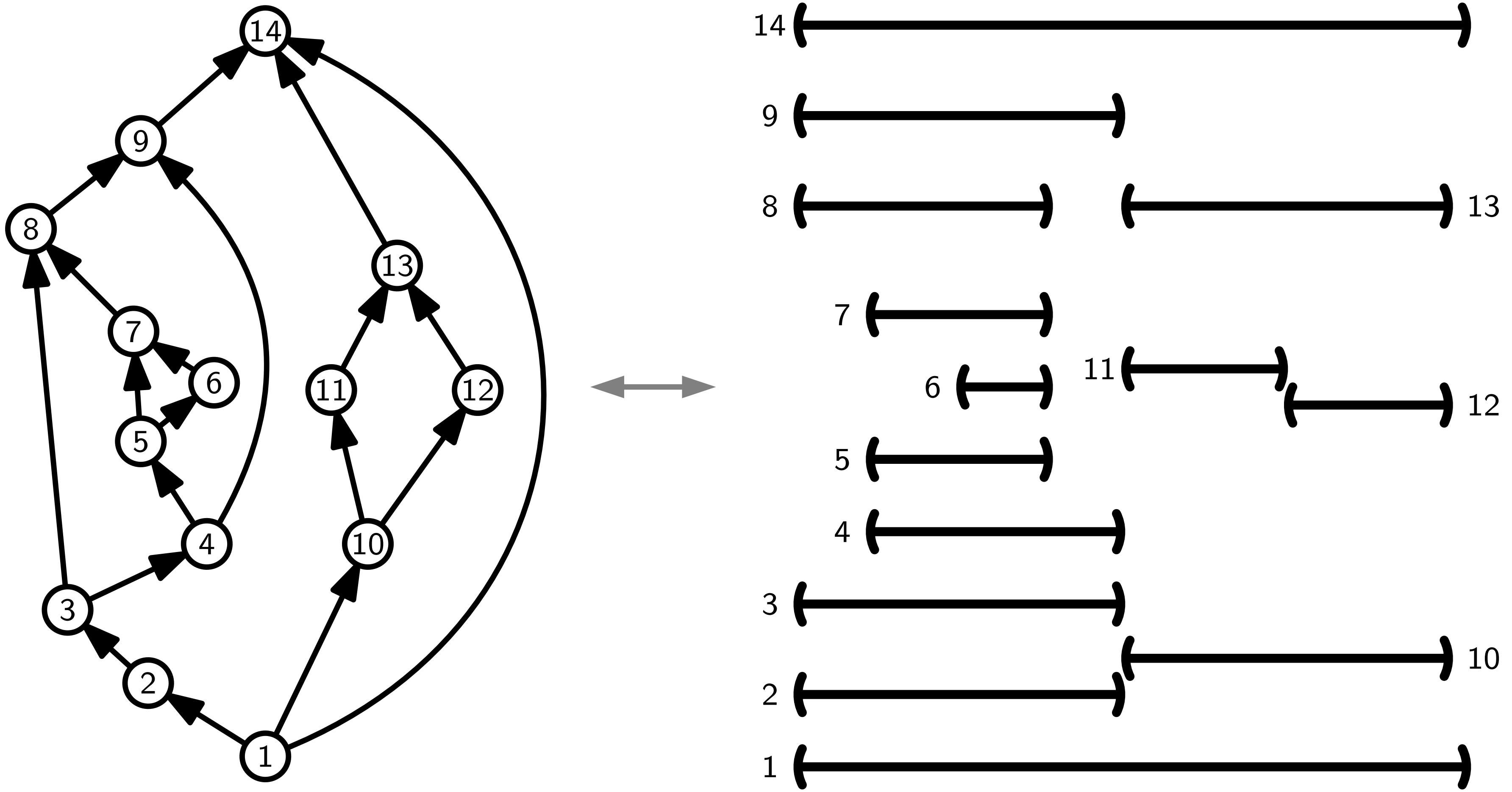

Results and outline

Theorem 1. [Chaplick et al. '18]
Rectangular ε-Bar Visibility Representation Extension can be solved in $\mathcal{O}(n \log^2 n)$ time for st-graphs.

- Dynamic program via SPQR-trees

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Theorem 3. [Chaplick et al. '18]
ε-Bar Visibility Representation Ext. is NP-complete for (series-parallel) st-graphs when restricted to the **integer grid** (or if any fixed $\varepsilon > 0$ is specified).

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Representation extension for st-graphs

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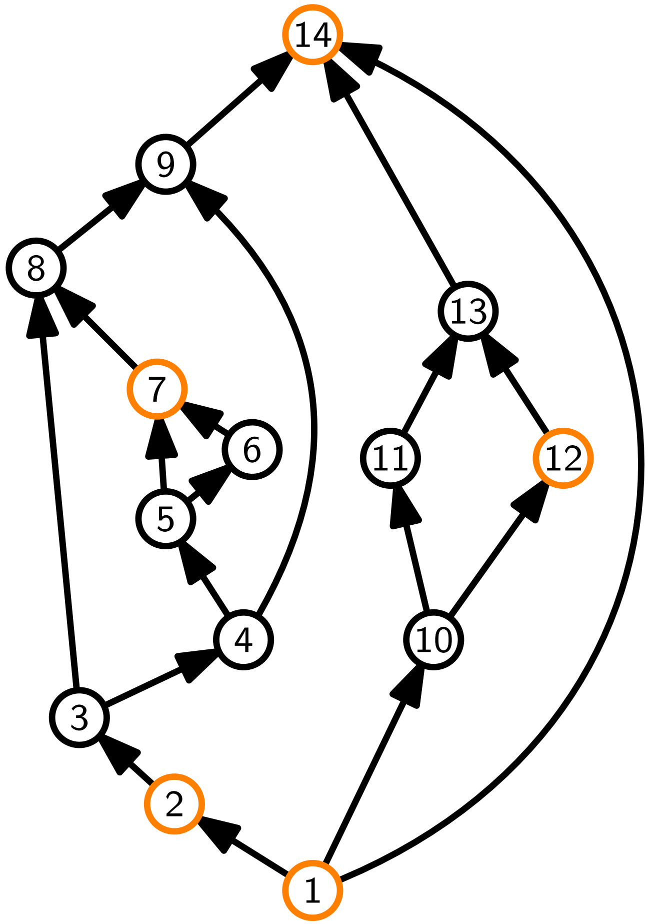

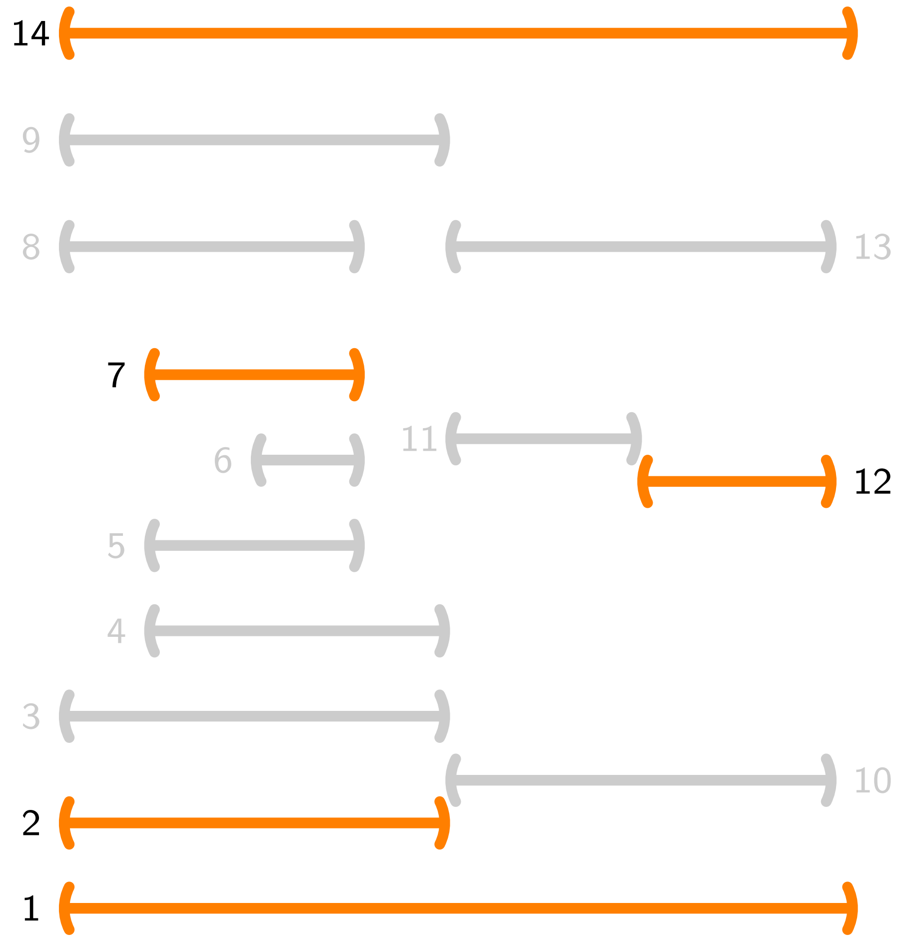

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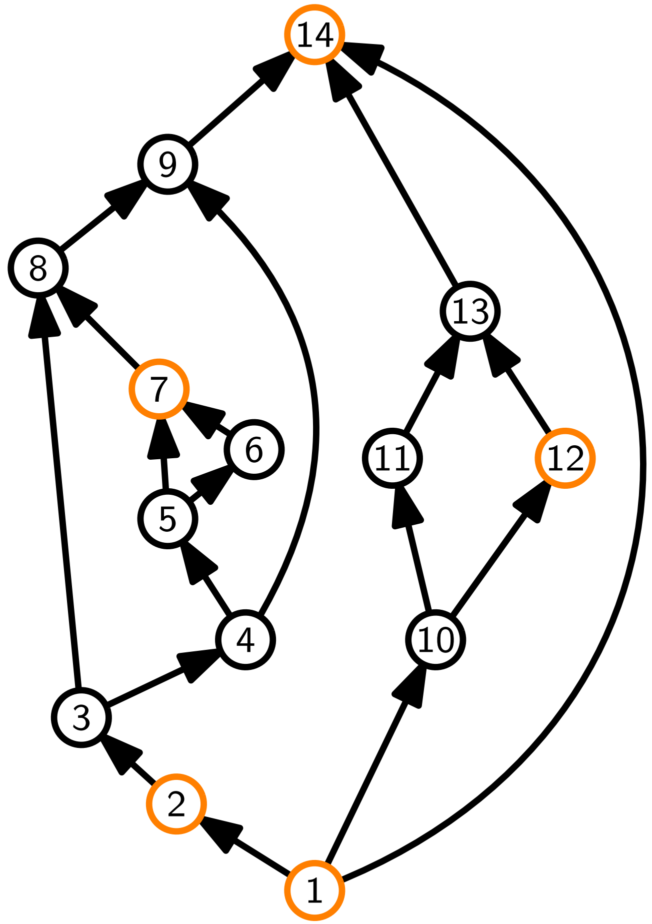
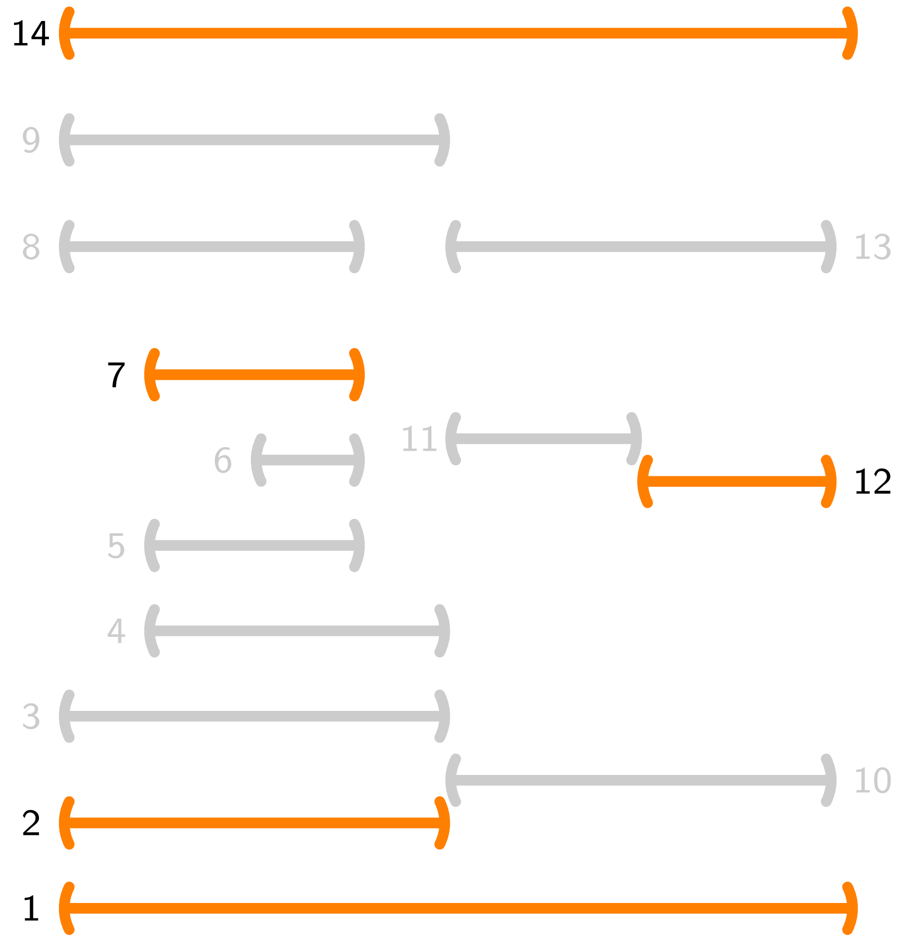


- Simplify with assumption on y-coordinates
- Look at connection to SPRQ-trees – tiling
- Solve problems for S, P and R nodes
- Dynamic program via SPQR-tree



y-coordinate invariant

- Let G be an st-graph, and ψ' be a representation of $V' \subseteq V(G)$.
- Let $y : V(G) \to \mathbb{R}$ such that
  - for each $v \in V'$, $y(v) =$ the y-coordinate of $\psi'(v)$.
  - for each edge (u, v), $y(u) < y(v)$.

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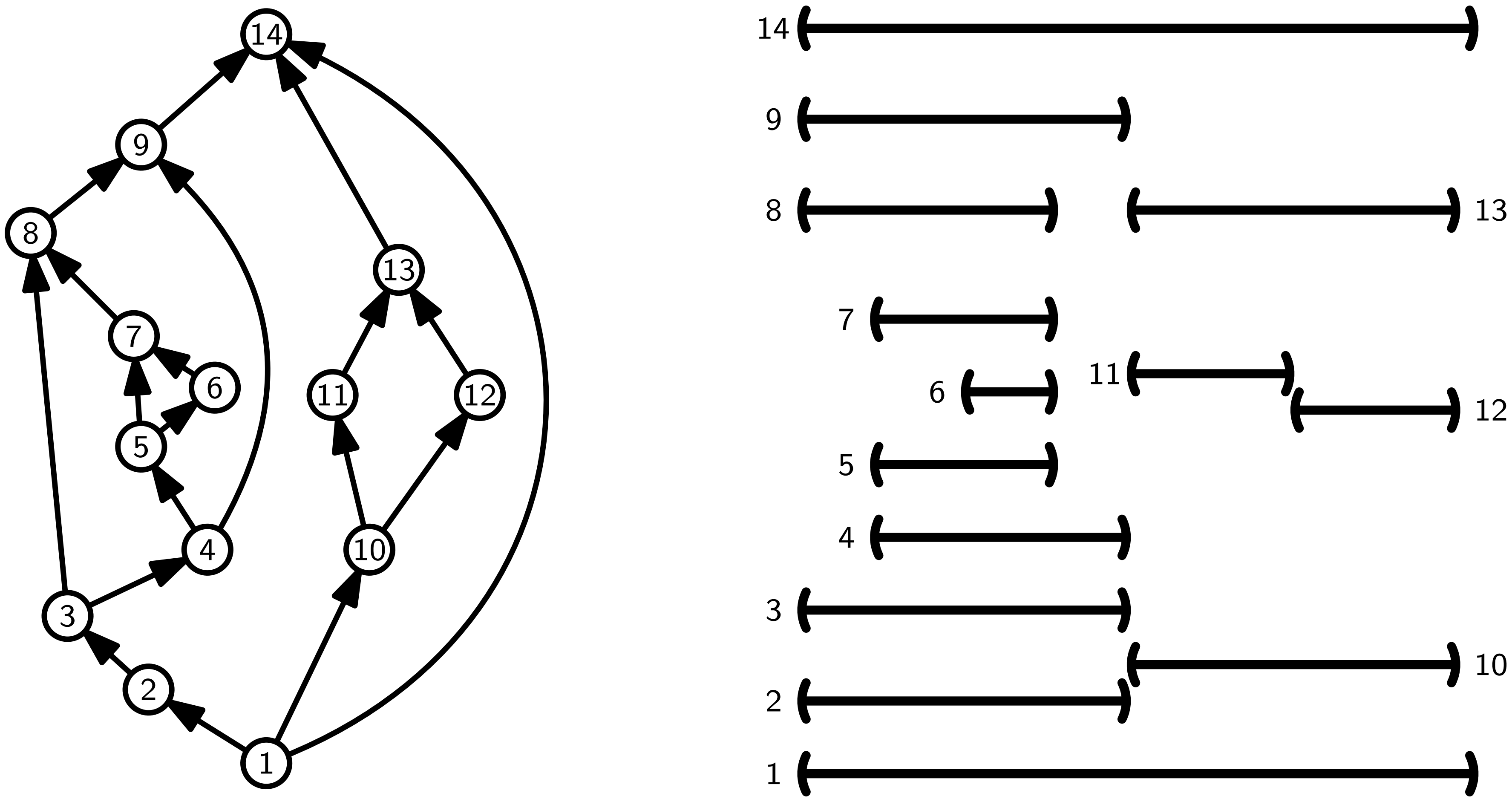
But why do SPQR-trees help?



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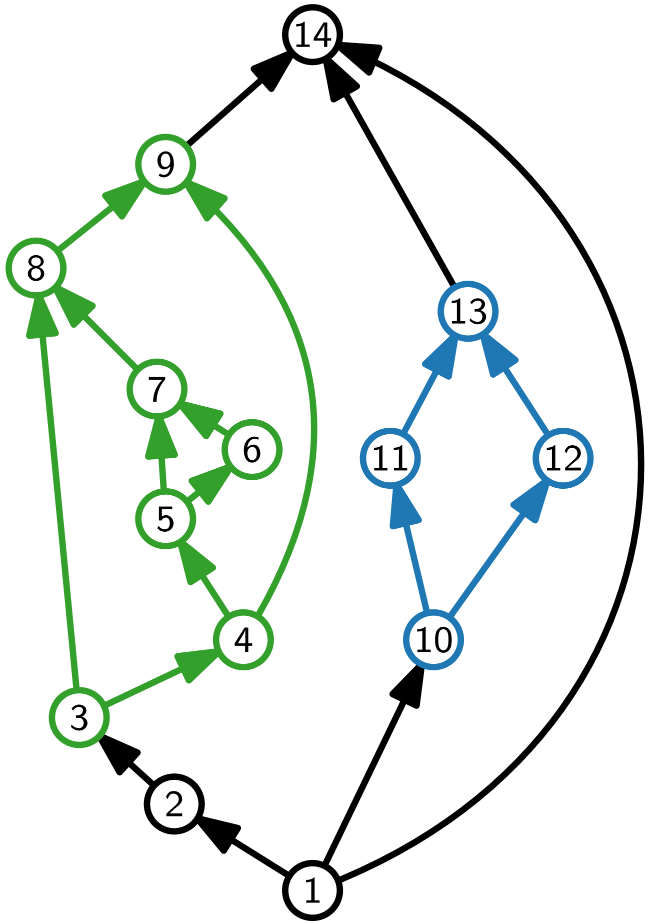

