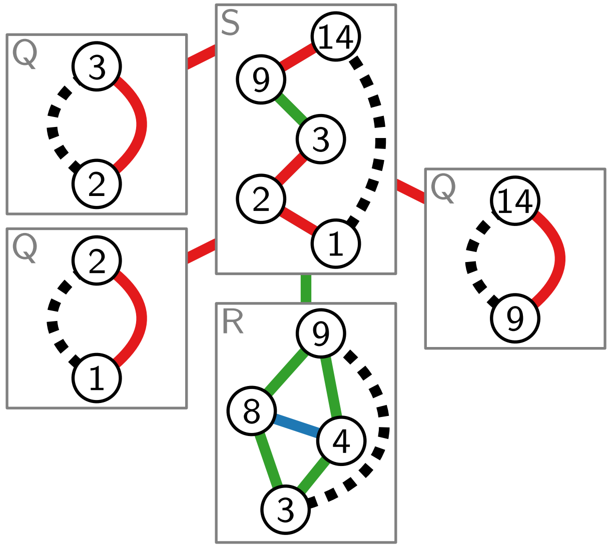
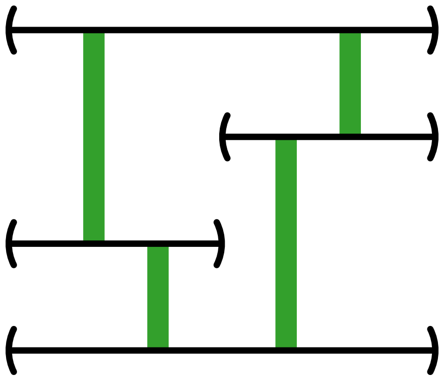
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Visualization of Graphs

Lecture 10:
Partial Visibility Representation Extension



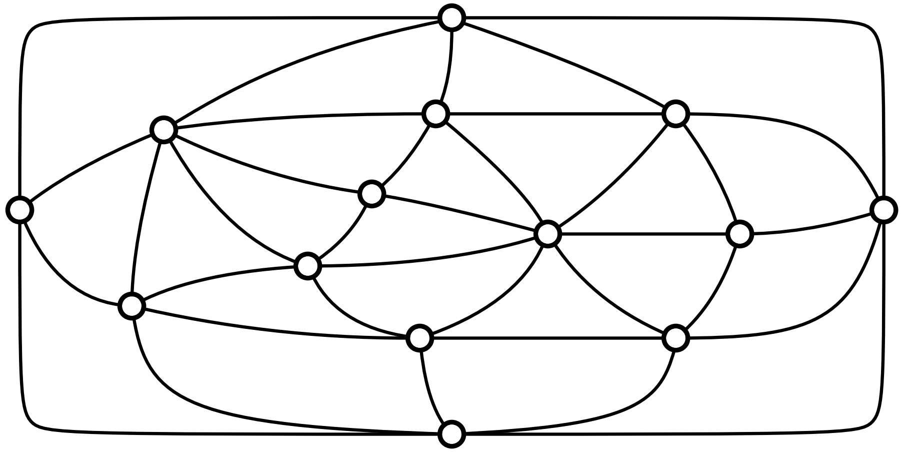


Alexander Wolff

Summer term 2026

Partial Representation Extension Problem

Let G be a graph.



