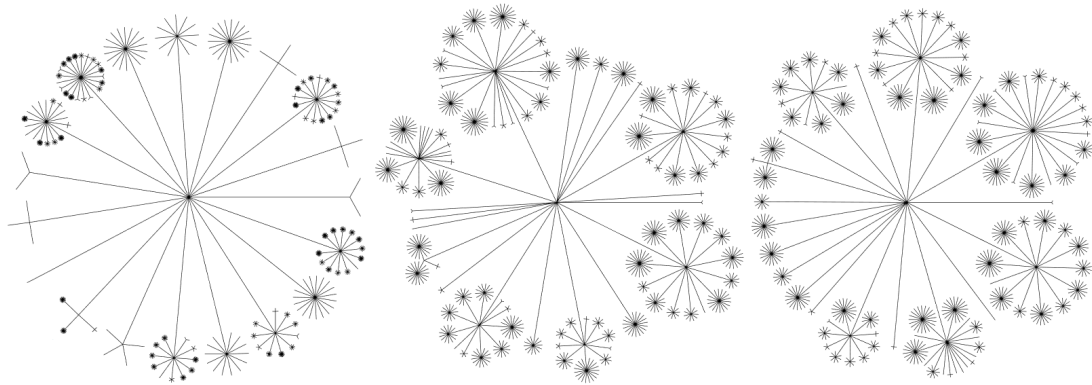


Exercise Sheet #1

Graph Visualization (SS 2026)

Exercise 1 – Drawing conventions & aesthetics of balloon layouts

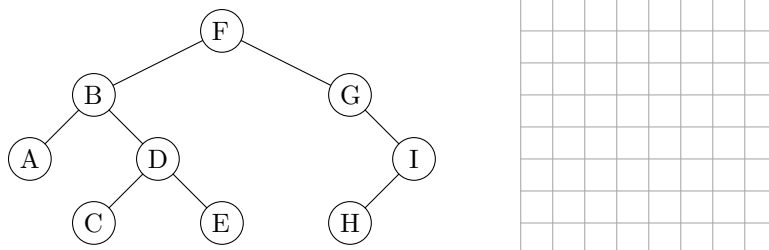
The three drawings of the same tree below are drawn with a *balloon layout*. Try to find at least two common drawing conventions and two possible drawing aesthetics to optimize for this layout style. **2 Points**



Exercise 2 – Binary trees with pre- and postorder coordinates

Let T be a binary tree with root r . For each $v \in V(T)$, let $x(v) := \text{preorder}(v)$ and $y(v) := \text{postorder}(v)$. Recall that $T(v)$ denotes the subtree rooted at v .

You may use the graph and grid below to try an example.



a) Prove that this coordinate assignment yields a planar drawing of T . **4 Points**

b) Give tight bounds on the area requirement of the generated drawing.

2 Points

- c) Prove that if you direct all edges of T such that they “point away” from r – that is, all vertices can be reached from r – then all arcs in the drawing point downwards. **2 Points**

Exercise 3 – Lower bound on the area of right-heavy HV-drawings

Prove that there are trees for which the right-heavy HV-layout algorithm from the lecture produces drawings with area $\Omega(n \log n)$, where n is the number of vertices in the tree. **4 Points**

Exercise 4 – Space-saving HV-drawings of complete binary trees

Let T be a *complete binary tree* of height h , that is, a binary tree where all vertices of depth $0, \dots, h - 1$ have exactly 2 children and all vertices of depth h are leaves. Consider the following HV-drawing algorithm.

Algorithm 1: BalancedHVDDraw(node v , height h)

```

if  $v == \text{nil}$  then return  $\emptyset$ 
 $v_l \leftarrow$  left child of  $v$ 
 $v_r \leftarrow$  right child of  $v$ 
 $\Gamma_1 \leftarrow$  BalancedHVDDraw( $v_l$ ,  $h - 1$ )
 $\Gamma_2 \leftarrow$  BalancedHVDDraw( $v_r$ ,  $h - 1$ )
if  $h$  odd then return horizontal combination of  $\Gamma_1$  and  $\Gamma_2$ 
if  $h$  even then return vertical combination of  $\Gamma_1$  and  $\Gamma_2$ 

```

- a) Prove that BalancedHVDDraw produces a drawing of T with area $O(n)$.
*Hint: use induction on the height of the tree with the following hypothesis. The area (width \times height) of the drawing for odd height is $(2\sqrt{n+1} - 3) \times (\frac{3}{2}\sqrt{n+1} - 2)$ and for even height is $(\sqrt{2(n+1)} - 2) \times (\frac{3}{2}\sqrt{2(n+1)} - 3)$. **4 Points***
- b) Give tight constant upper and lower bounds on the aspect ratio (i.e., the ratio between the width and the height) of the drawing generated with input T . **2 Points**

This assignment is due at the beginning of the next lecture, that is, on April 24 at 10:15 am. Please submit your solutions via WueCampus. The solutions will be discussed on April 29.