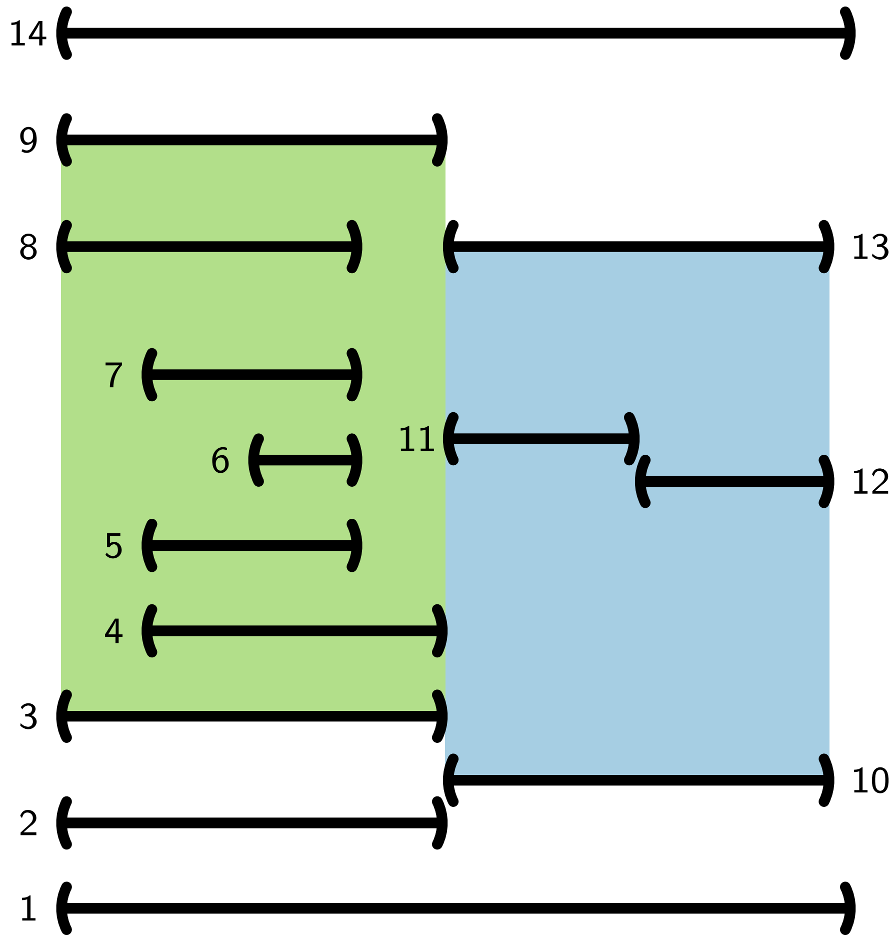

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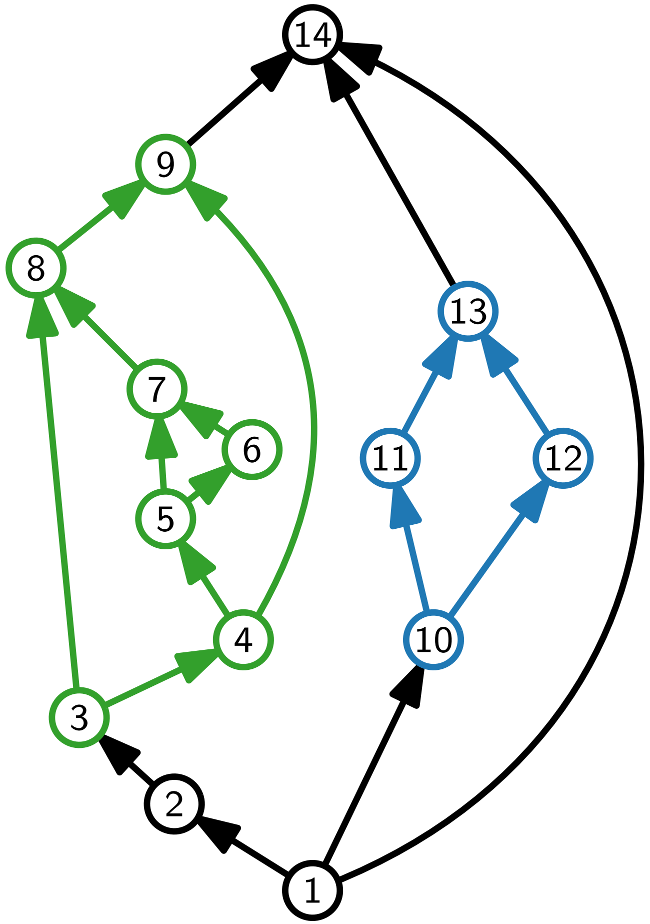

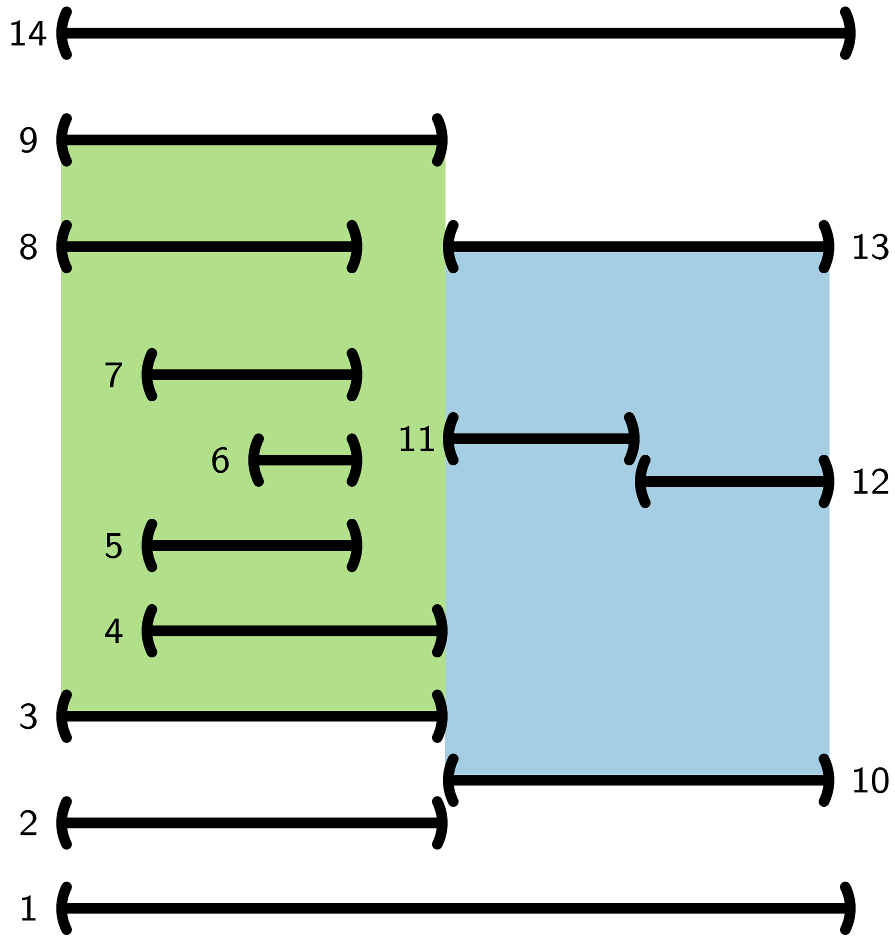

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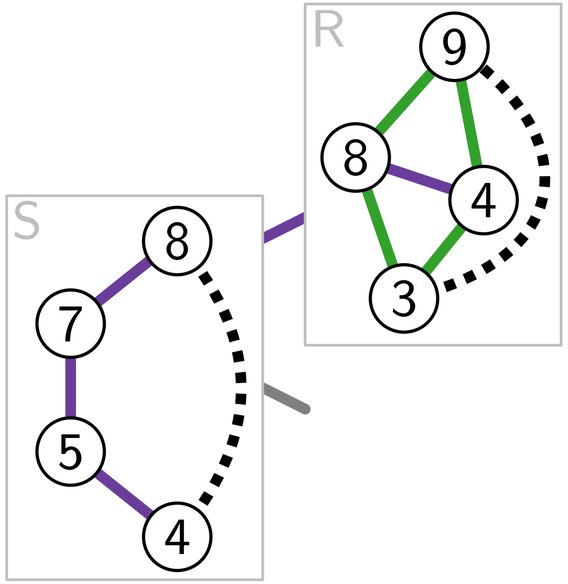
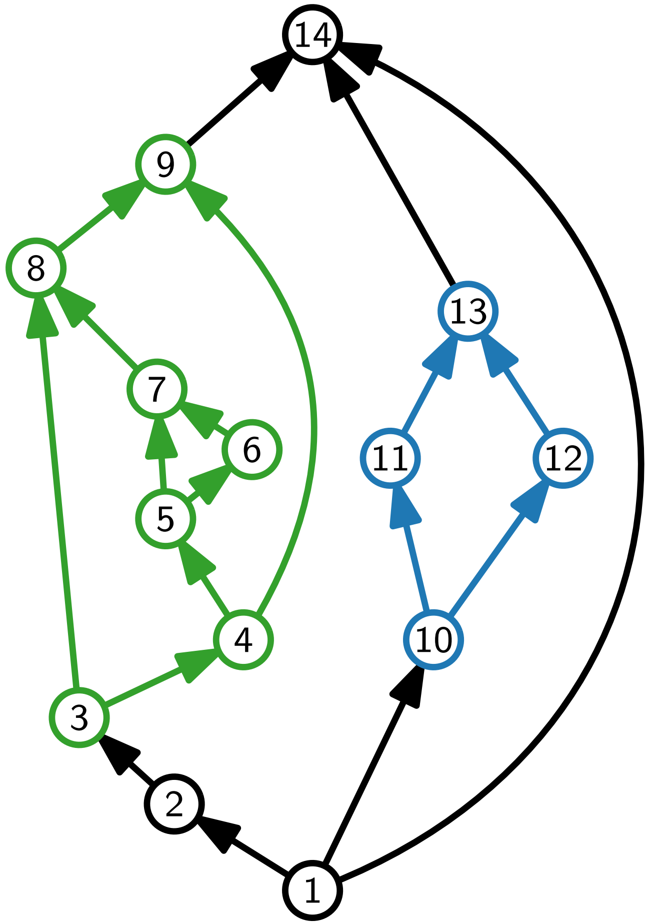
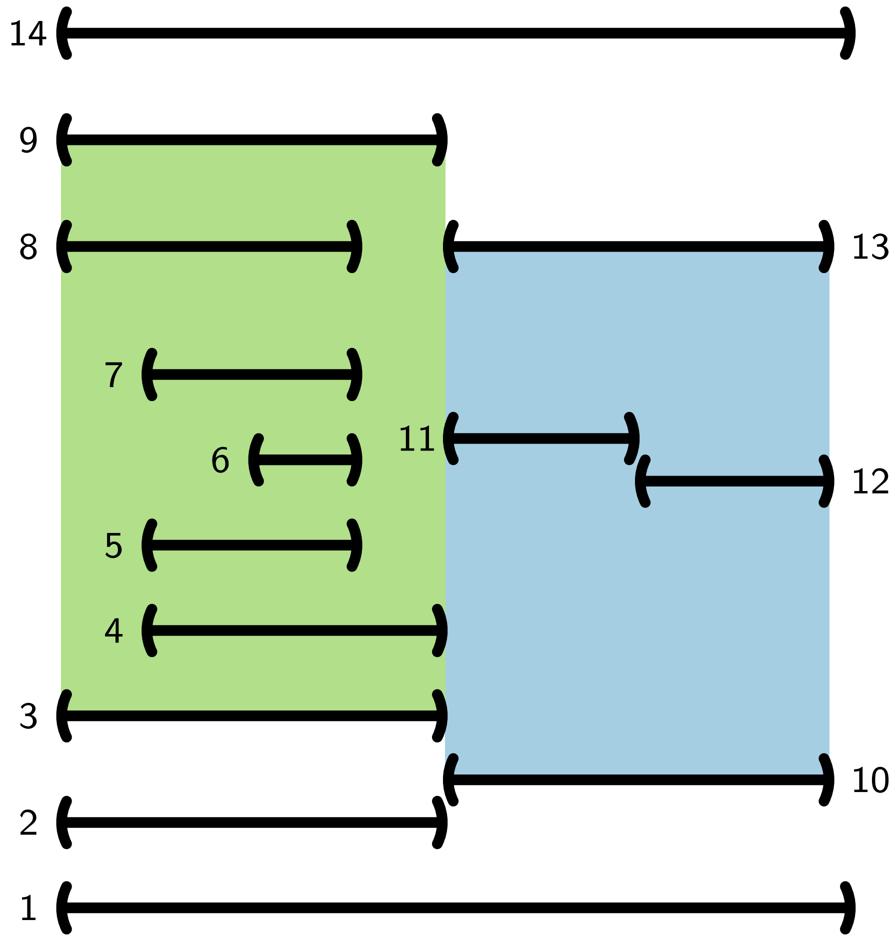


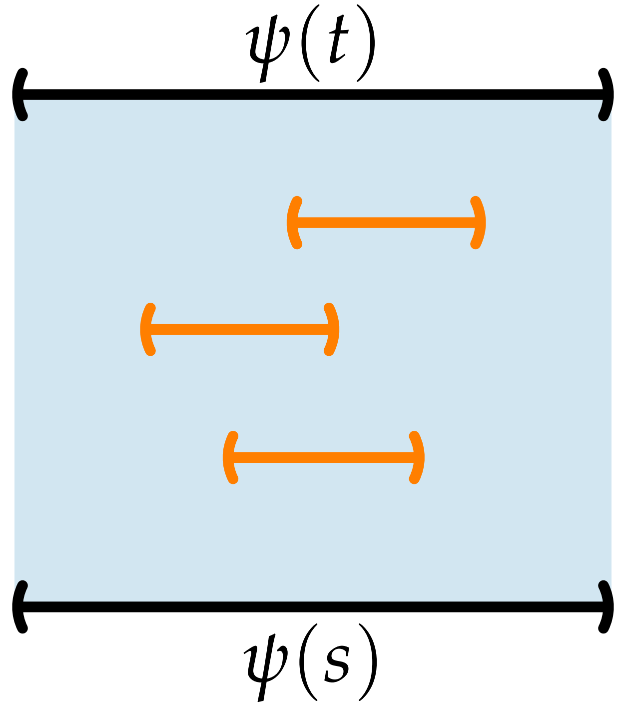
Solve tiles bottom-up





Tiles

Convention. **Orange** bars are from the partial representation



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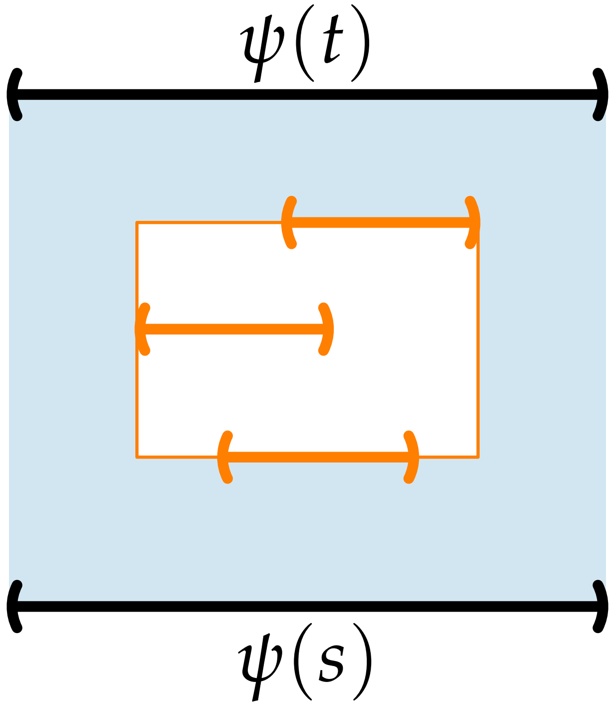

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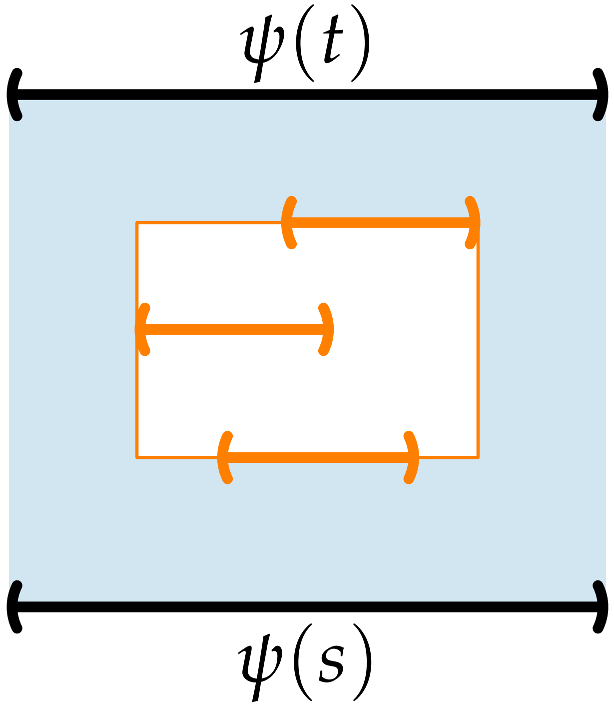

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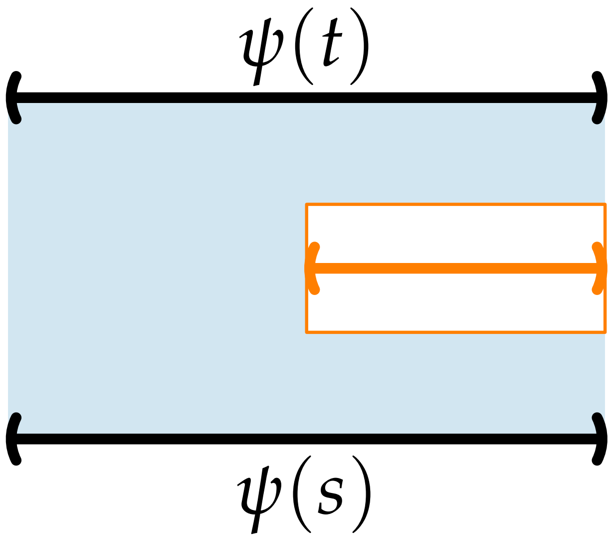
How many **different** tiles can we really have?

Types of tiles



- Right **F**ixed – due to the orange bar
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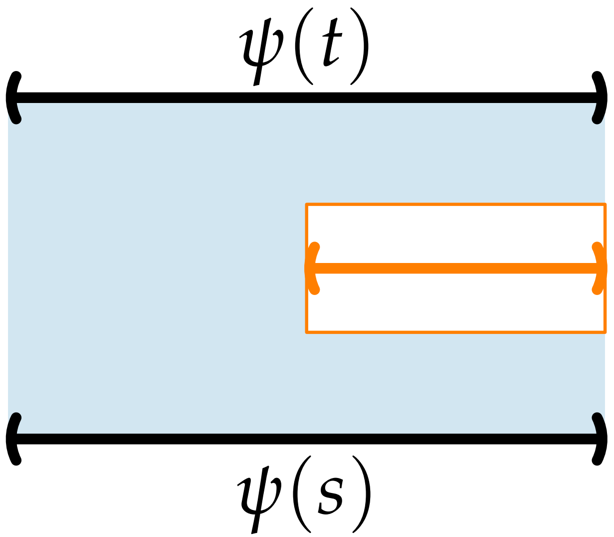

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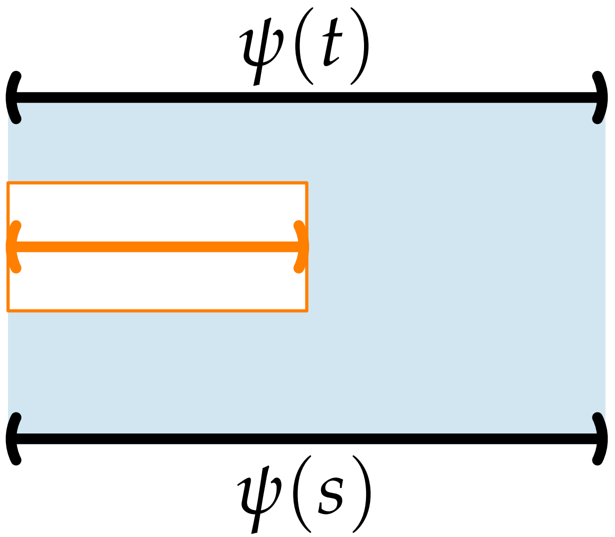

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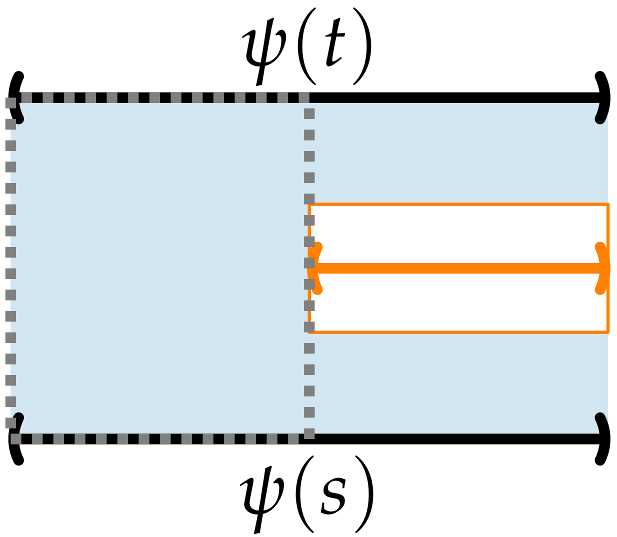

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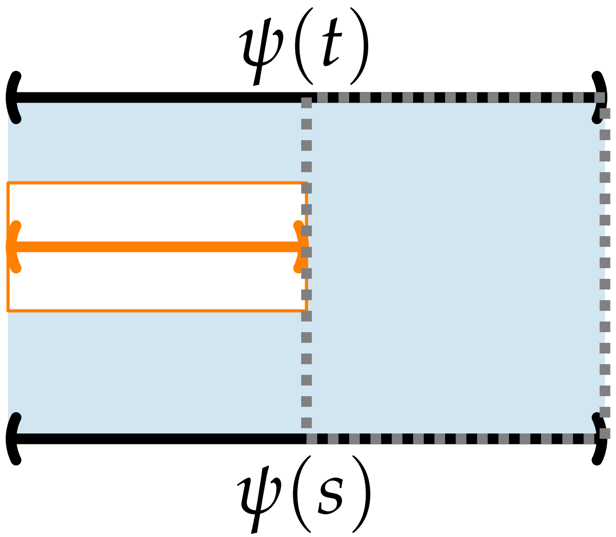

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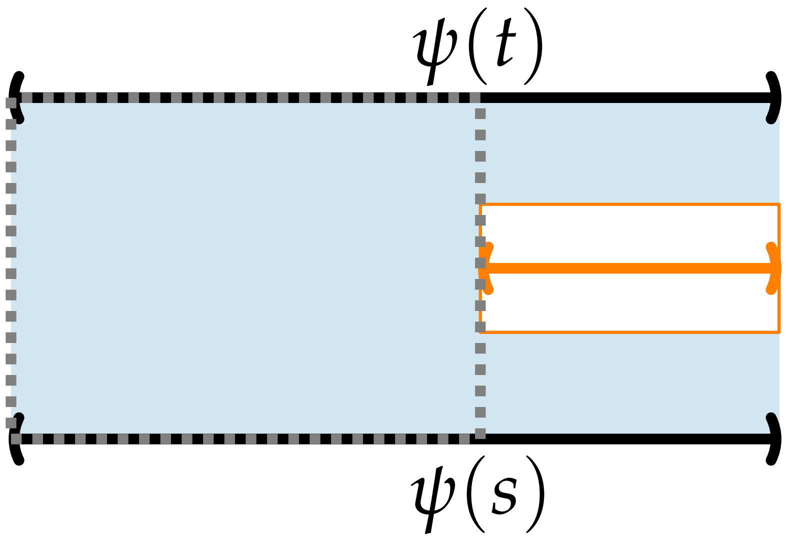