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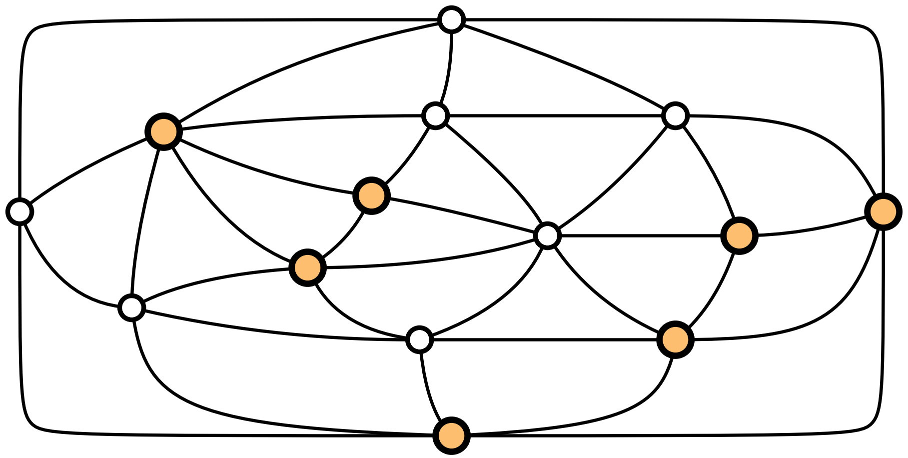
Let $V' \subseteq V(G)$



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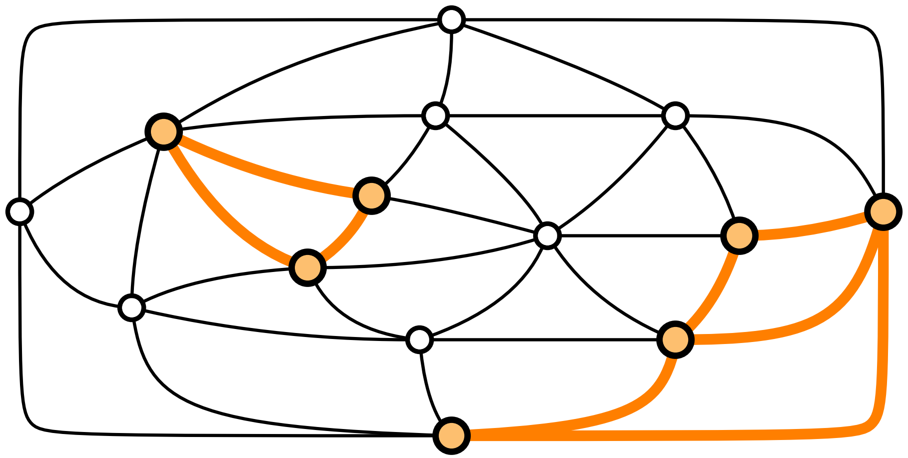

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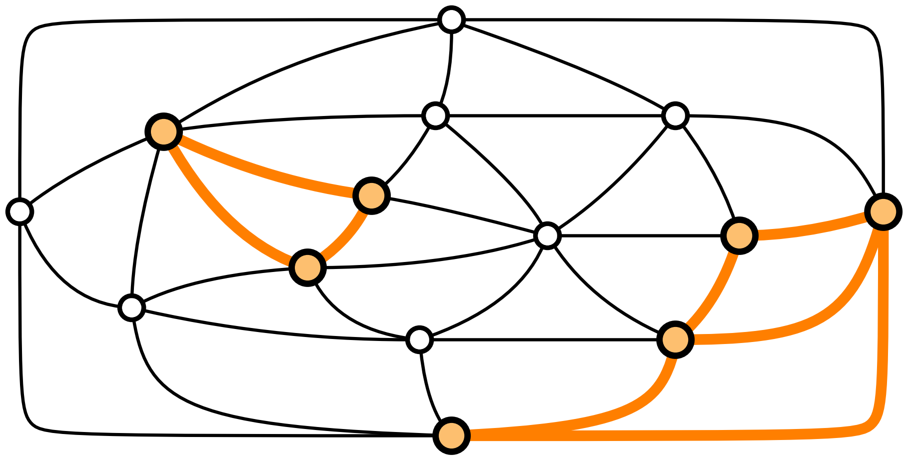

