

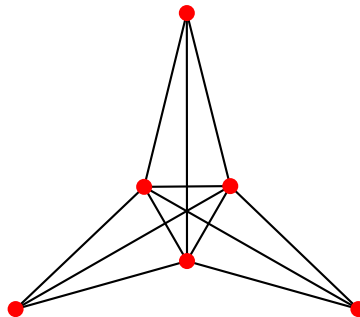
Exercise Sheet #9

Graph Visualization (SS 2026)

Exercise 1 – Finding a crossing number

What is the crossing number of the graph below and why?

3 Points



Exercise 2 – Adding an edge with a minimum number of new crossings

Suppose that you are given a planar drawing Γ of a biconnected graph G where each edge is represented by a polygonal line (i.e., a sequence of line segments) with a constant number of bends. Let u and v be a pair of non-adjacent vertices of G .

Devise an efficient algorithm that adds the edge uv to the existing drawing causing as few crossings as possible. The new edge should also be drawn as a polygonal path, but the number of bends is not restricted. Show the correctness of your algorithm. Can you come up with a linear-time implementation?

7 Points

Exercise 3 – Fixed linear crossing number

In the lecture we talked about the problem *fixed linear crossing number*, which is the crossing number for a fixed linear layout: For a graph G with given vertex numbering $V(G) = \{v_1, v_2, \dots, v_n\}$, vertex v_i has position $(i, 0)$ and every edge is drawn as a semi-circle. The only decision when drawing an edge is, therefore, whether it is drawn in the halfplane above or below the x -axis. Given a graph G with numbered vertices and an integer k , it is NP-hard to decide whether a fixed linear layout with at most k crossings exists.

- a) Devise an algorithm that decides for a graph with given vertex numbering whether a fixed linear layout with zero crossings exists. Can you implement your algorithm such that it runs in linear time? **6 Points**
- b) Show that, when restricting the input graphs to matchings (i.e., all vertices have degree 1), the decision problem for the fixed linear crossing number problem is still NP-hard. **4 Points**

Exercise 4 – Tightness of the asymptotic lower bound on the crossing number

To show that the bound $cr(G) \in \Omega(m^3/n^2)$ from the lecture is asymptotically tight, find a family of graphs that contains, for every combination of n and m that makes sense, a graph G with n vertices and m edges that admits a drawing with $O(m^3/n^2)$ crossings.

4 Extra points

Note that the family of complete graphs is *not* a solution since, for every n , there is just one graph in the family, and it has $m \in \Theta(n^2)$ many edges. In other words, all graphs of the family have the same density. You should find a graph family for any (integer) edge density $k = m/n$.

This assignment is due at the beginning of the next lecture, that is, on July 03 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 24 at 16:00 and the solutions will be discussed two weeks after that on July 08.