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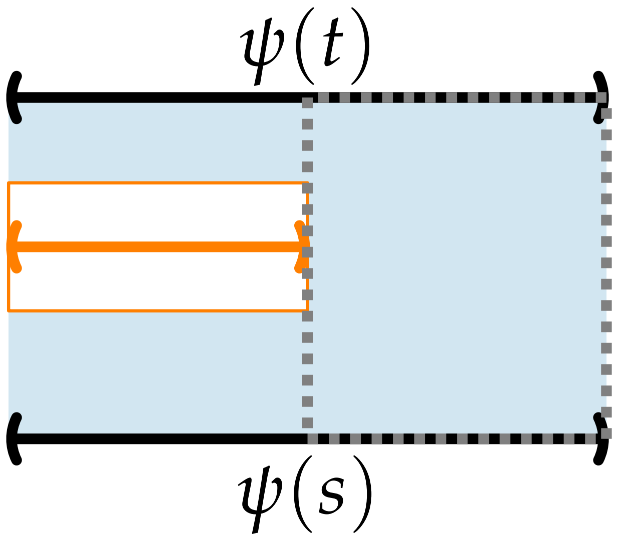

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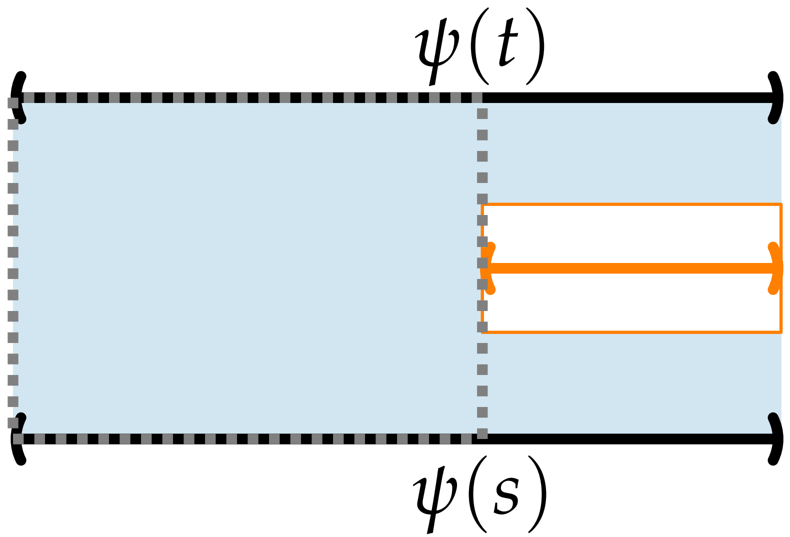

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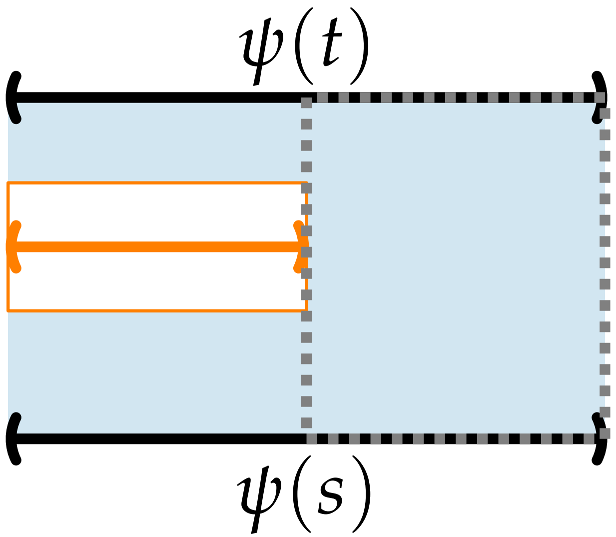

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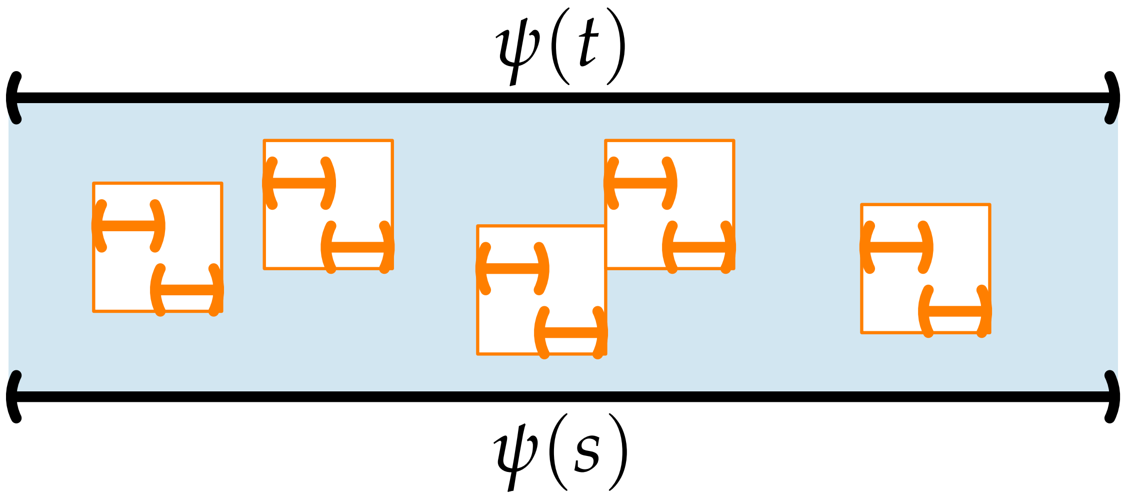
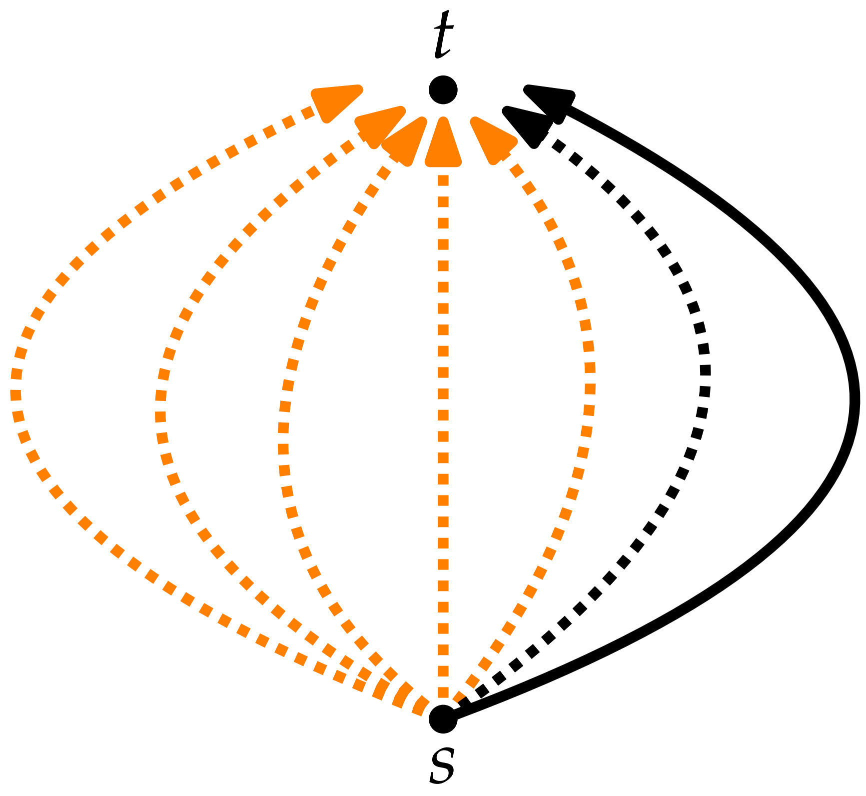
Four different types: **FF, FL, LF, LL**

P nodes





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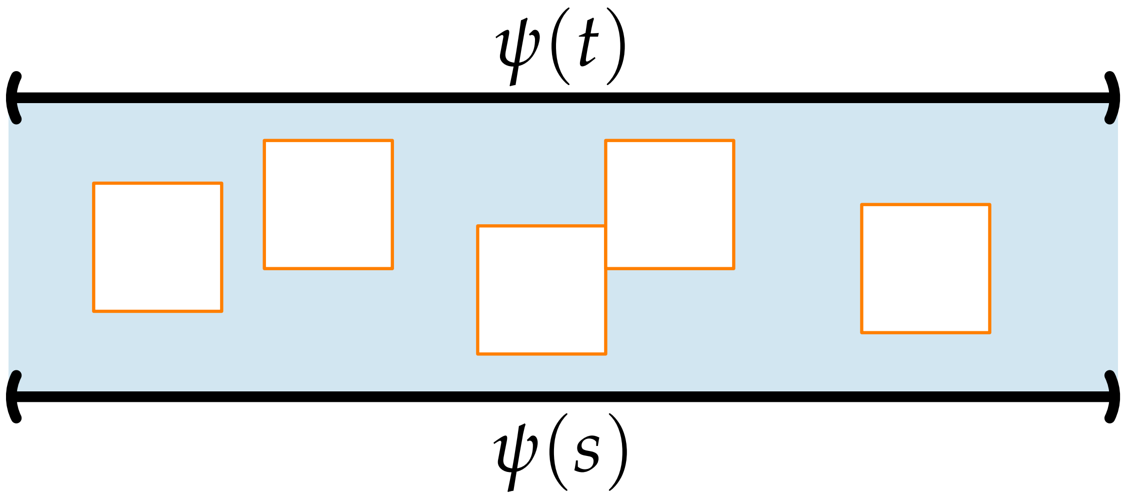

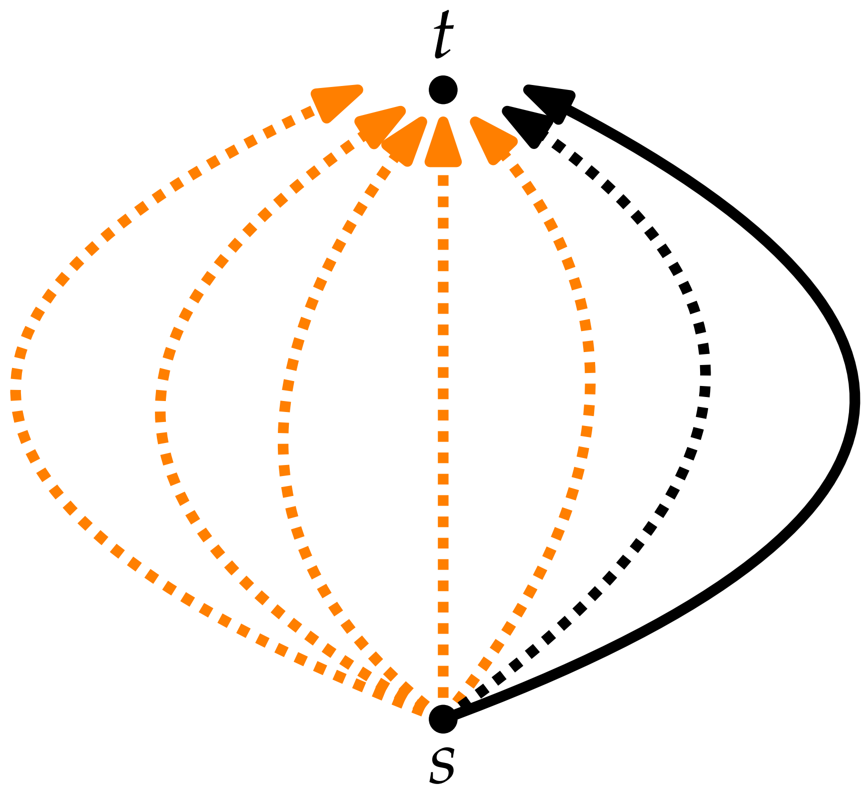

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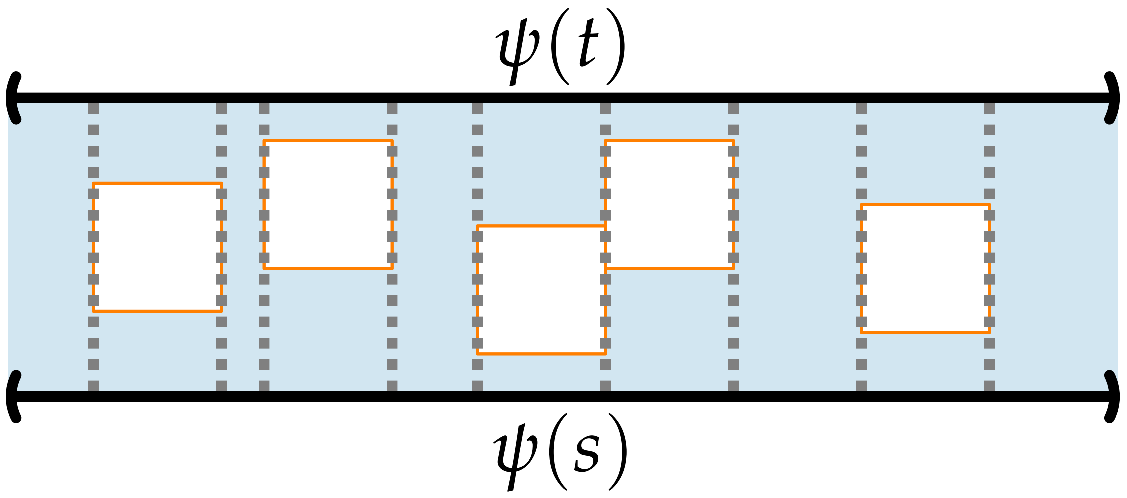

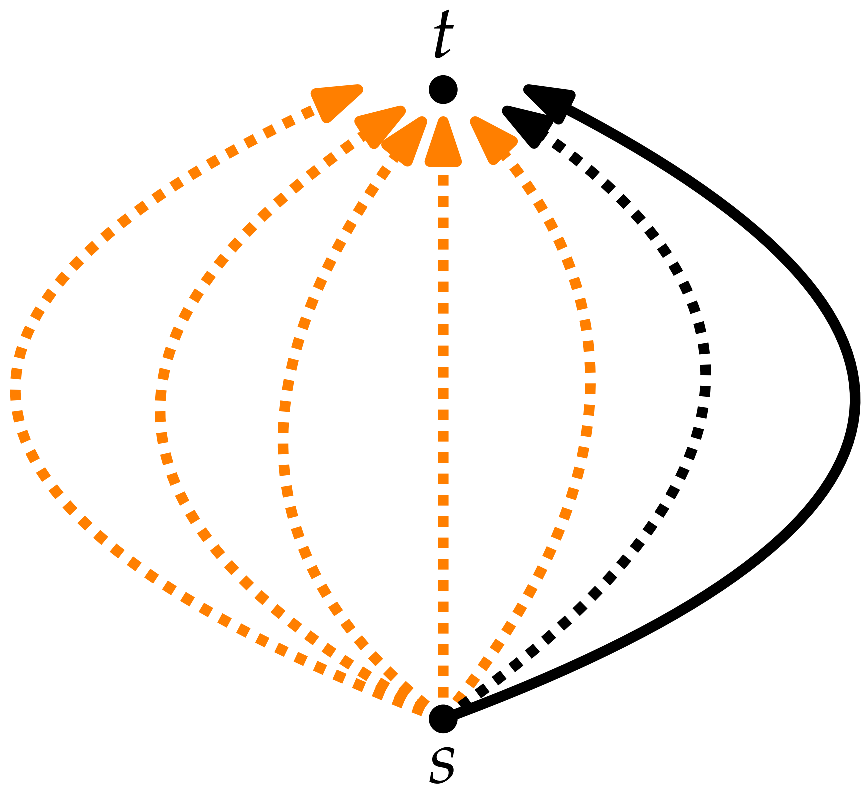

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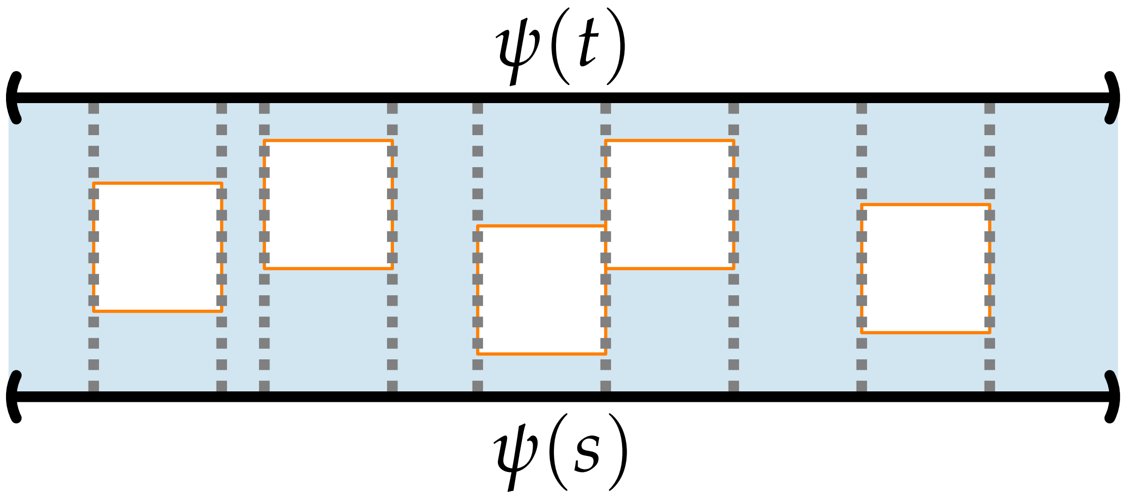

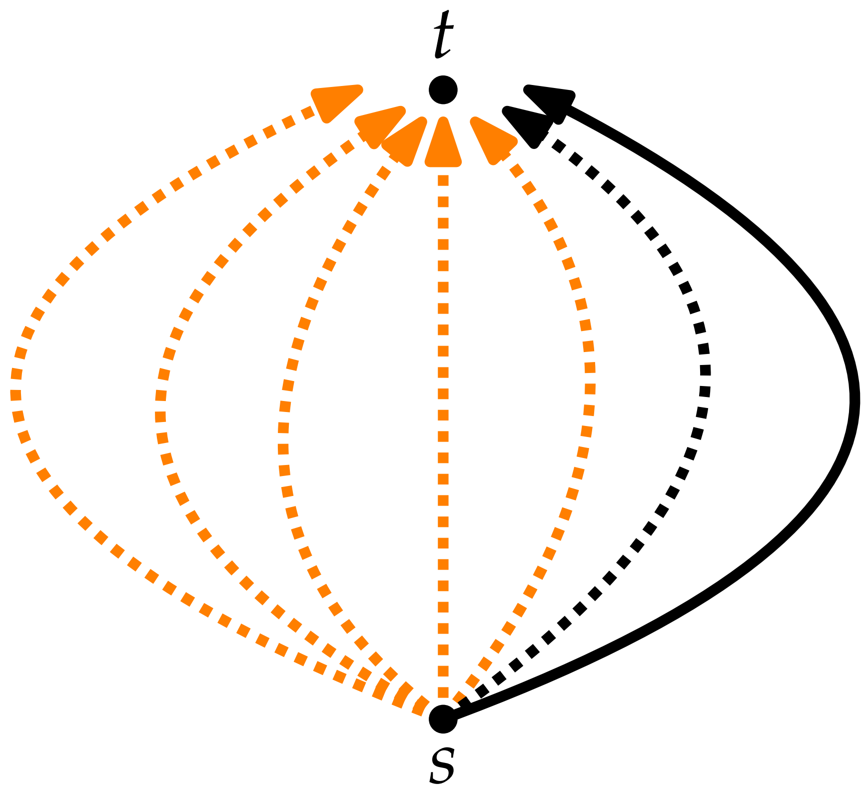

- Children of P node with prescribed bars occur in given left-to-right order
- But there might be some gaps. . .