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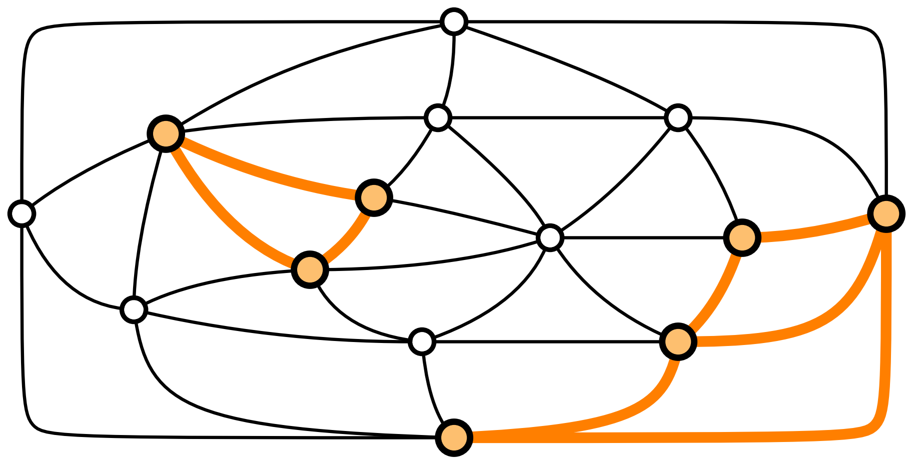

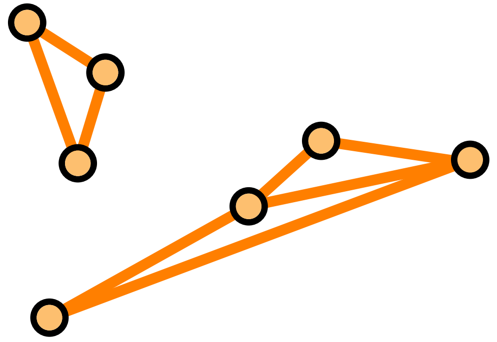

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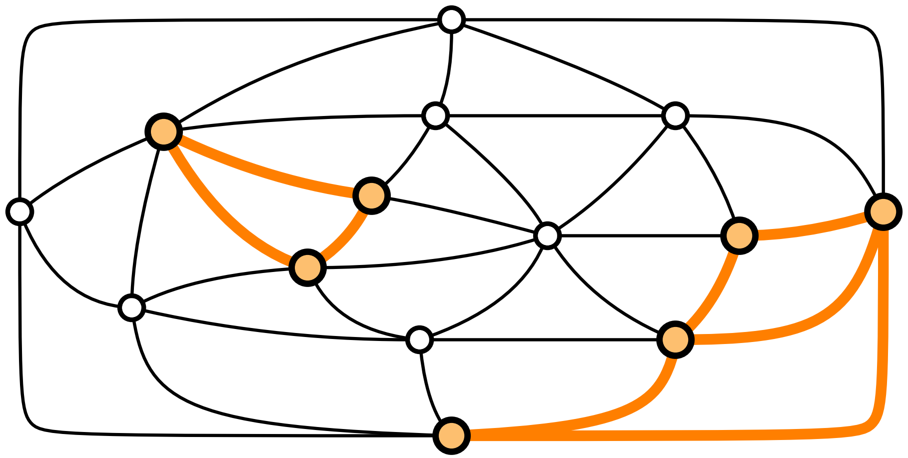