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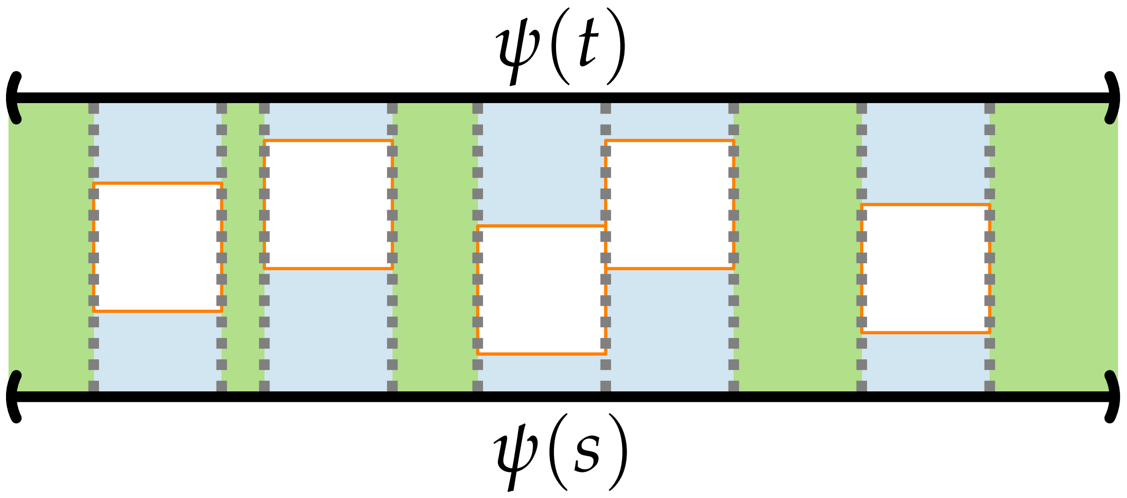

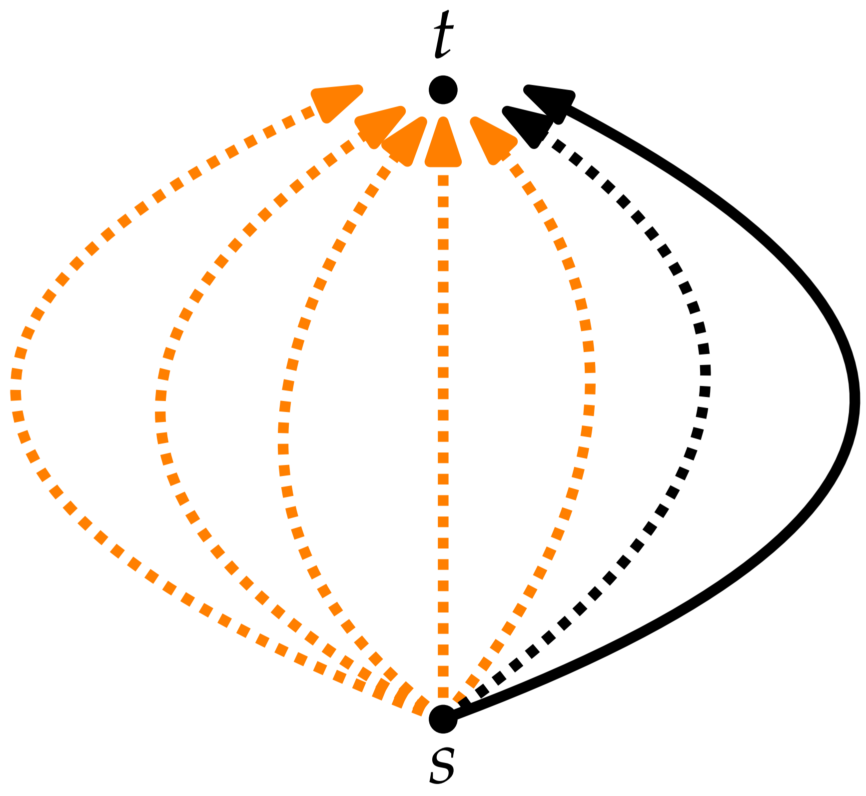

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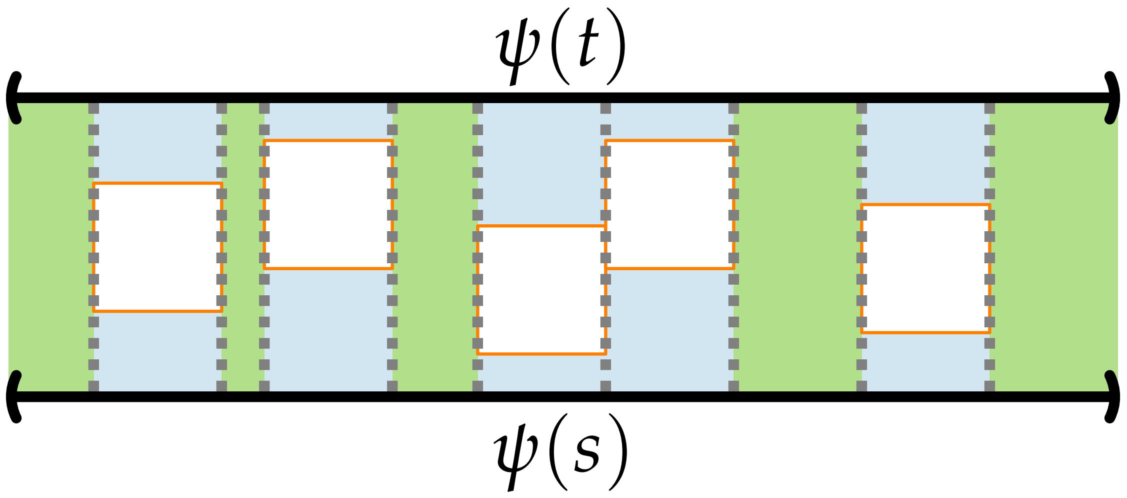

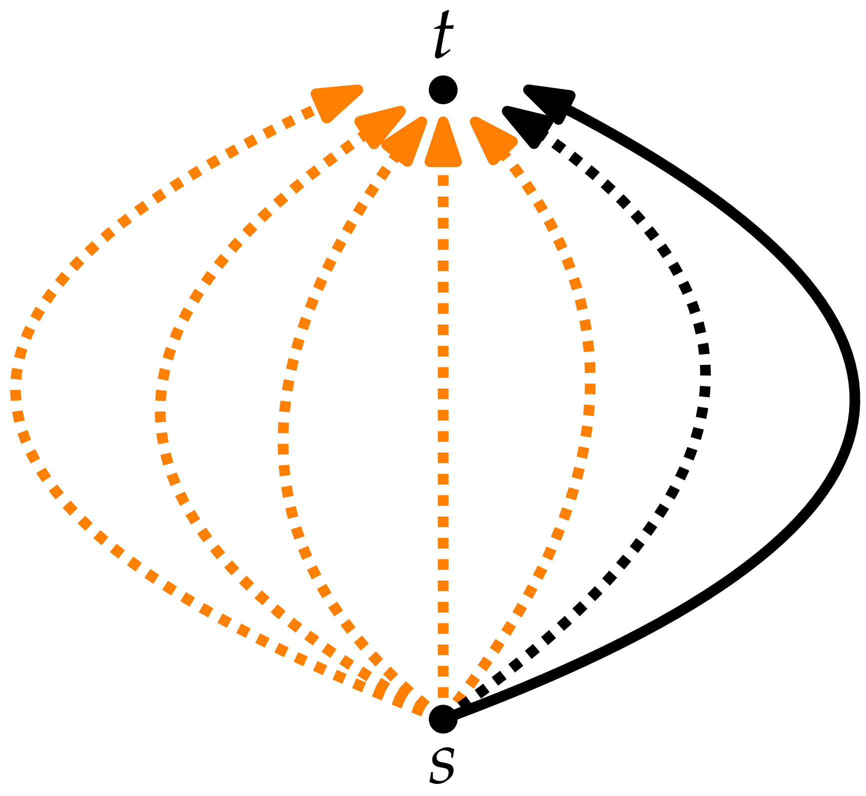

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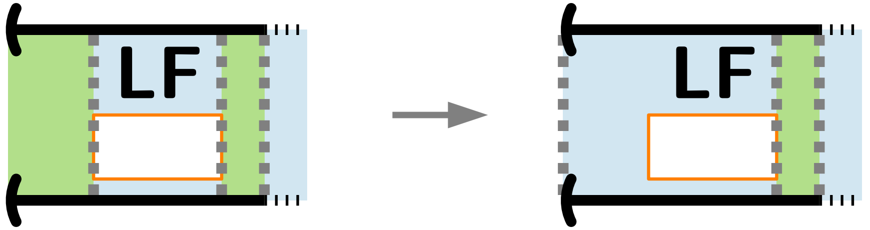
Idea.

Greedily *fill* the gaps by preferring to "stretch" the children with prescribed bars.



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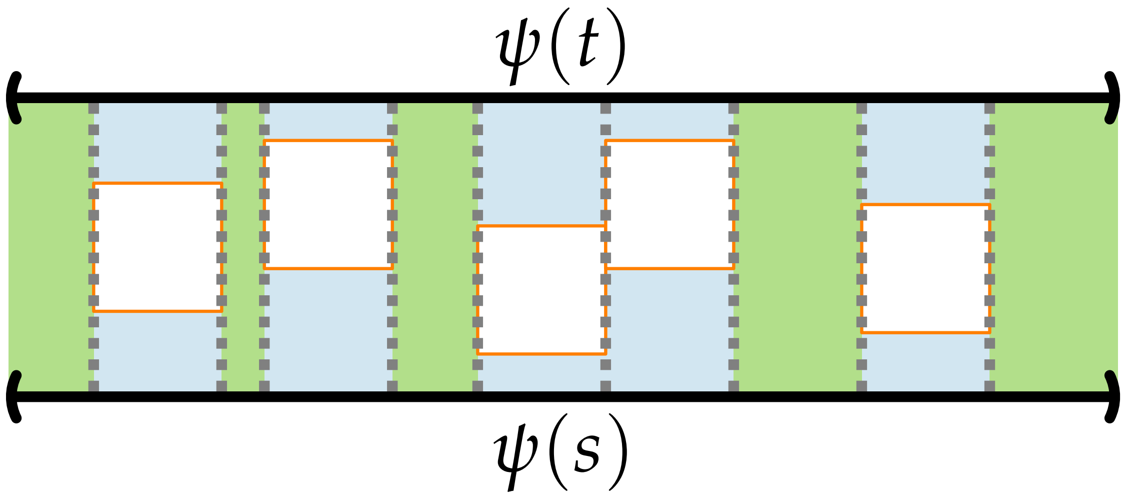

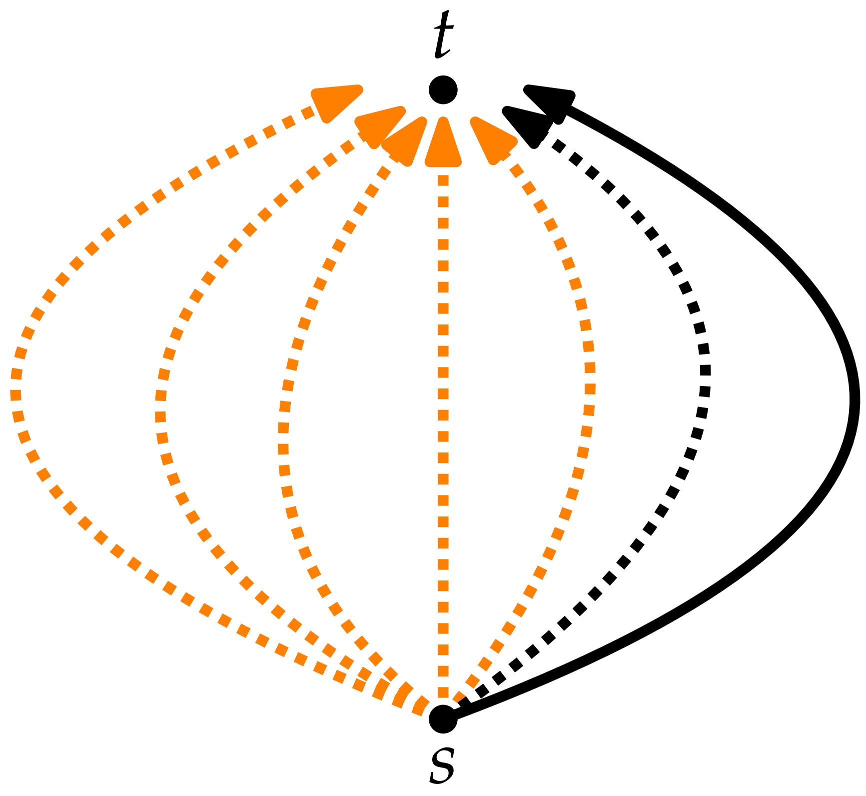

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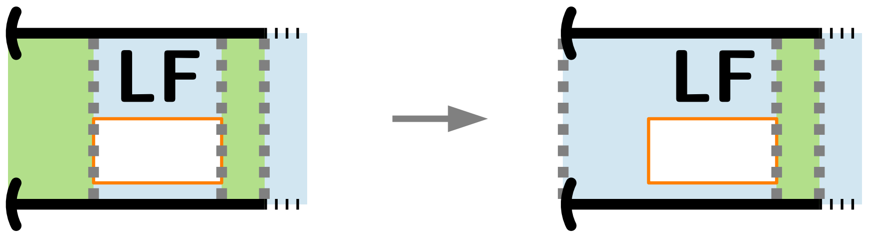
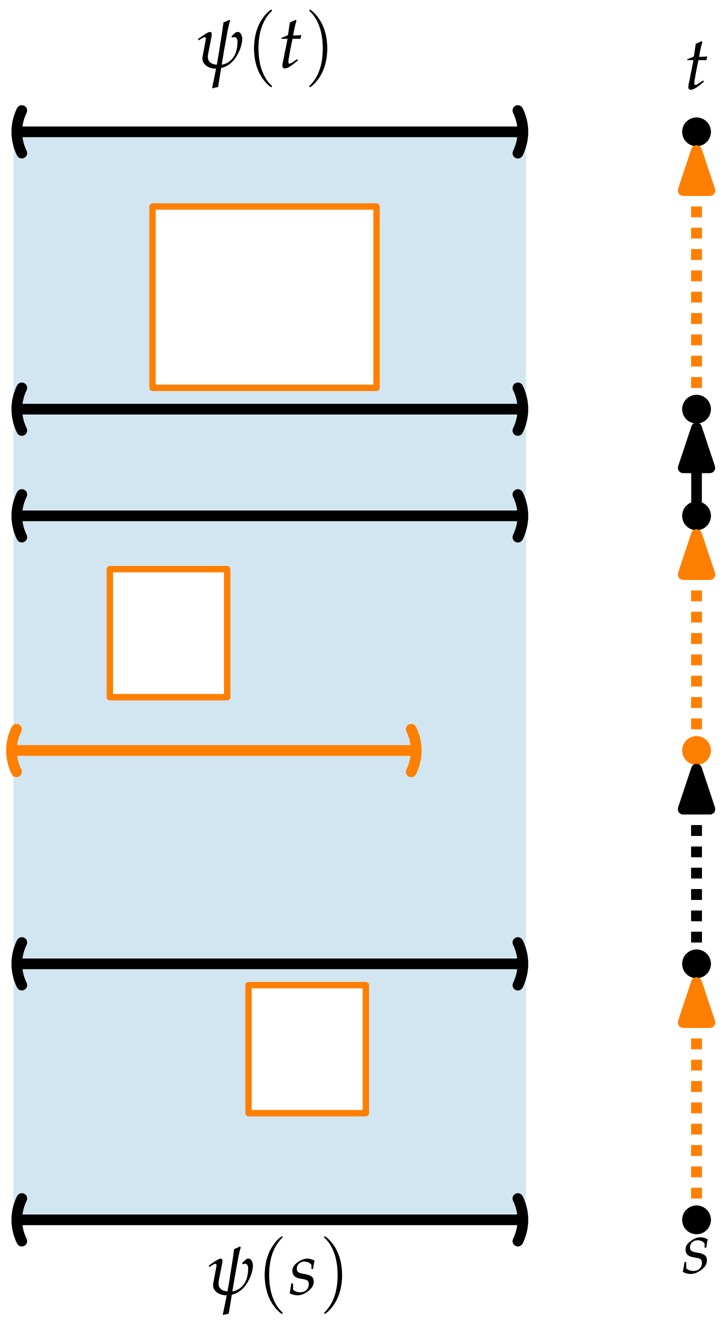


Outcome.

After processing, we must know the valid types for the corresponding subgraphs.

S nodes



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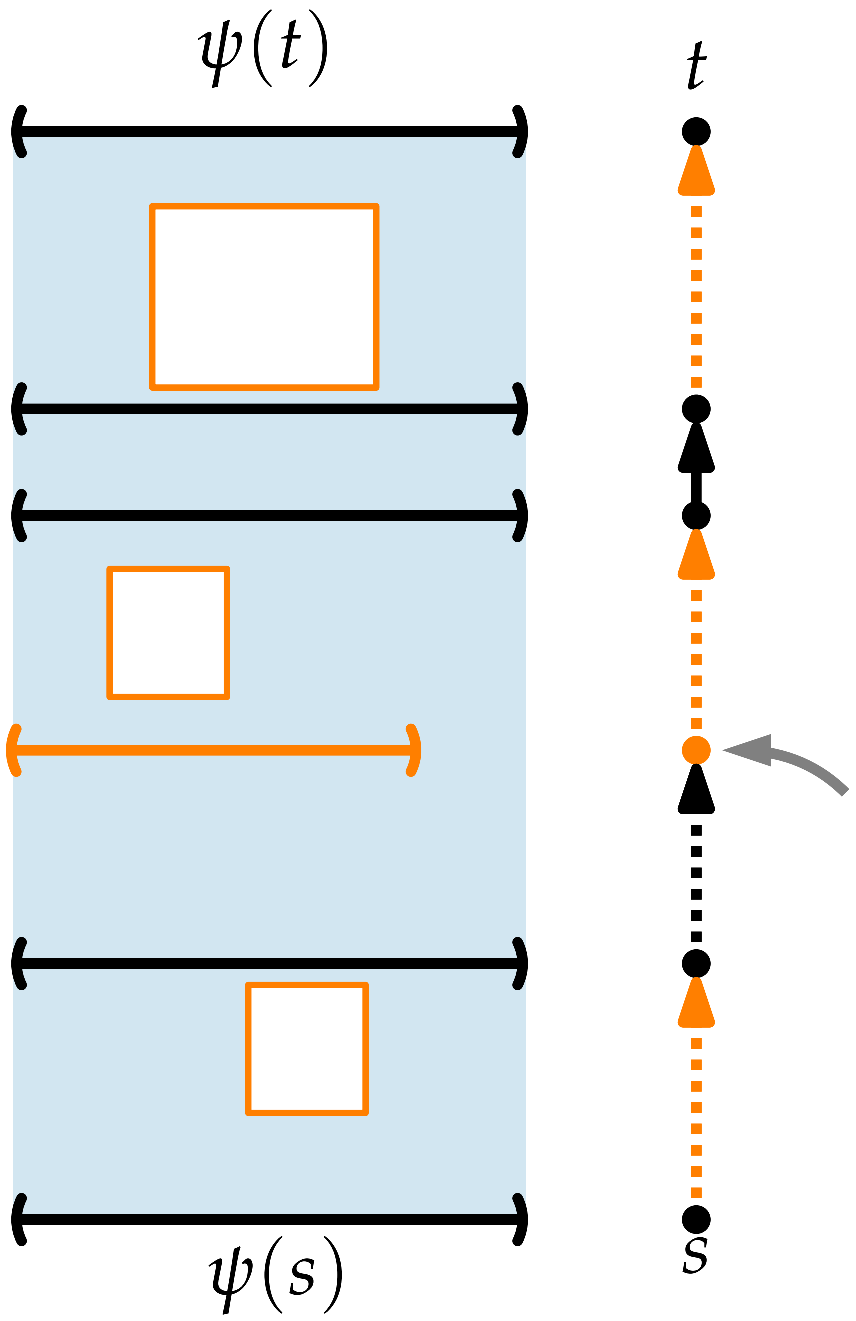

This fixed vertex means we can only make a Fixed-Fixed representation!

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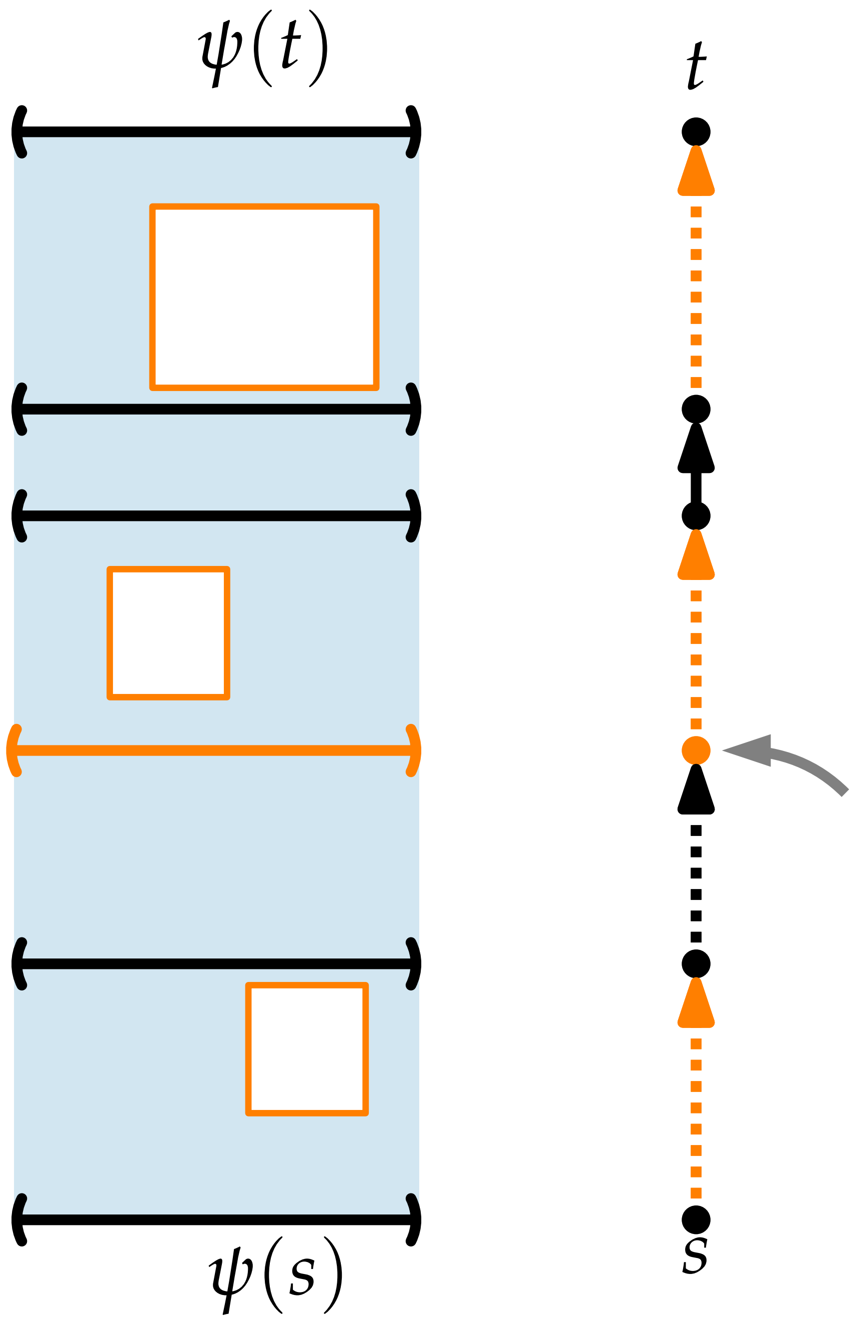