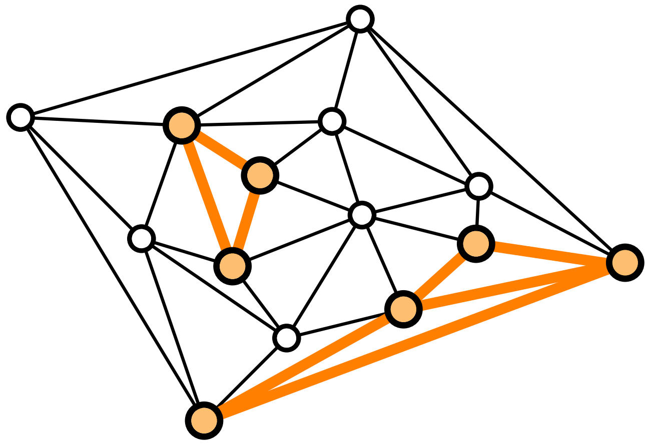

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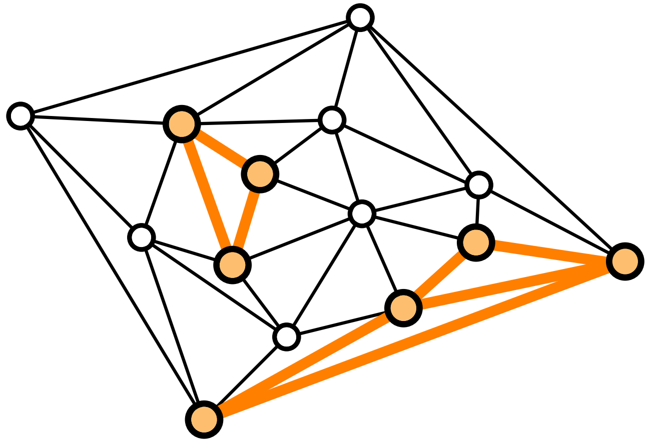

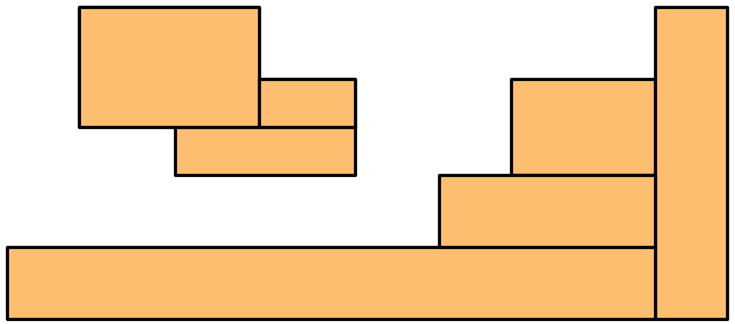

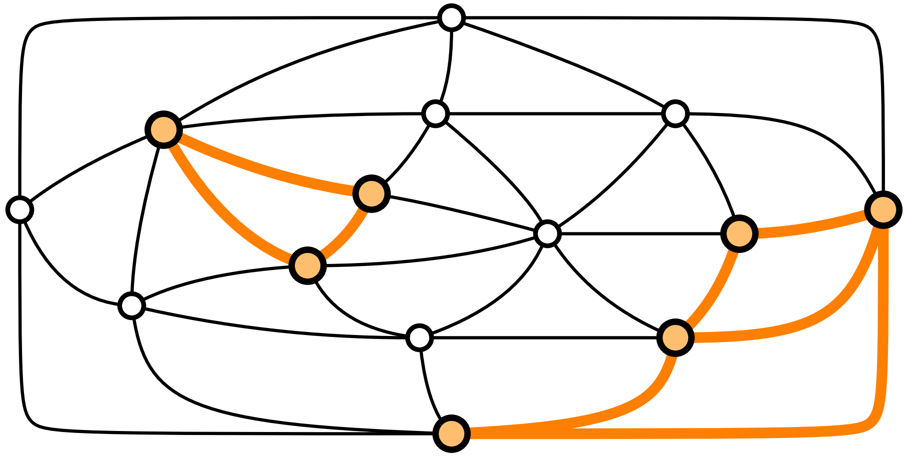

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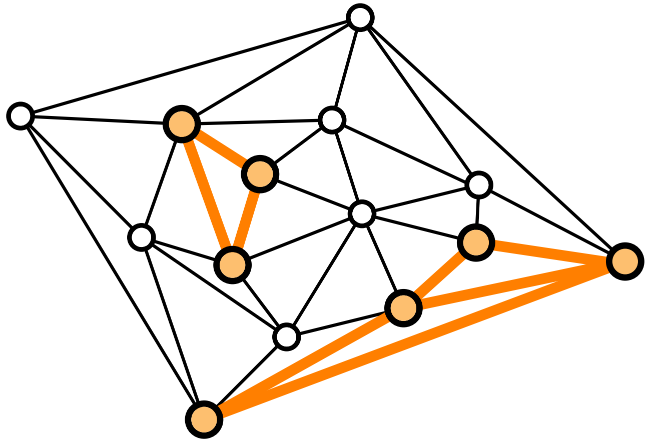

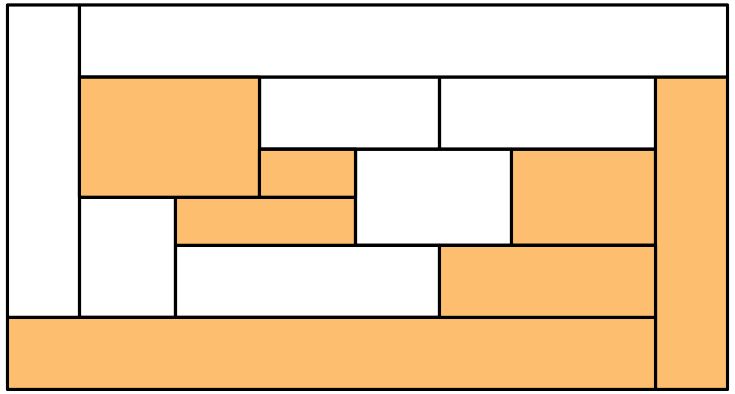

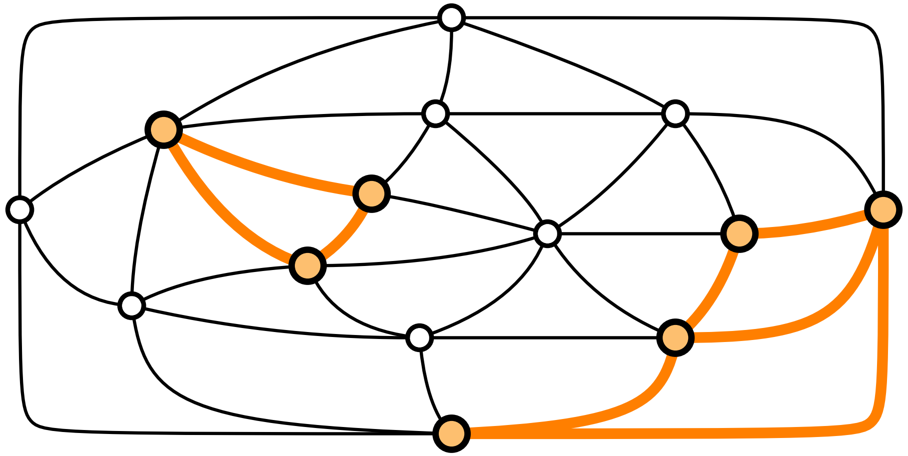

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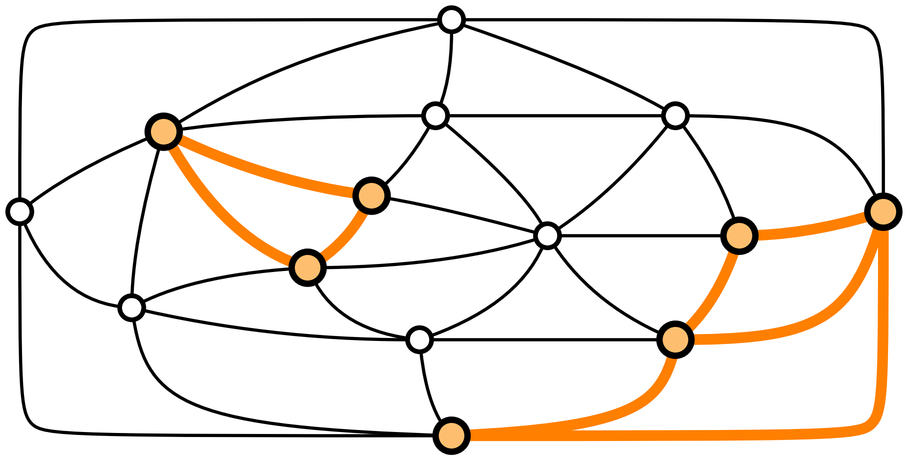