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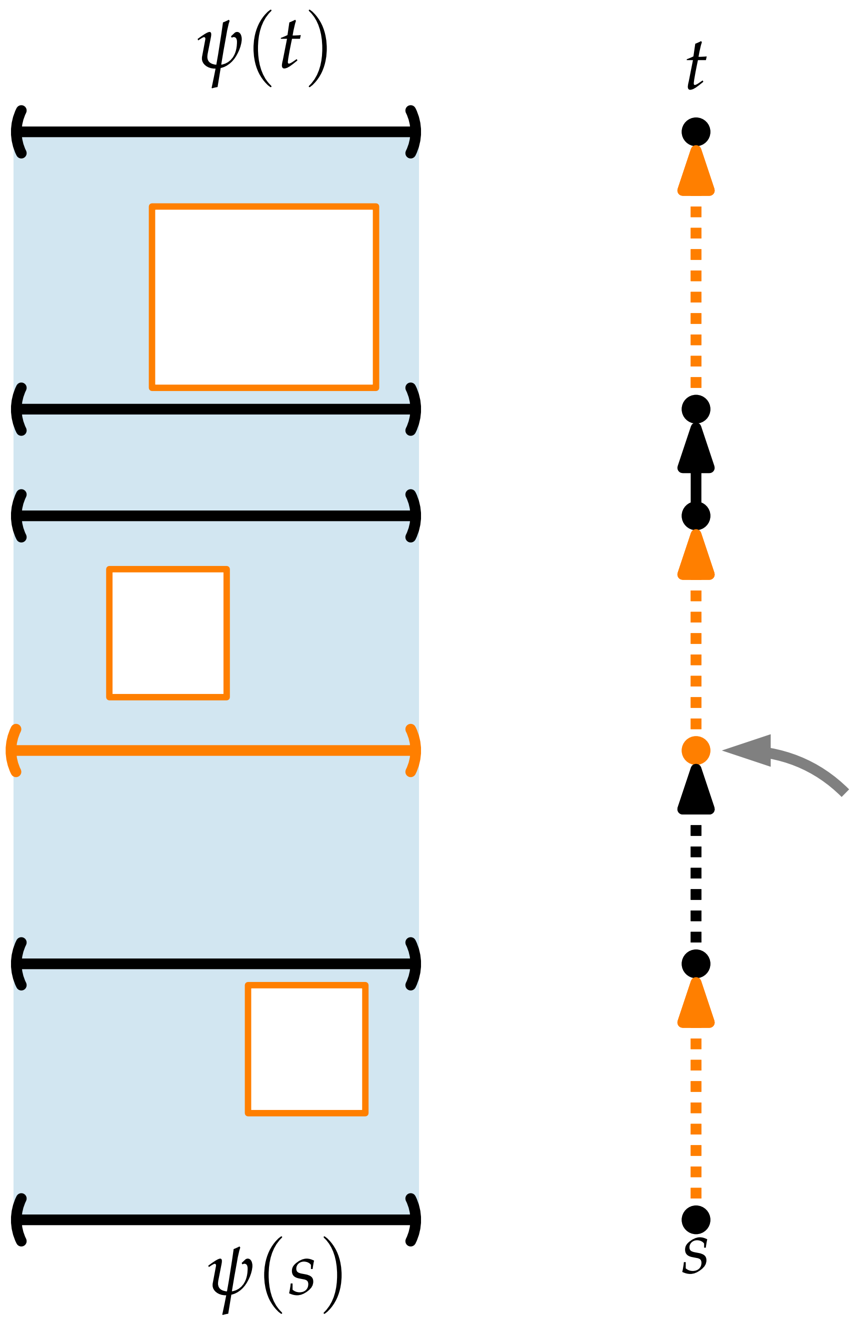
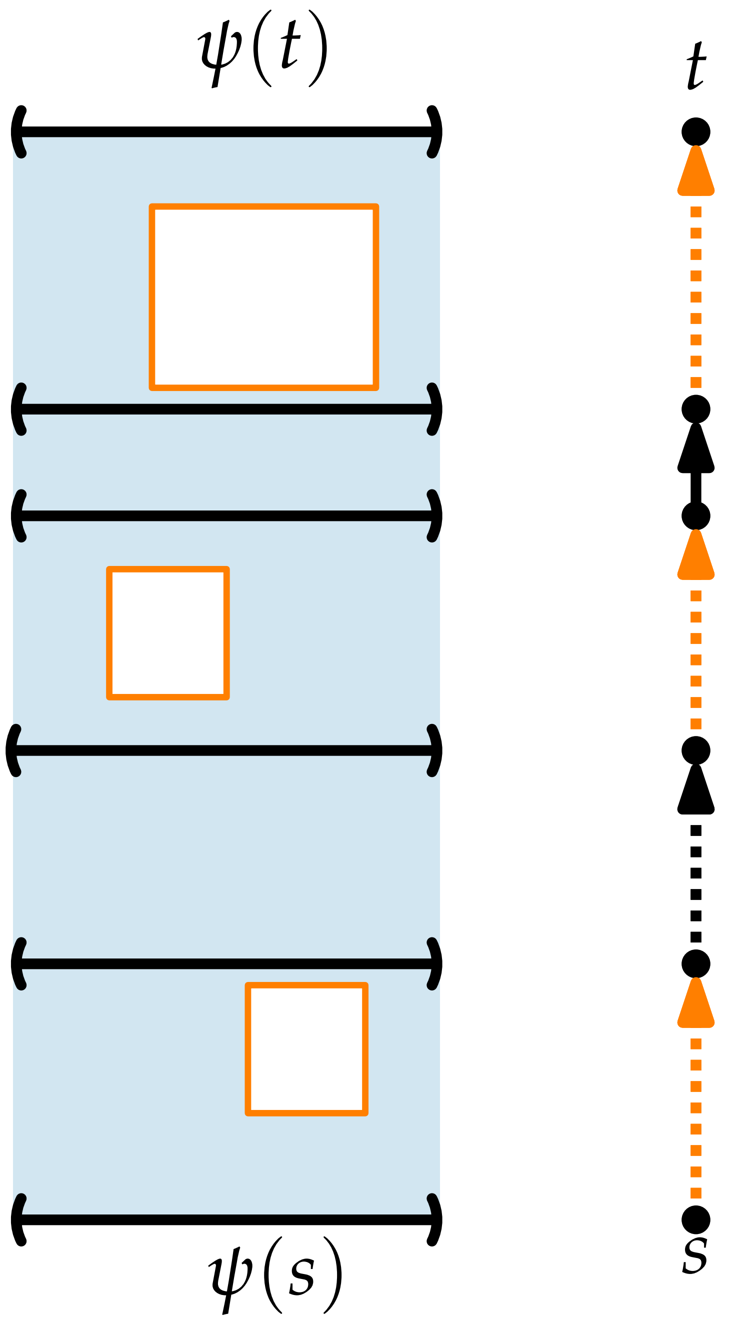


Here we have a chance to make all (LL, FL, LF, FF) types.

This fixed vertex means we can only make a Fixed-Fixed representation!



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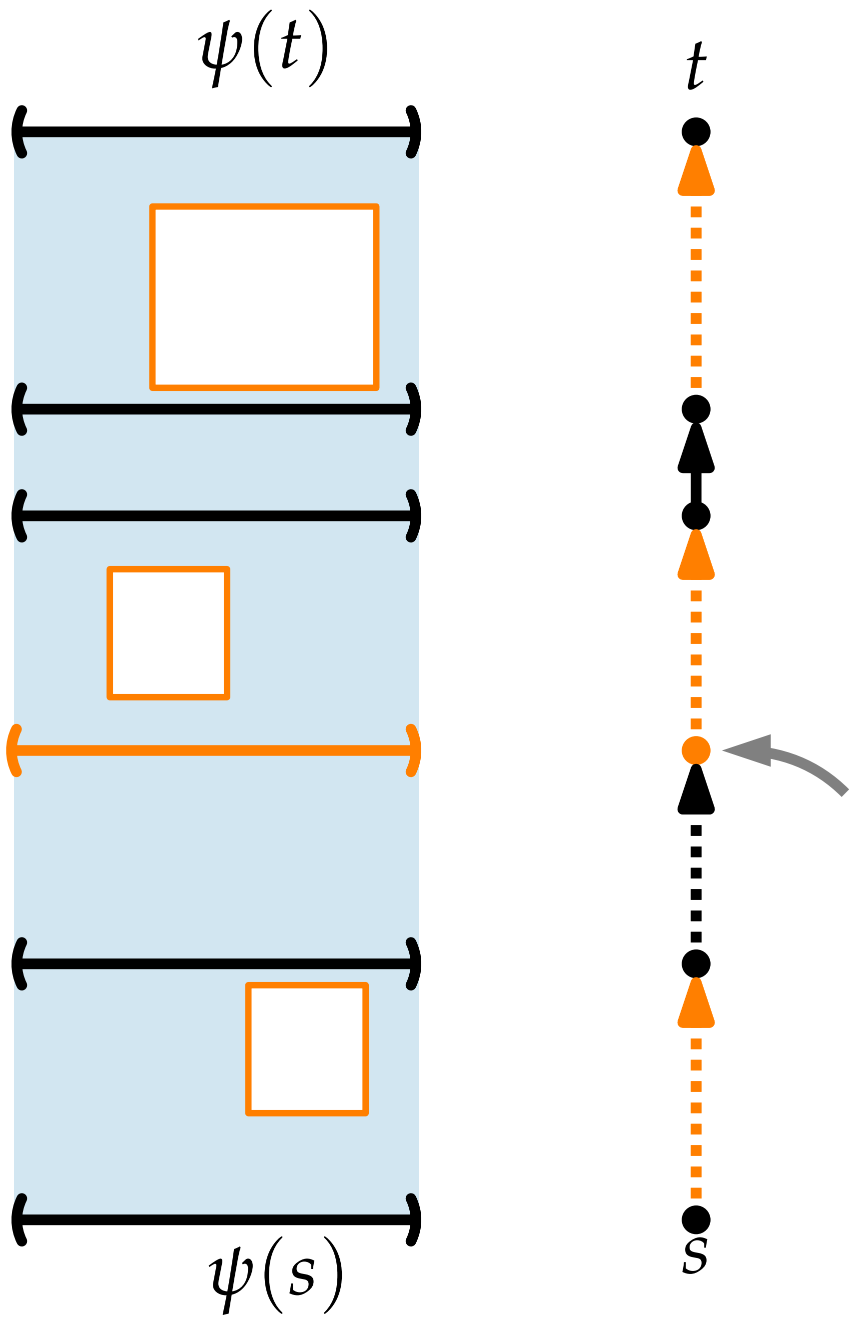

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How does this work?

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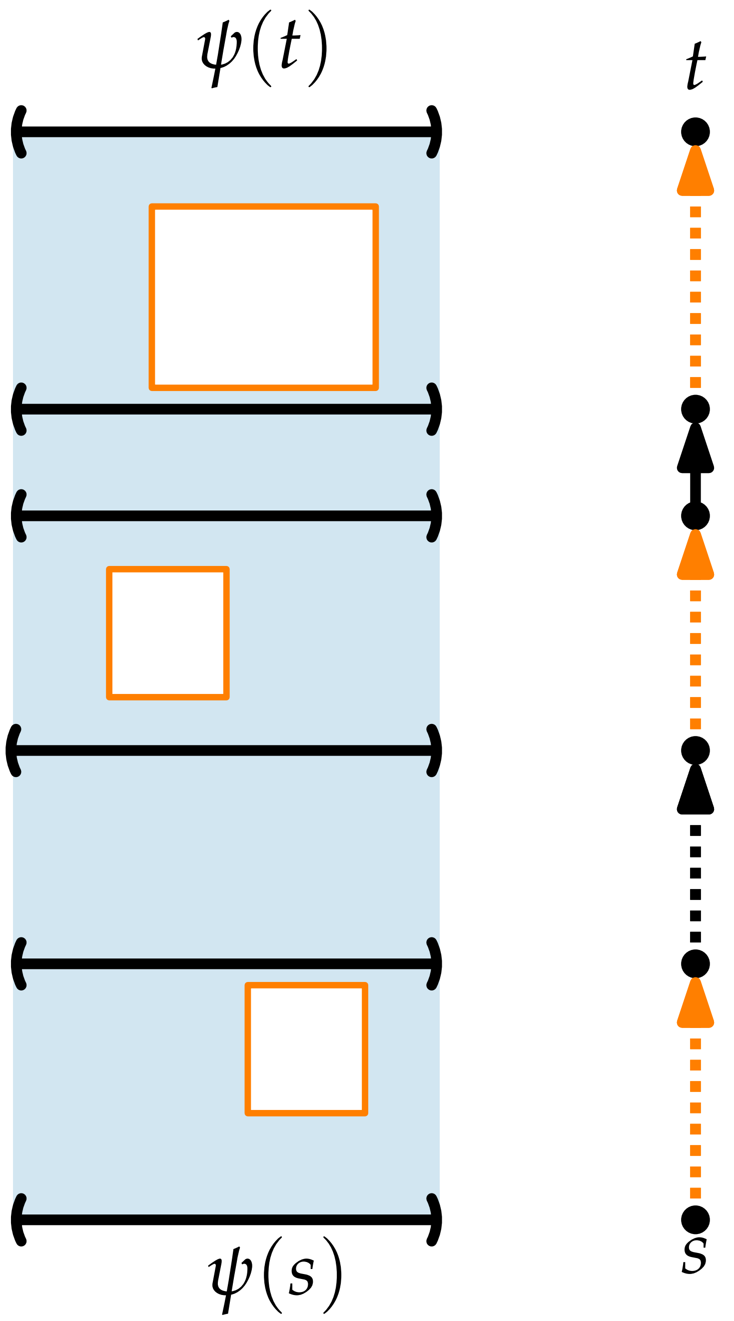
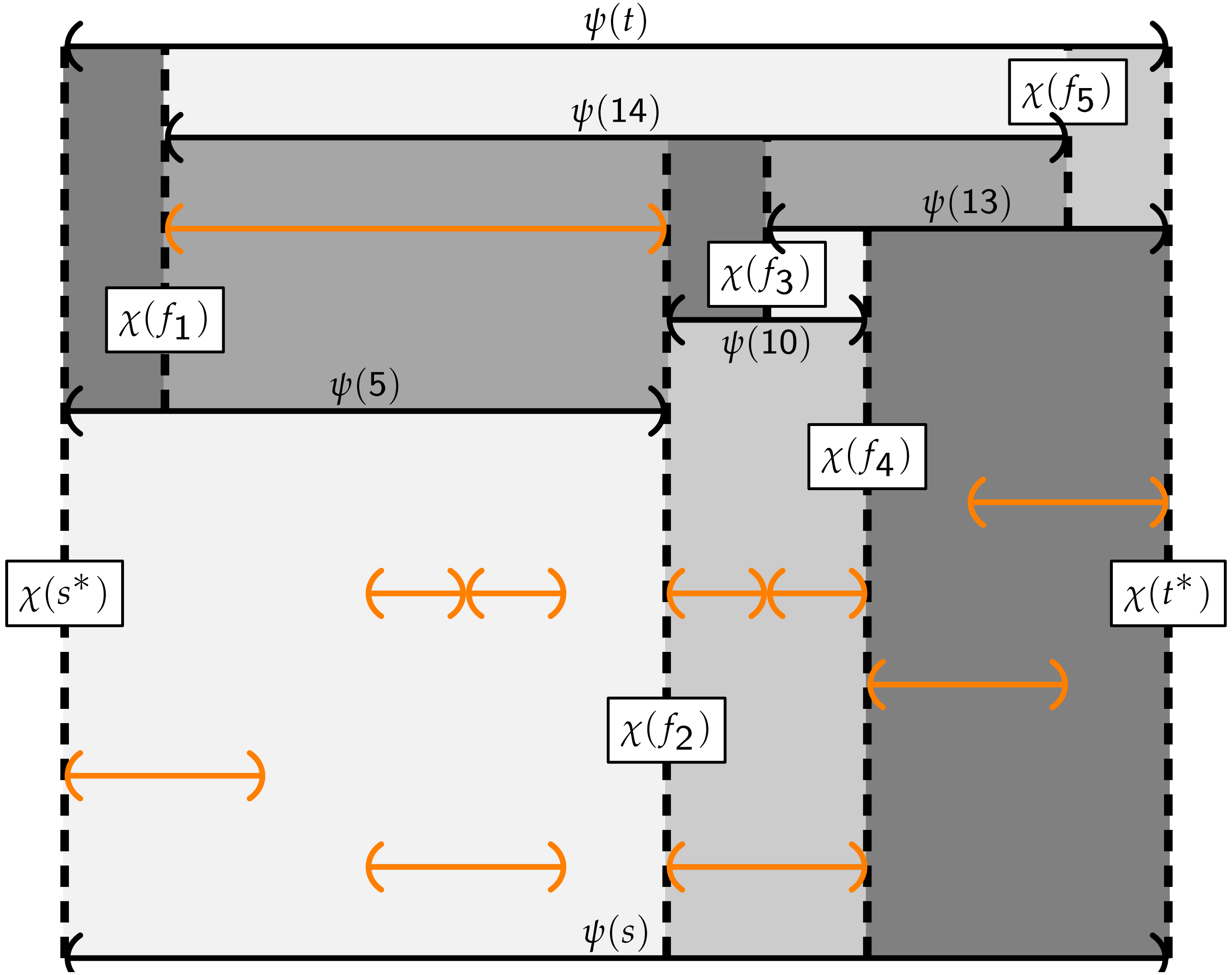
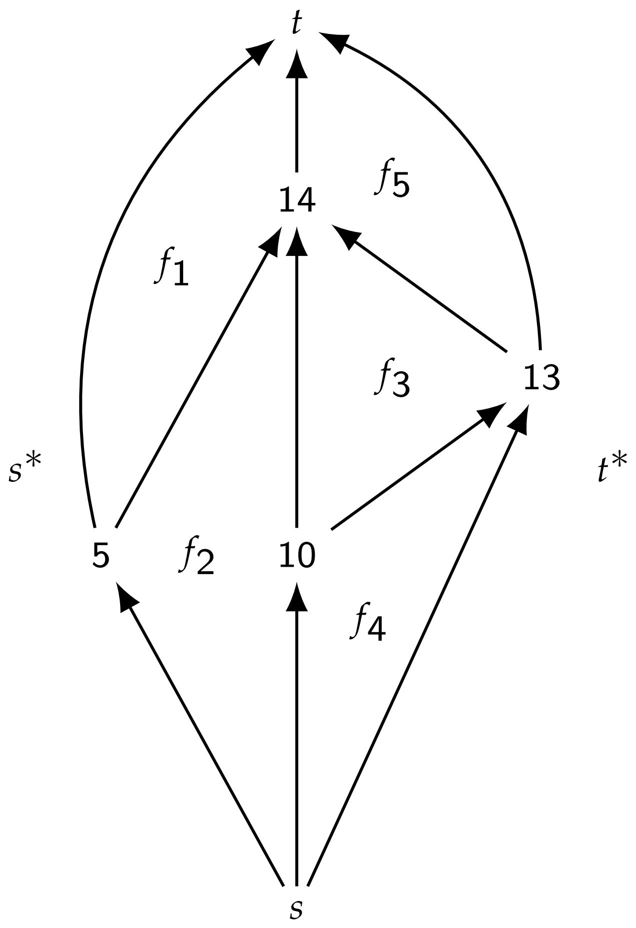