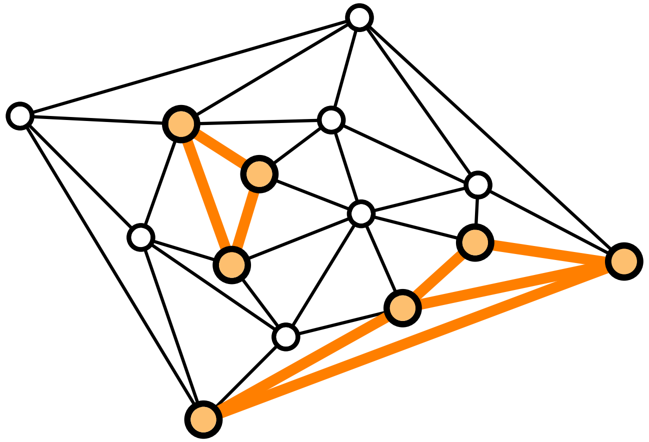

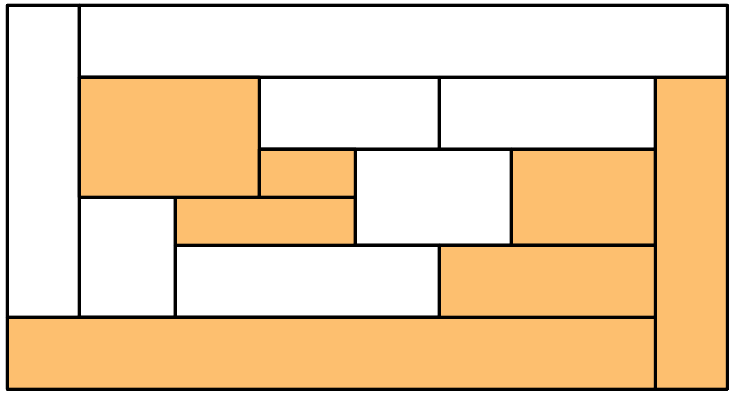

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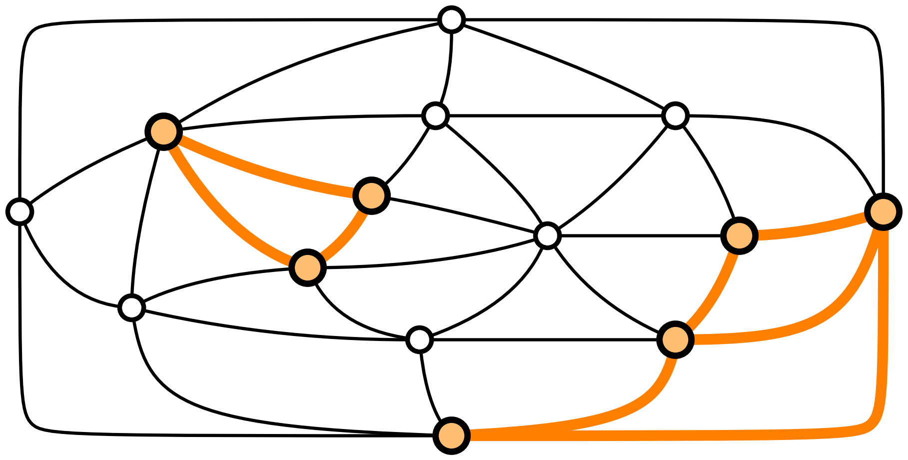

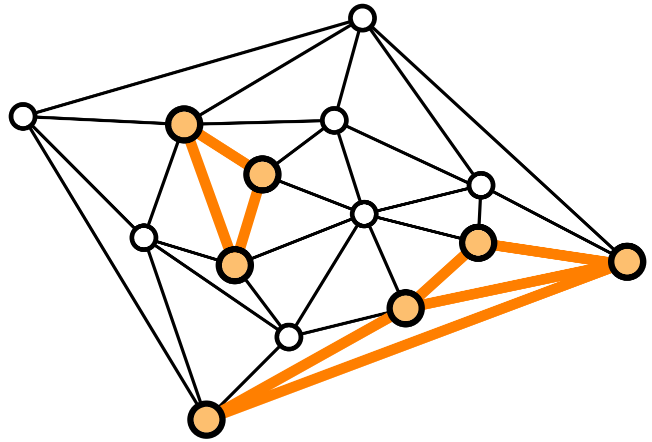

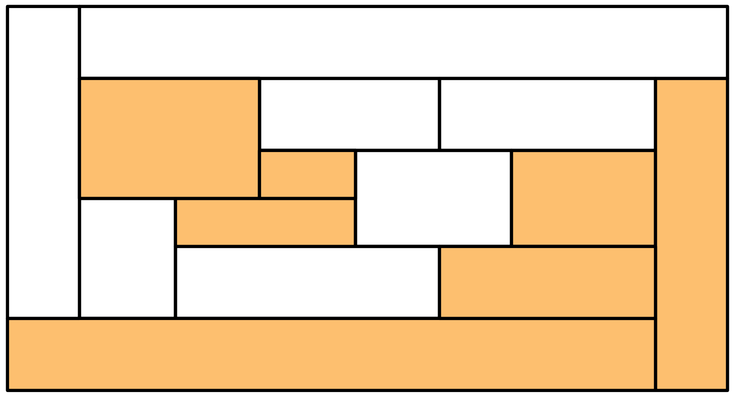

Polytime for:

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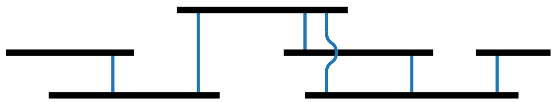

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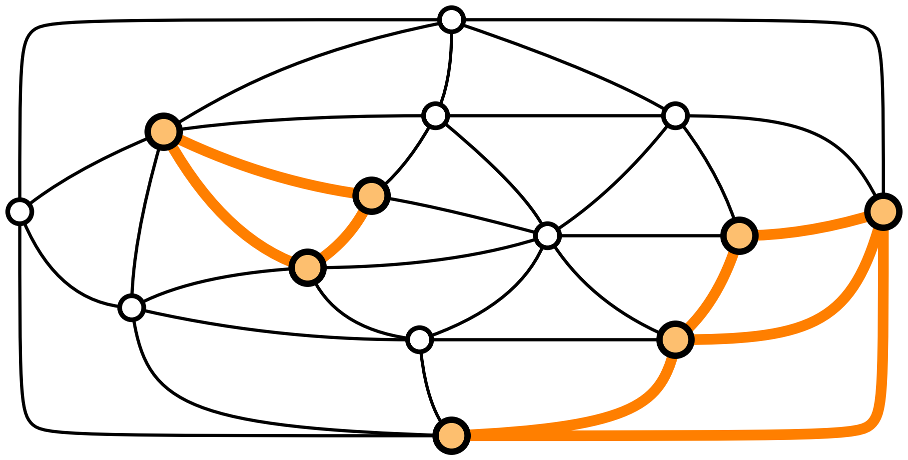

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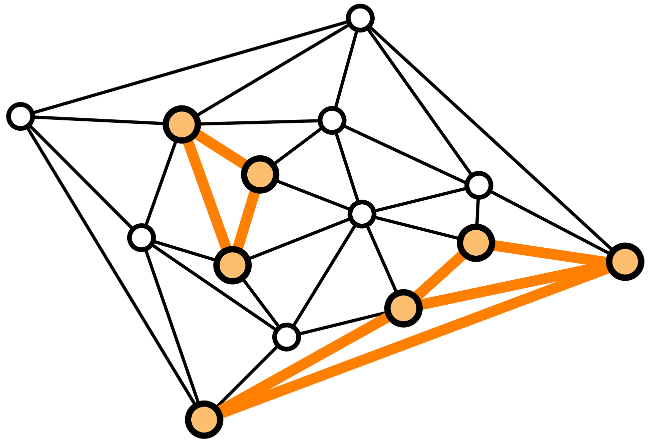