R nodes

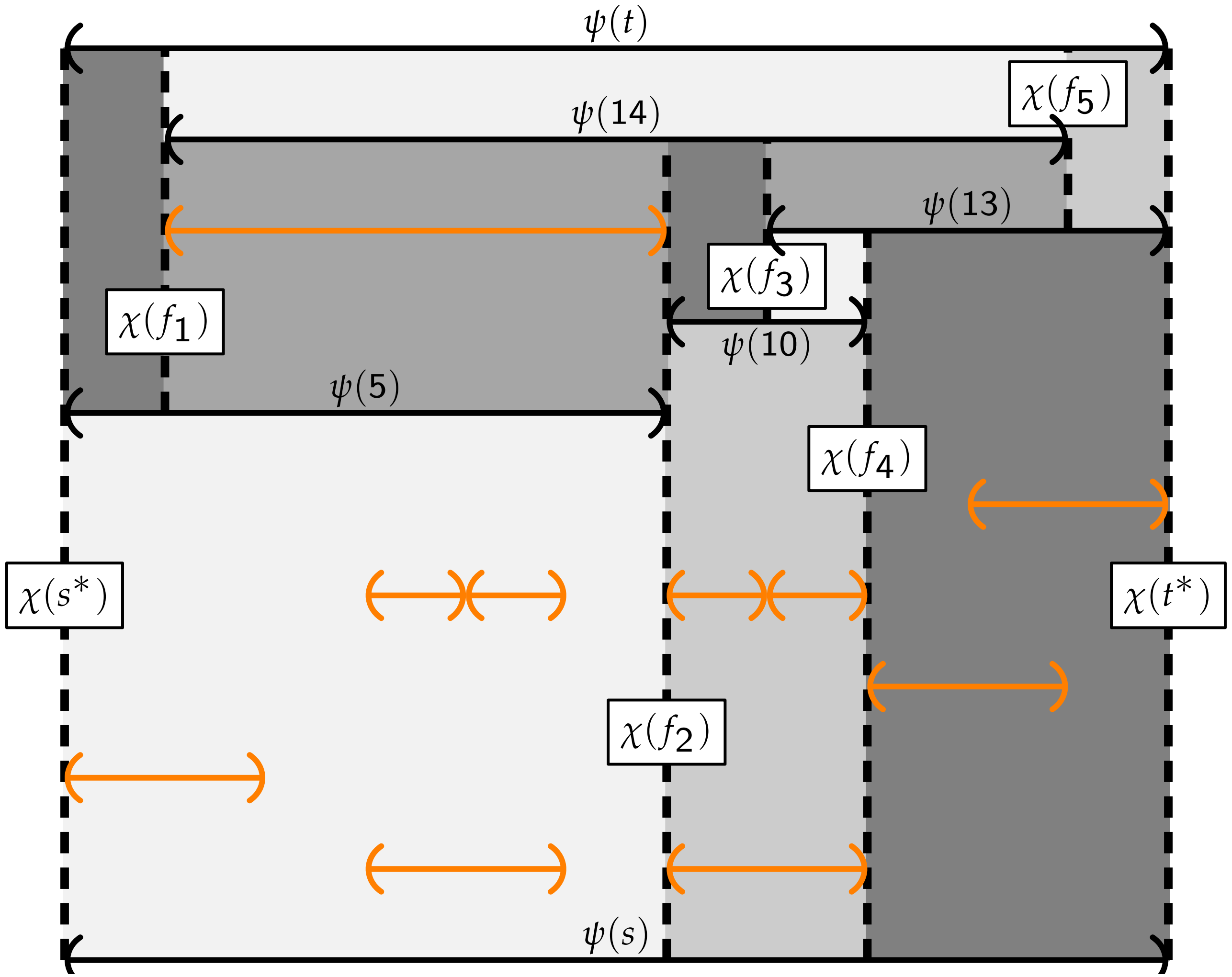


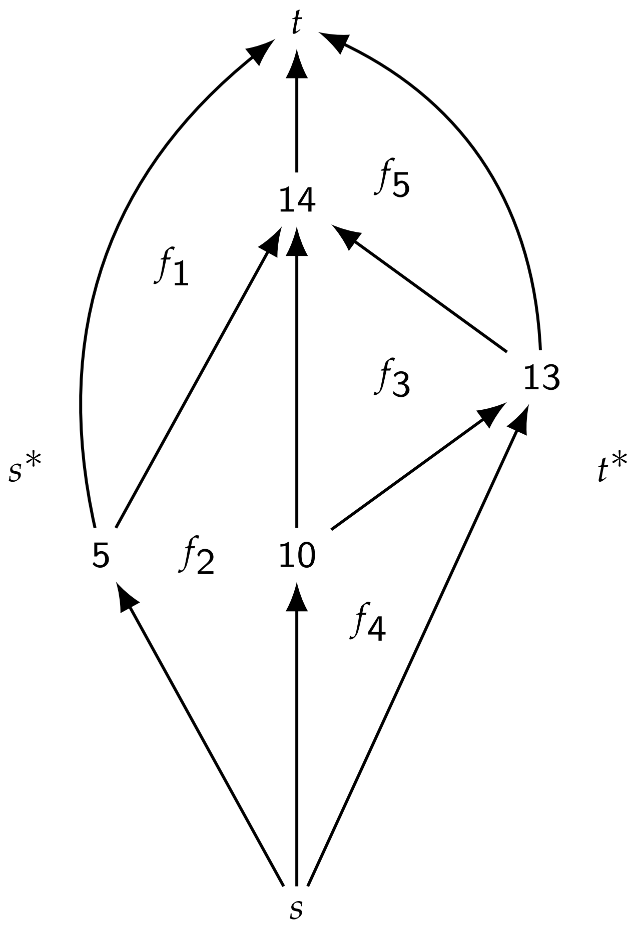
$\psi(t)$
$\chi(f_5)$
$\psi(14)$
$\psi(13)$
$\chi(f_3)$
$\chi(f_1)$
$\psi(10)$
$\psi(5)$
$\chi(f_4)$
$\chi(s^*)$
$\chi(t^*)$
$\chi(f_2)$
$\psi(s)$



t
14
f_5
f_1
f_3
13
s^*
t^*
5
f_2
10
f_4
s

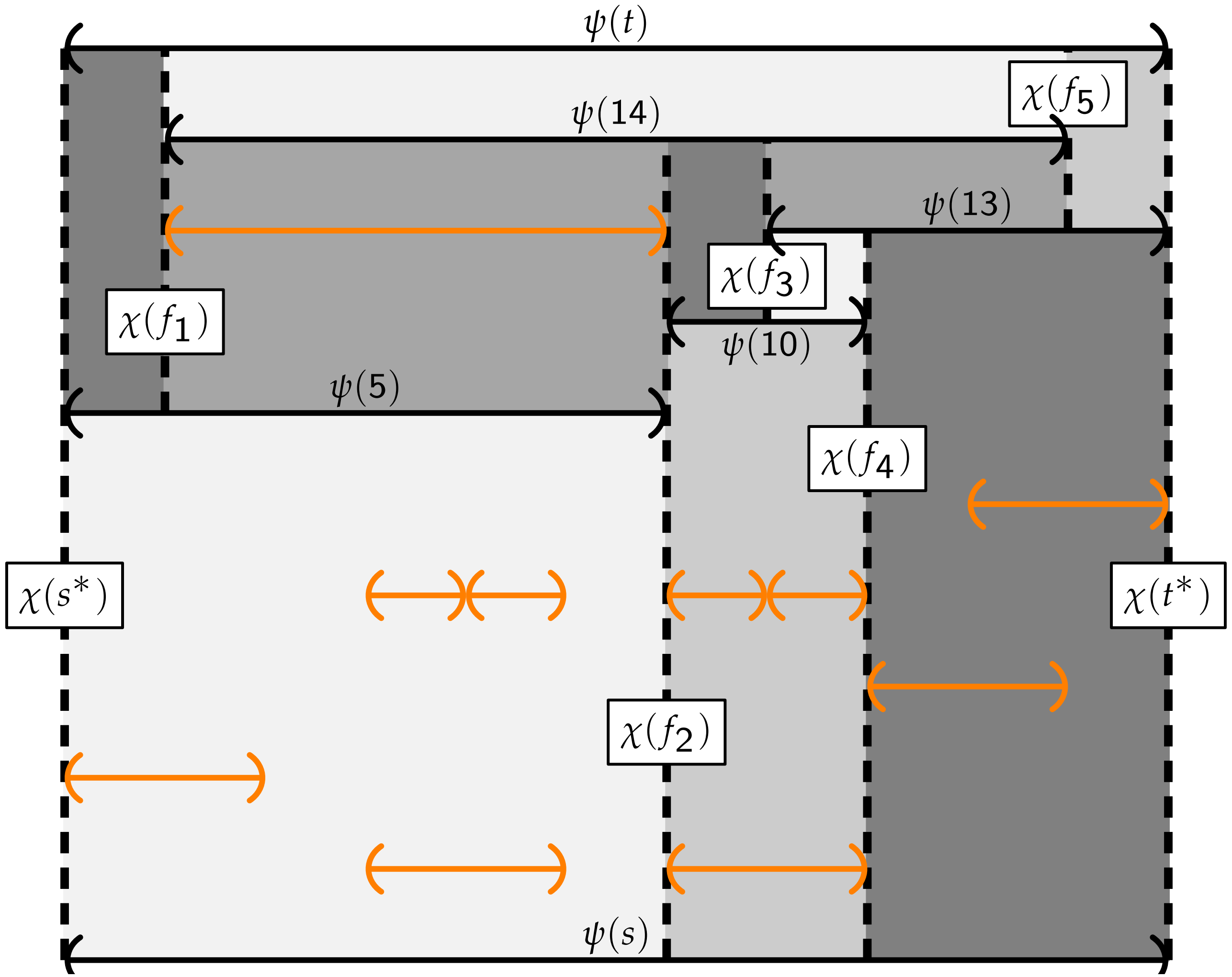
R nodes with 2-SAT formulation

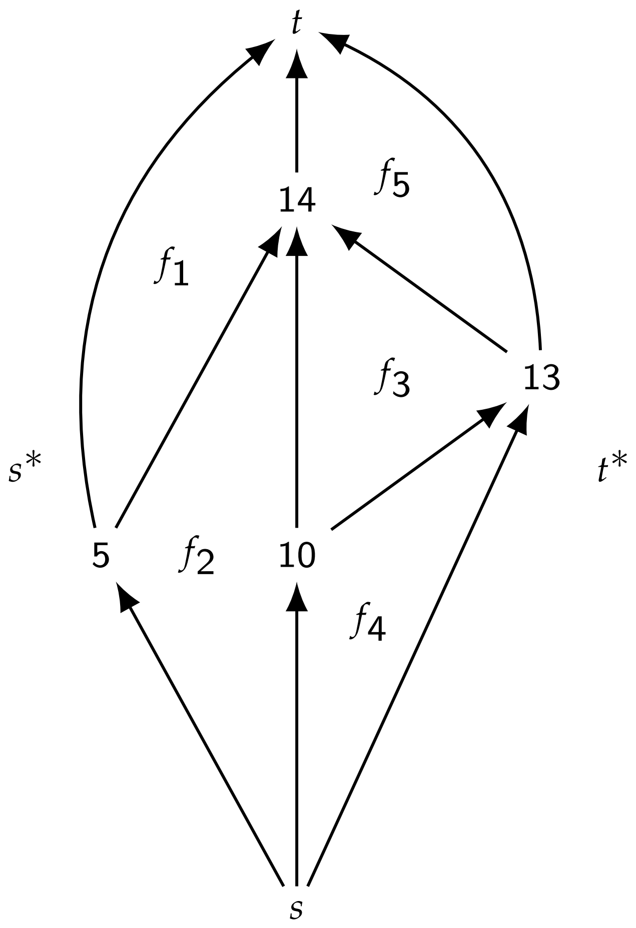




R nodes with 2-SAT formulation

- for each child
  - 2 variables encoding fixed/loose type of its tile
  - restriction clauses to subsets of {FF,FL,LF,LL}





R nodes with 2-SAT formulation

- for each child
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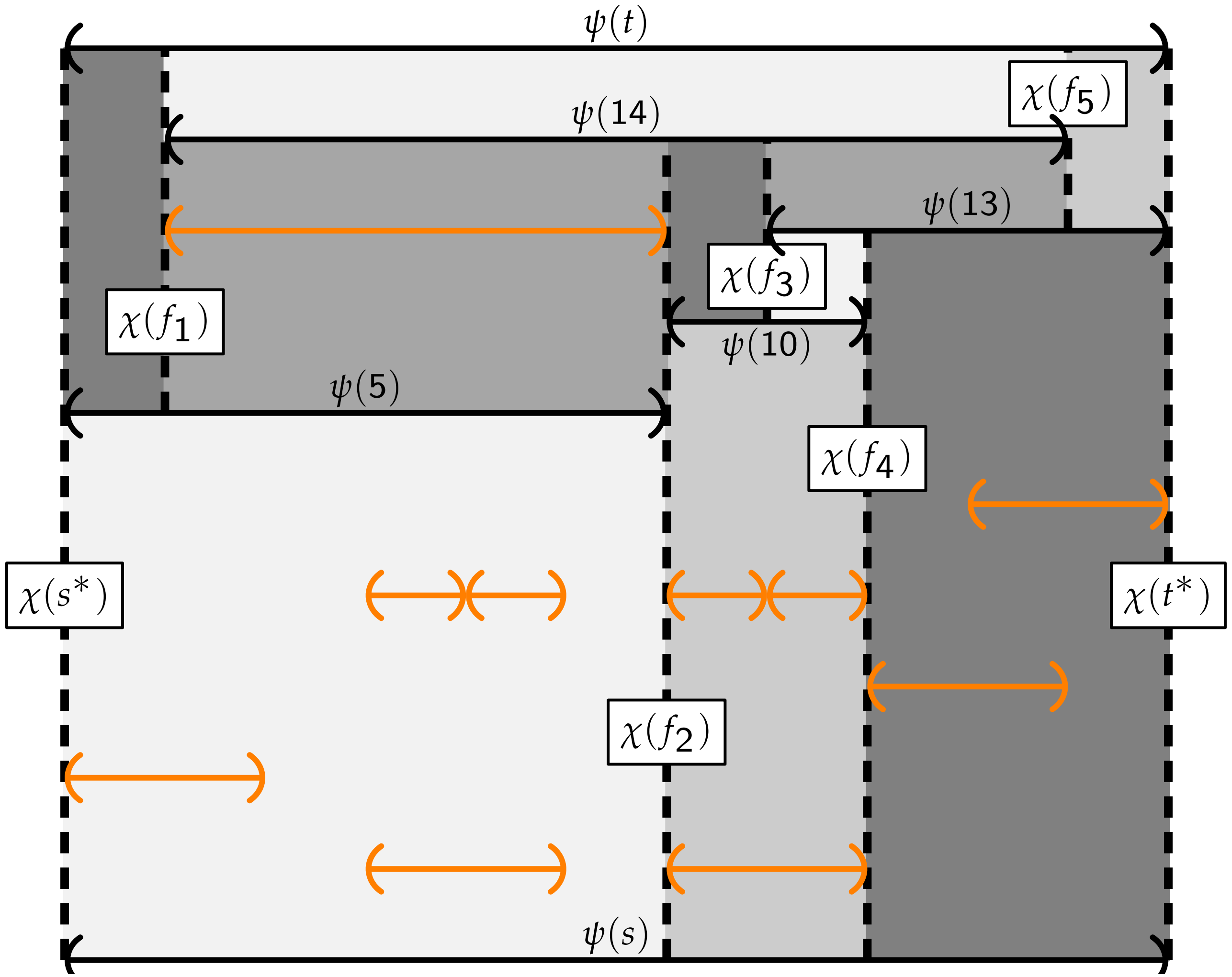
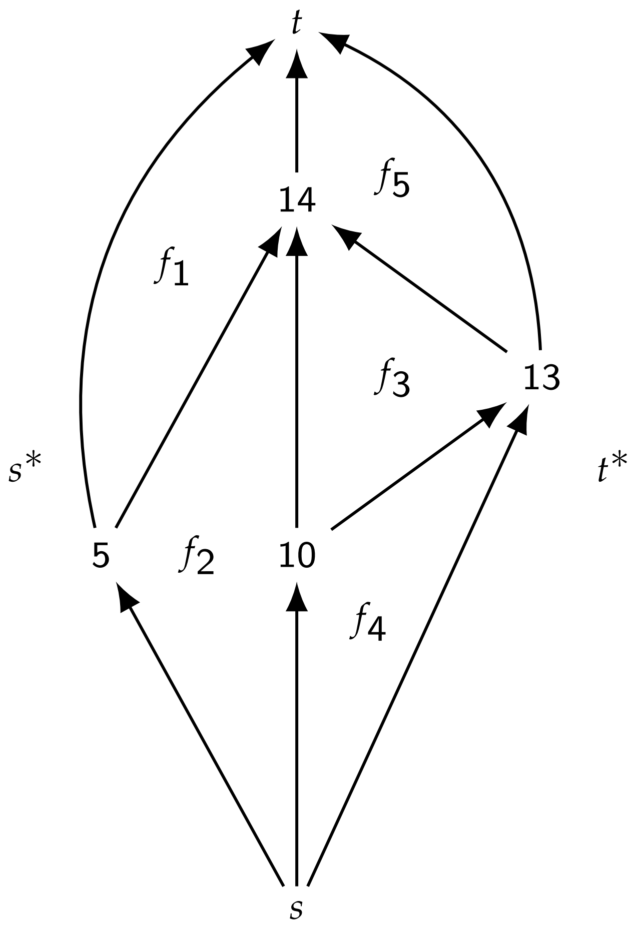
- for each face
  - 2 variables encoding position of the splitting line
  - consistency clauses





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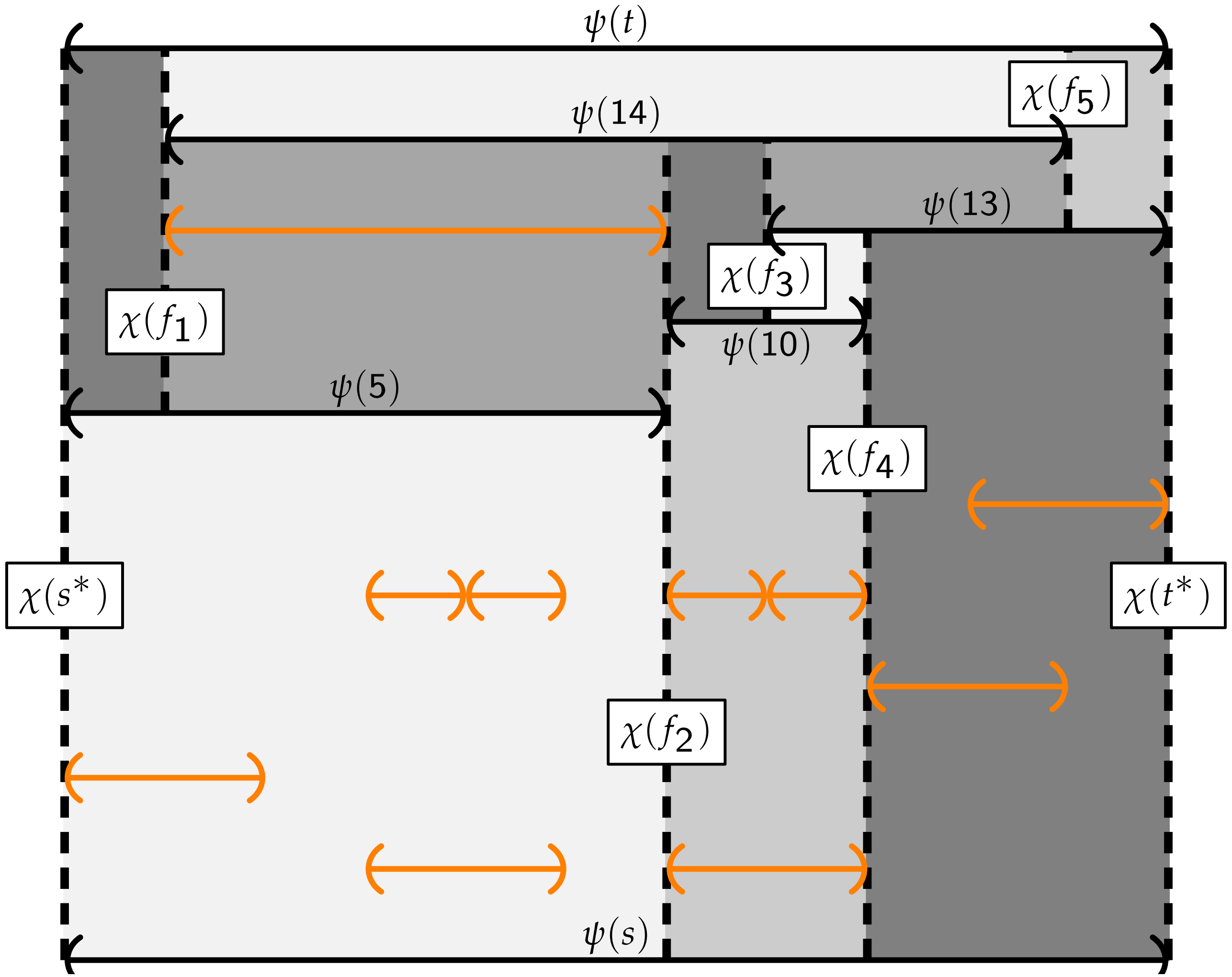

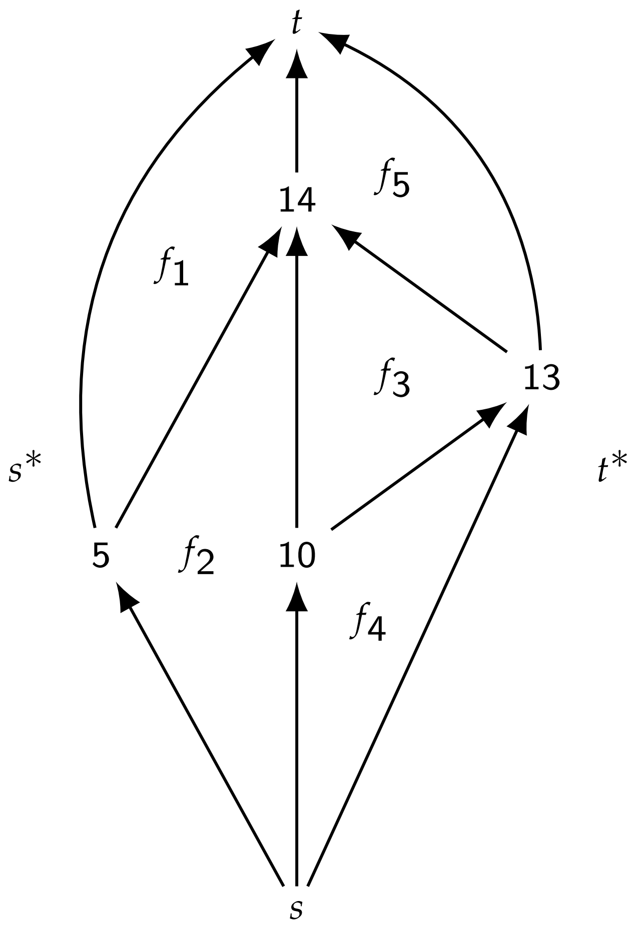

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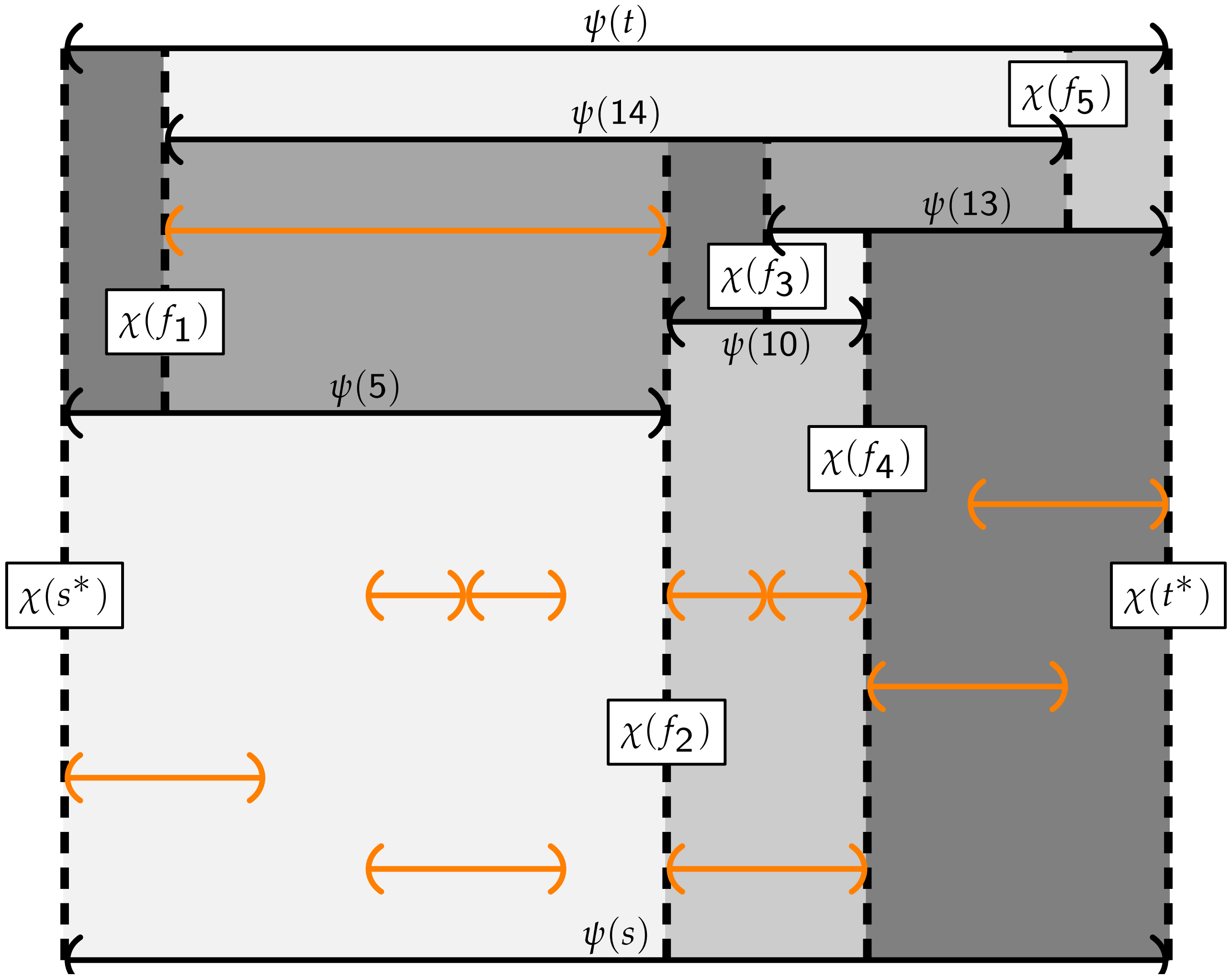
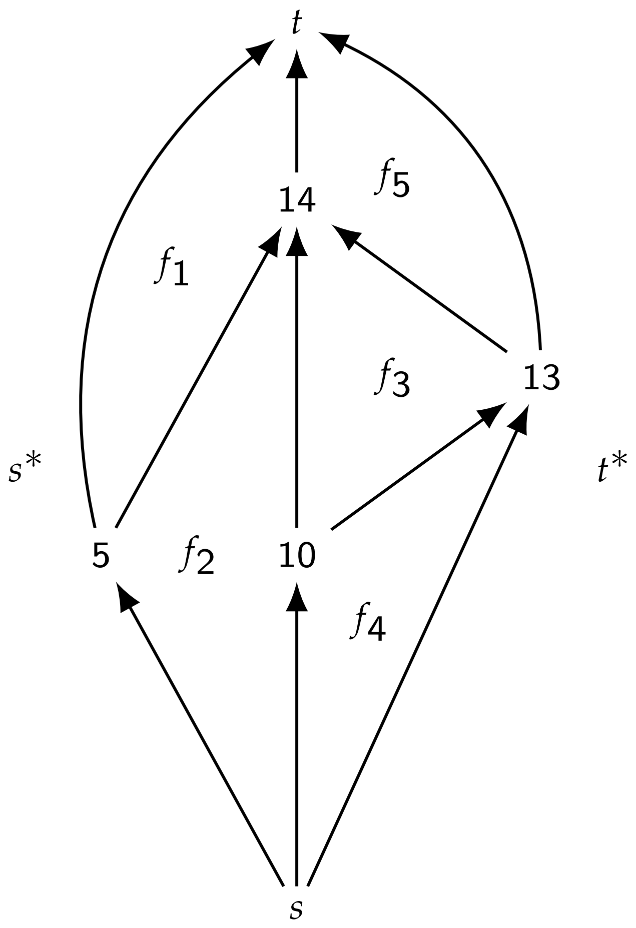
    - quadratically many
    - tricky part: use only $O(n \log^2 n)$ clauses





NP-hardness of RepExt in general case

Theorem 2.
ε-Bar Visibility Representation Ext. is NP-complete.

- Reduction from Planar Monotone 3-SAT

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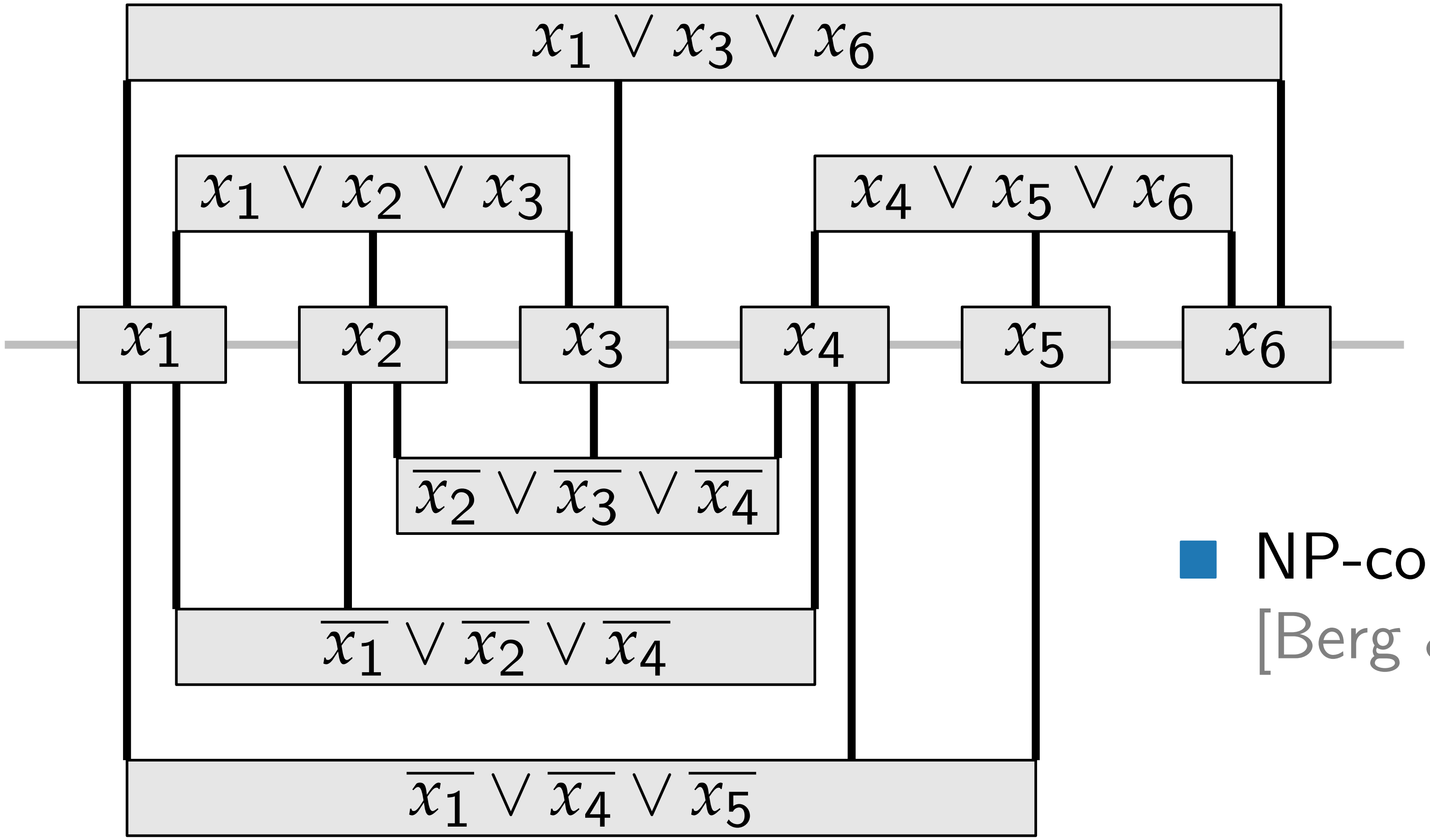
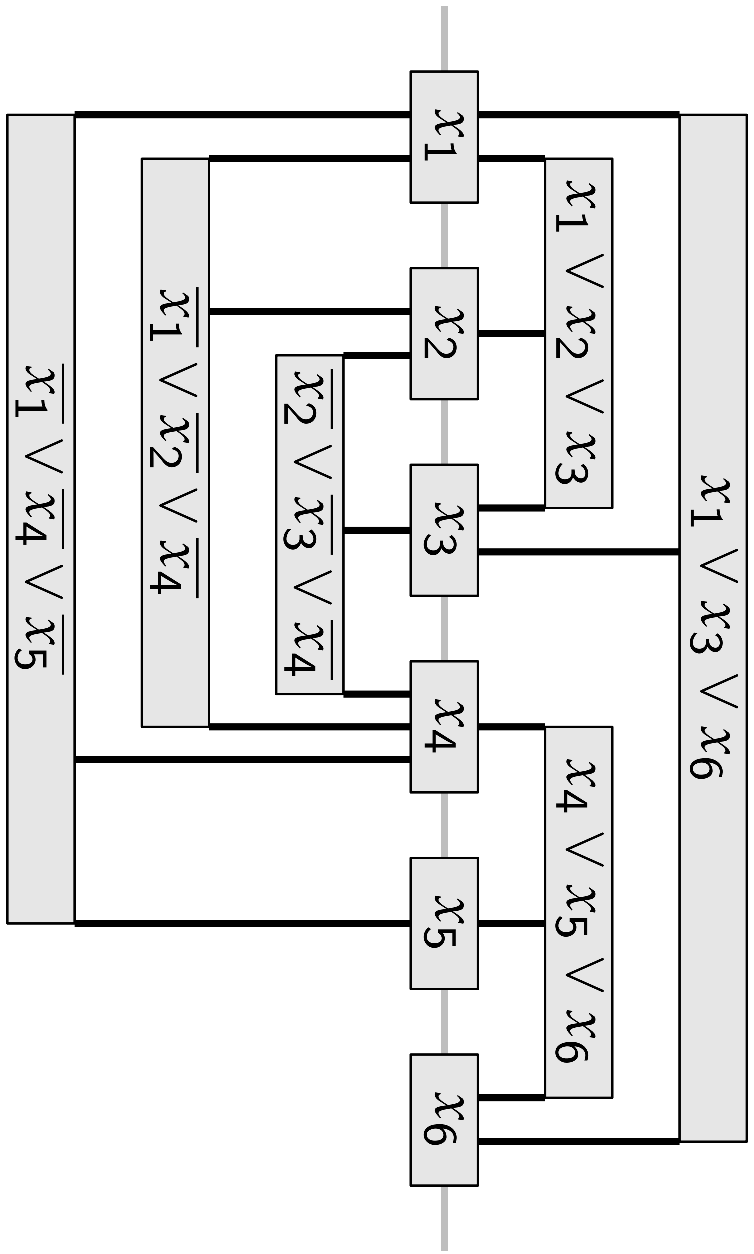


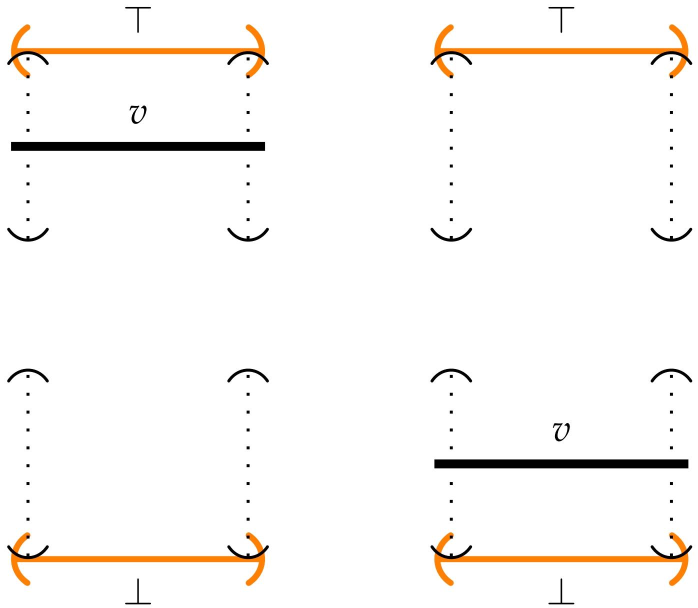
- NP-complete
  [Berg & Khosravi '10]

NP-hardness of RepExt in general case



Wire Transmission

transmitting
true and false



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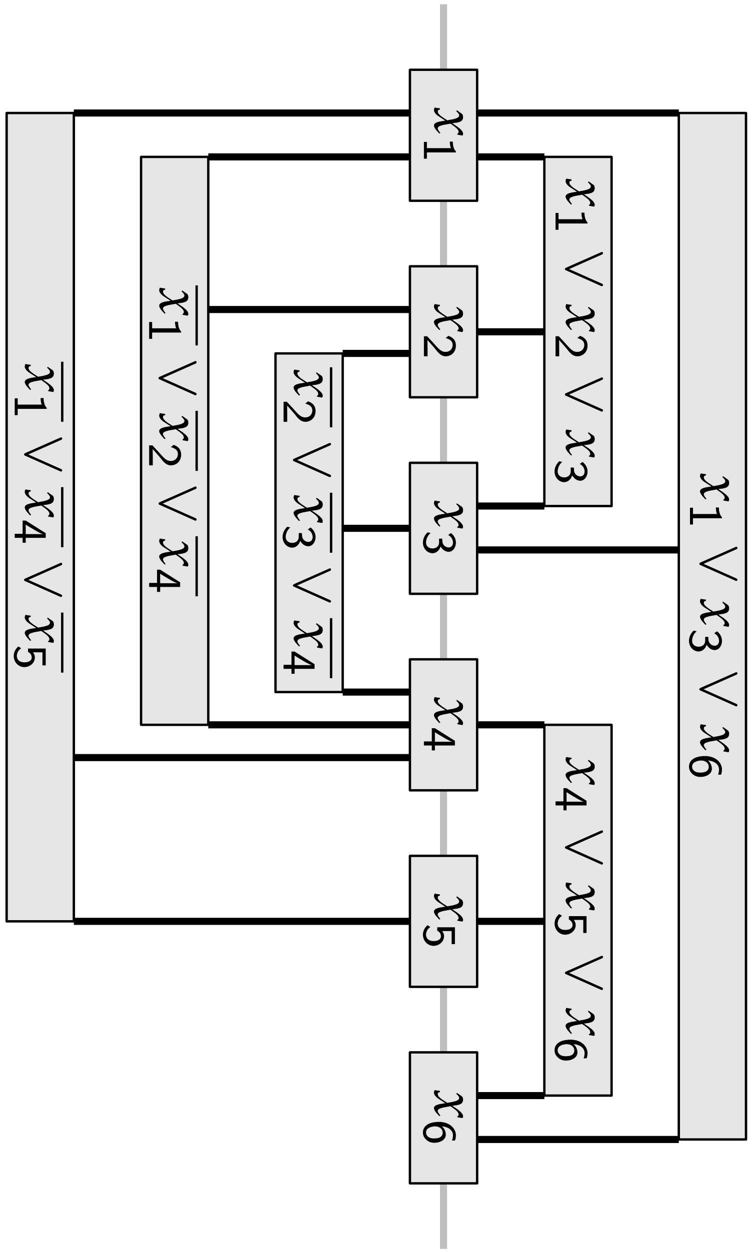

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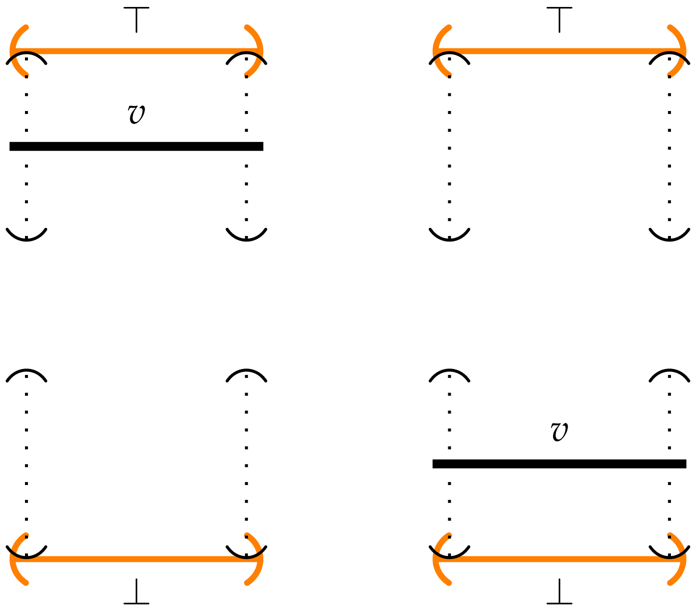
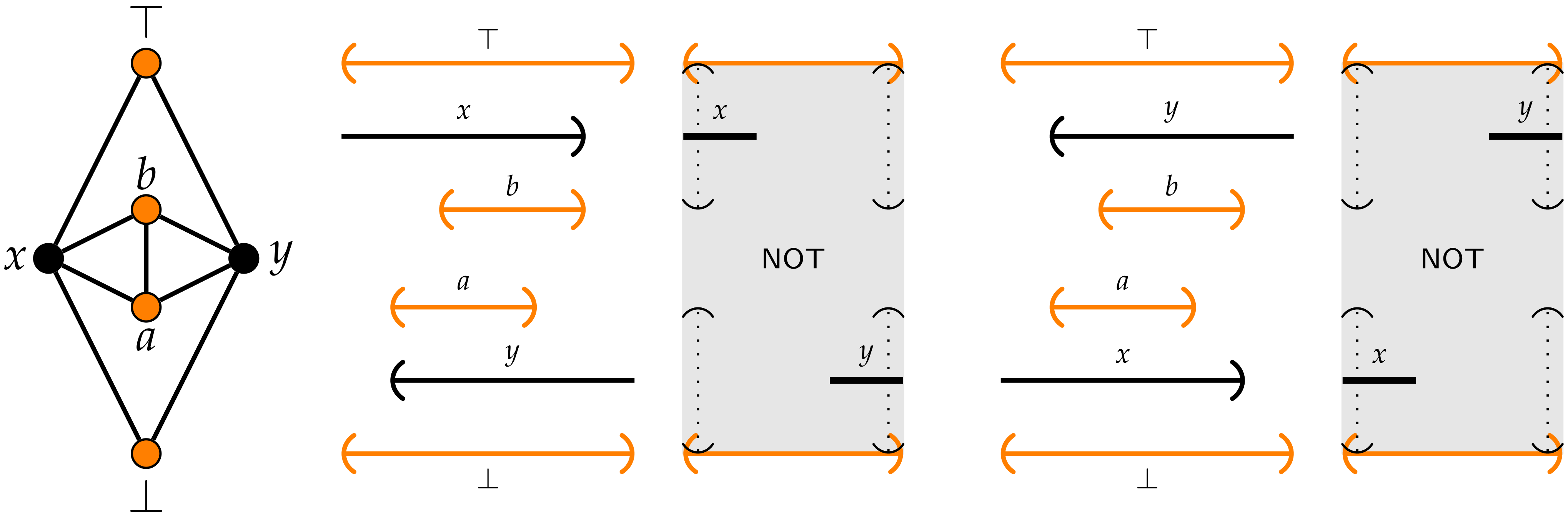


Remark. The following details omit the copying gadgets used for multiple occurrences of the variables

NP-hardness of RepExt in general case

NOT gate



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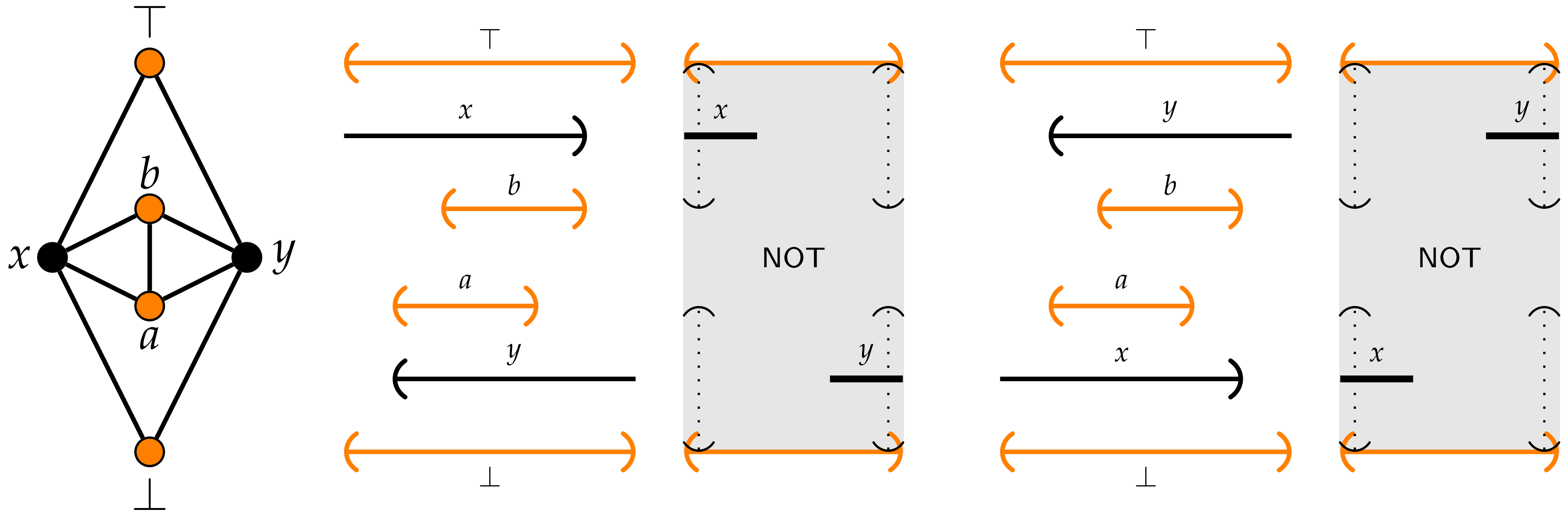
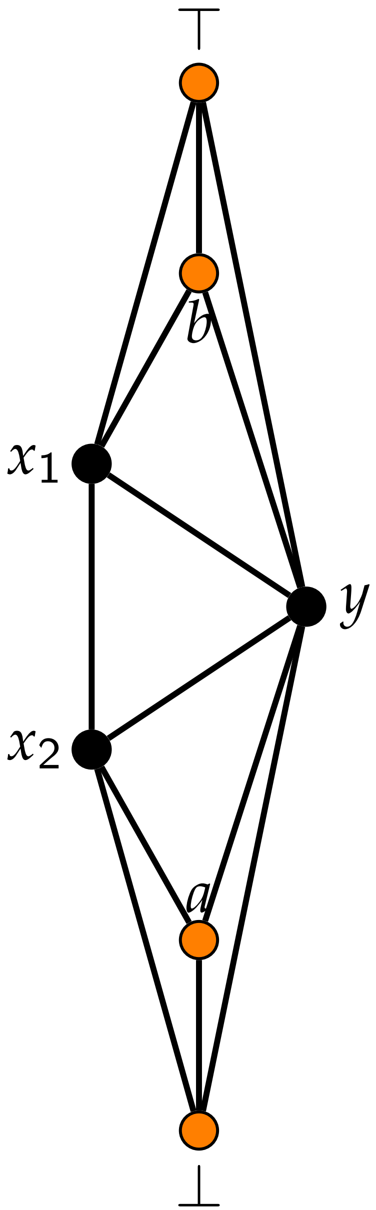
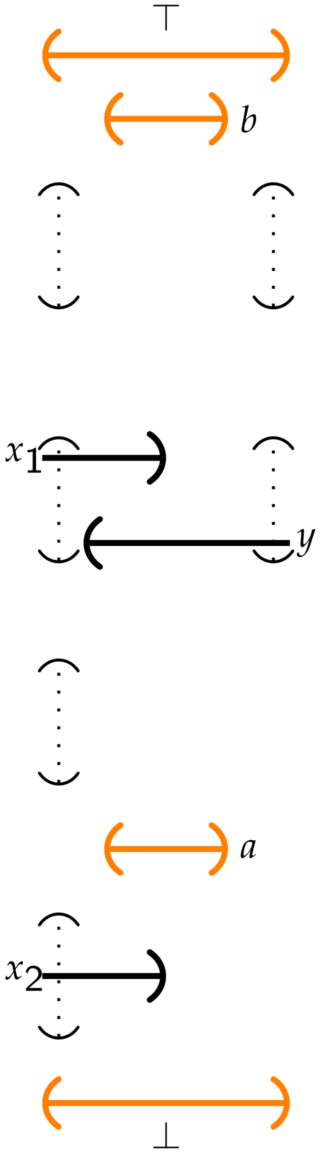


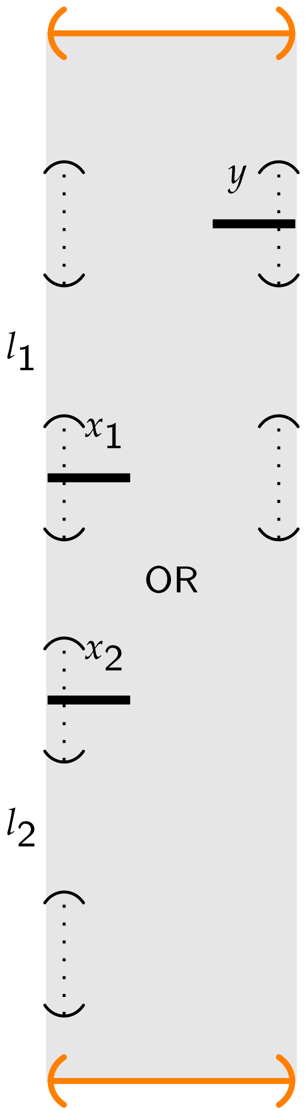
Note: the bars of x and y cannot occur between a and b since a and b are not supposed to be adjacent to either of $\bot$ and $\top$

NP-hardness of RepExt in general case

OR gate







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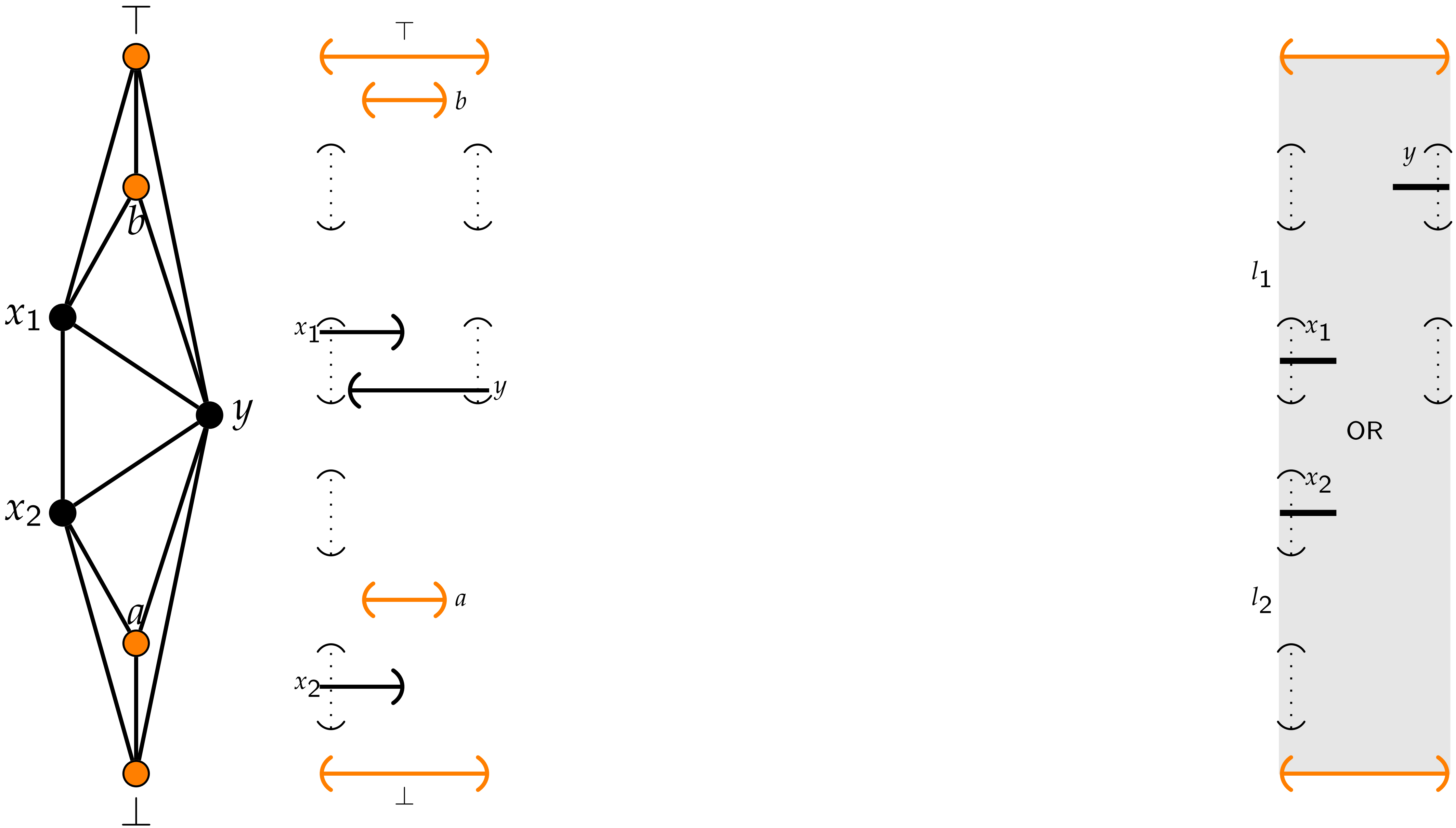

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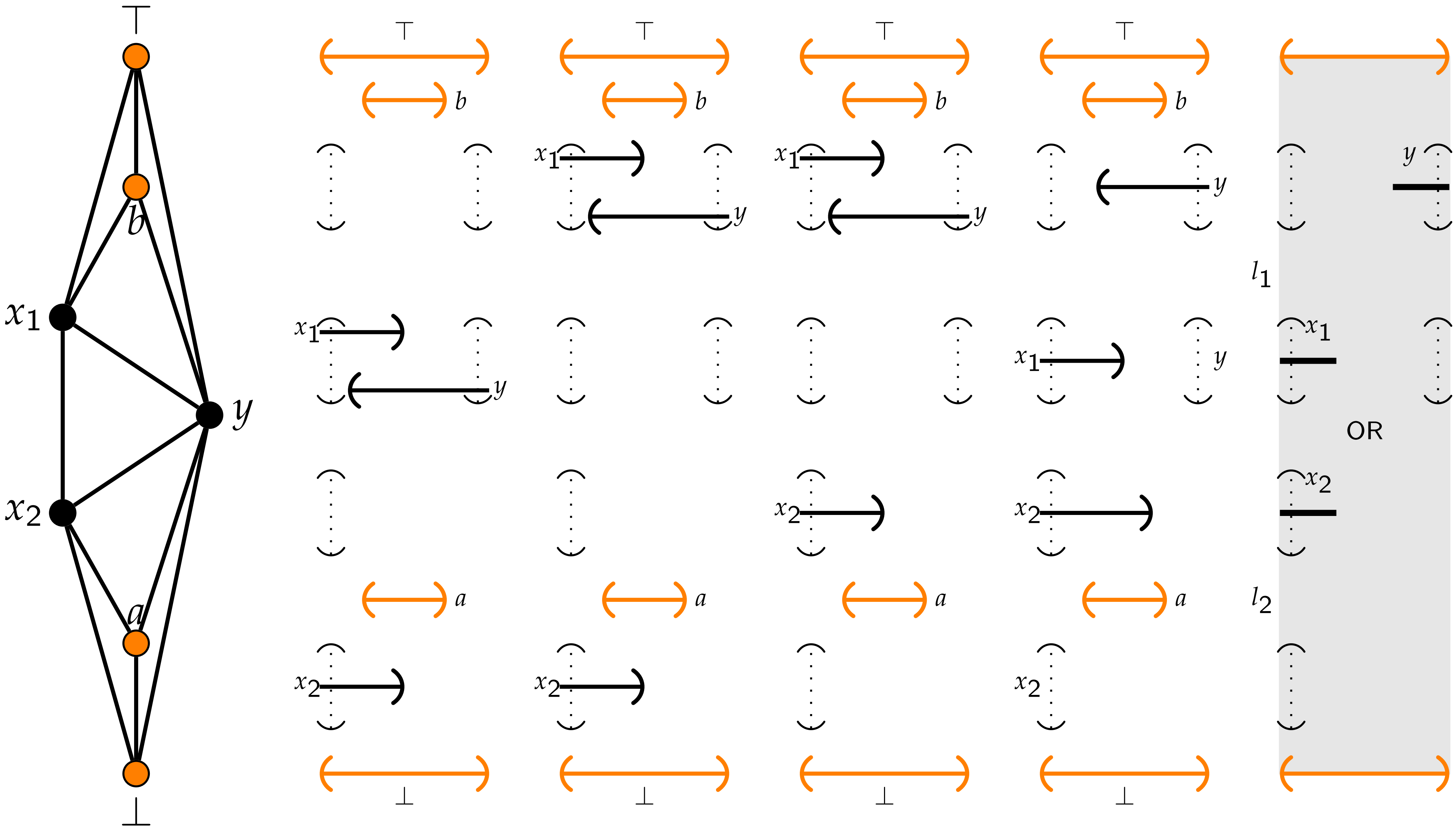

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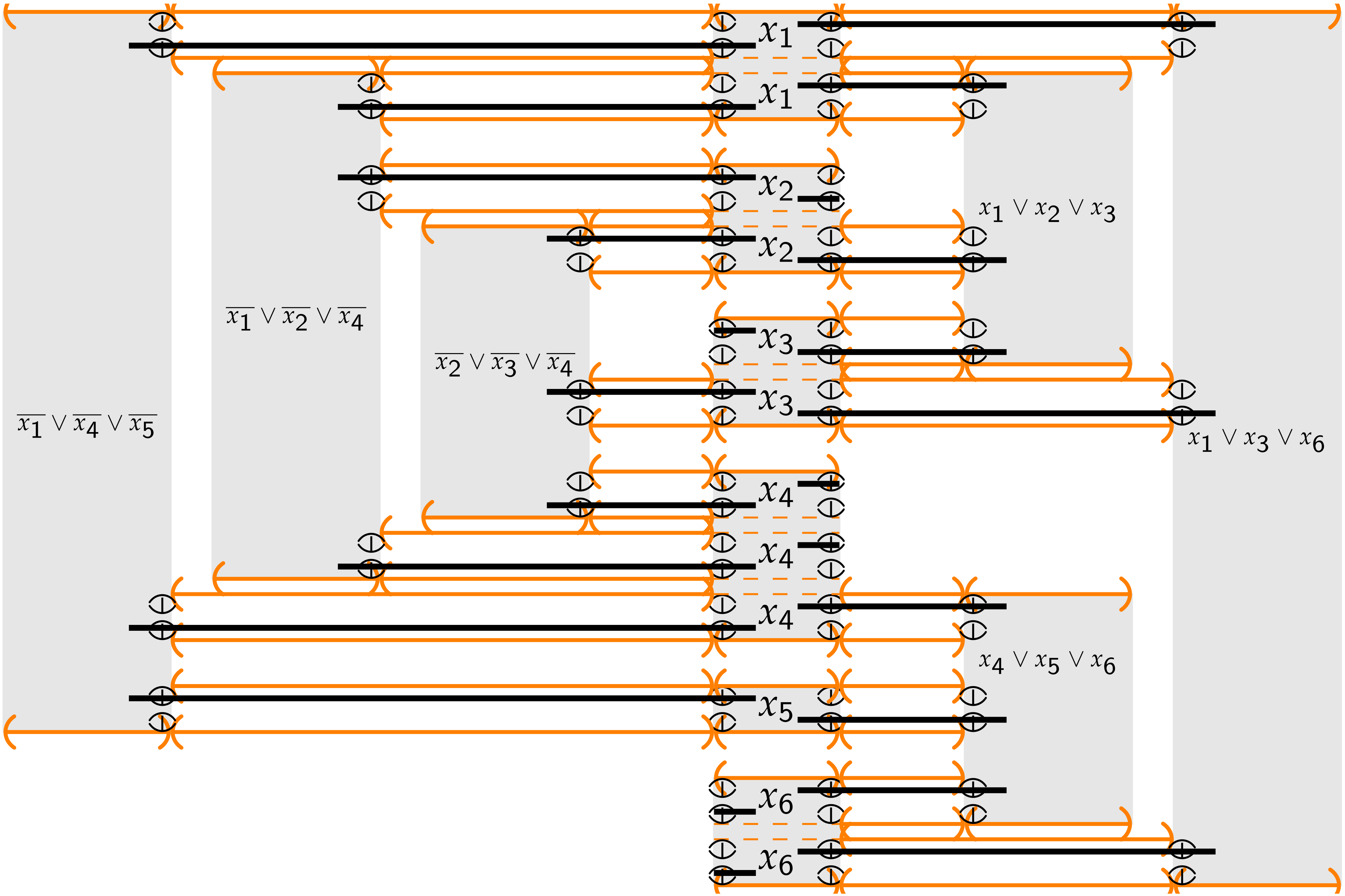

NP-hardness on the Integer Grid (or fixed ε)

Theorem 3.
ε-Bar Visibility Representation Ext. is NP-complete for (series-parallel) st-graphs when restricted to the **integer grid** (or if any fixed $\varepsilon > 0$ is specified).

- Reduction from 3-Partition

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3-Partition.
Input: A set of positive integers $w, a_1, a_2, \ldots, a_{3m}$ such that for each $i = 1, \ldots, 3m$, we have $\frac{w}{4} < a_i < \frac{w}{2}$.
Question: Can $\{a_1, \ldots, a_{3m}\}$ be partitioned into m triples such that the total sum of each triple is exactly w?

- Strongly NP-complete [Garey & Johnson '79]

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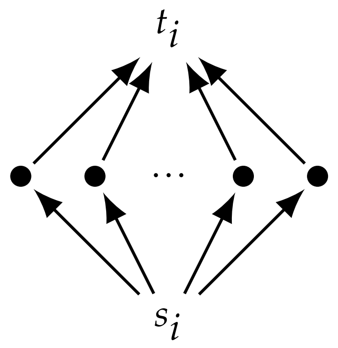
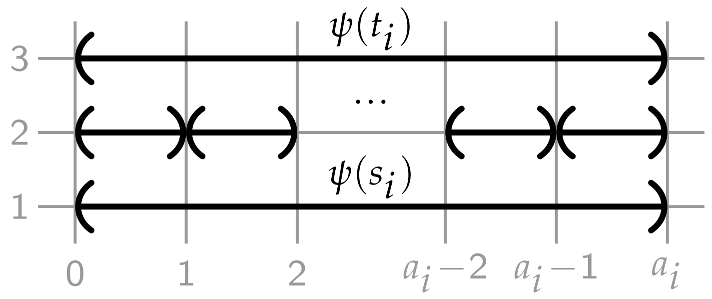
$a_i \rightarrow$  H_i



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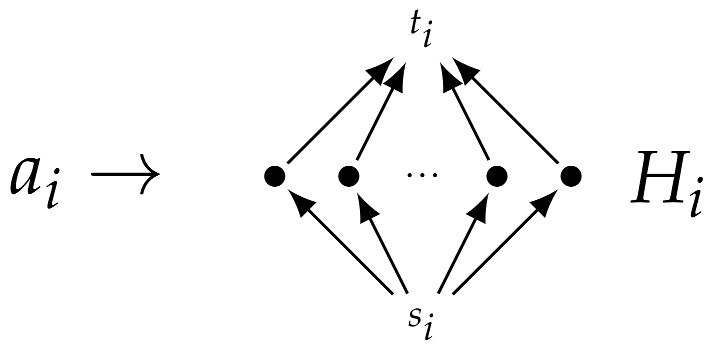

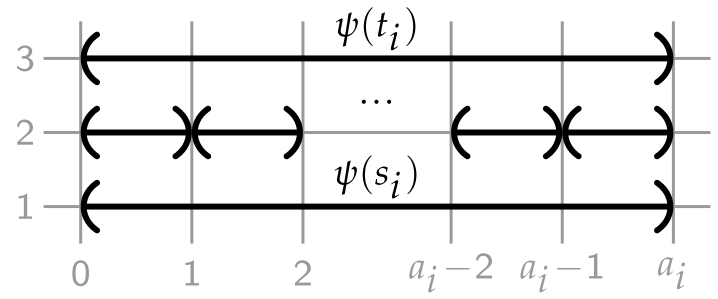

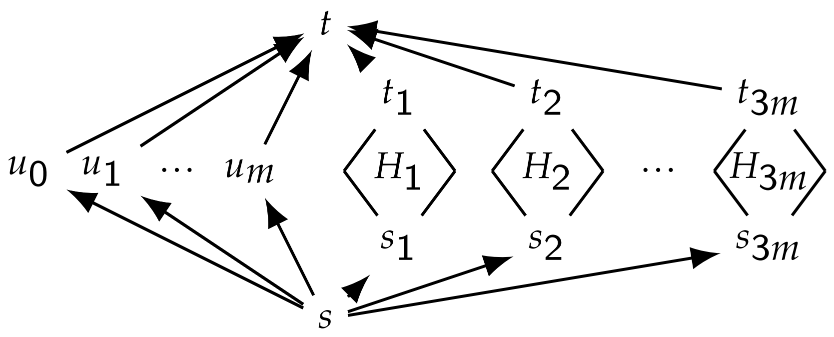

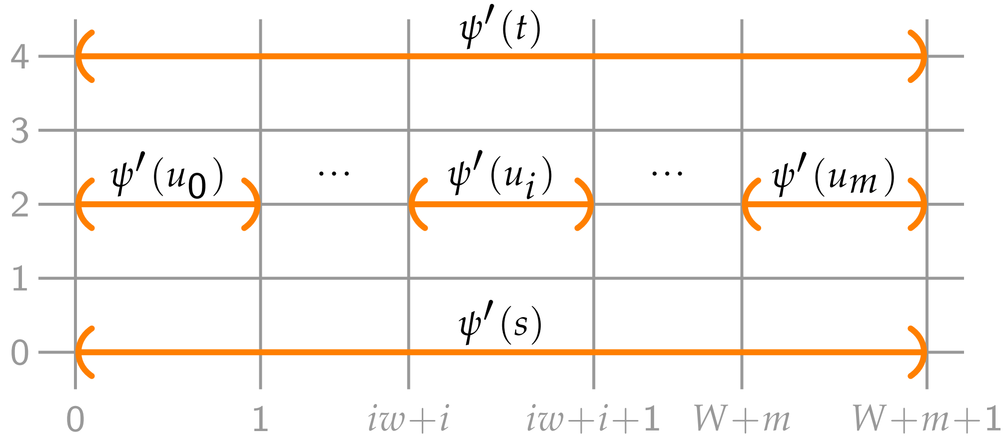

Discussion

- *rectangular* ε-Bar Visibility Representation Extension can be solved in $O(n \log^2 n)$ time for st-graphs.
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Open Problems:

- Can ~~*rectangular*~~ ε-Bar Visibility Representation Extension can be solved in polynomial time on st-graphs? DAGs?
- Can **Strong** Bar Visibility Recognition / Representation Extension can be solved in polynomial time on st-graphs?

Literature

Main source:

- [Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18] The Partial Visibility Representation Extension Problem

Referenced papers:

- [Gutwenger, Mutzel '01] A Linear Time Implementation of SPQR-Trees
- [Wismath '85] Characterizing bar line-of-sight graphs
- [Tamassia, Tollis '86] Algorithms for visibility representations of planar graphs
- [Andreae '92] Some results on visibility graphs
- [Chaplick, Dorbec, Kratchovíl, Montassier, Stacho '14] Contact representations of planar graphs: Extending a partial representation is hard