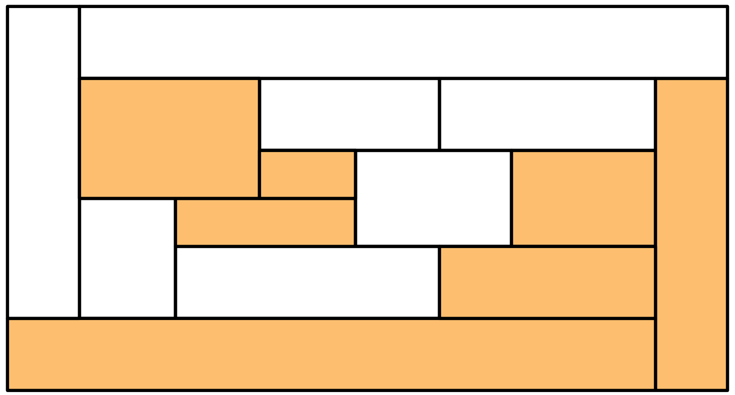

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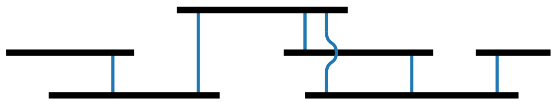

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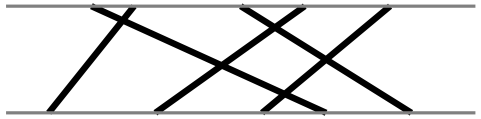

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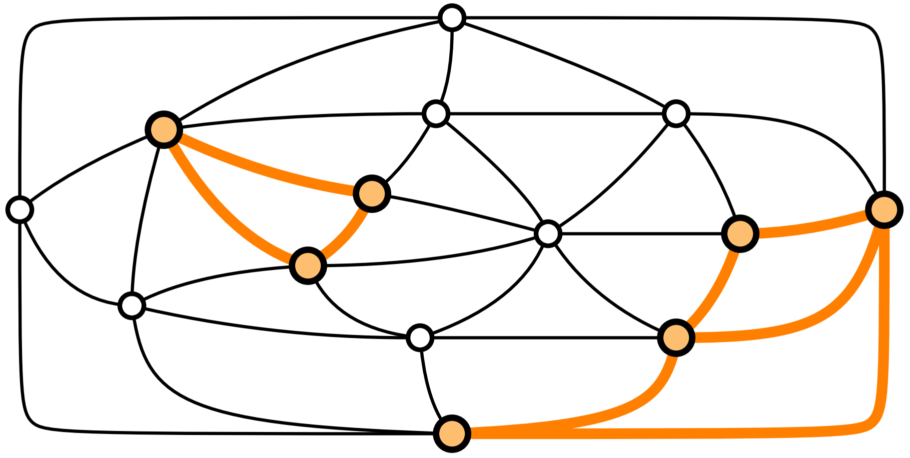

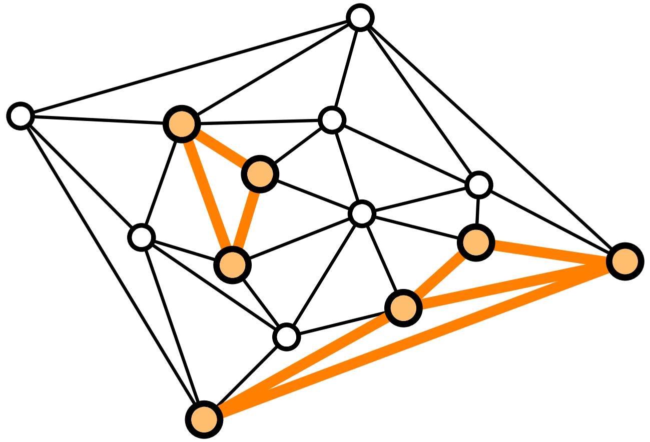

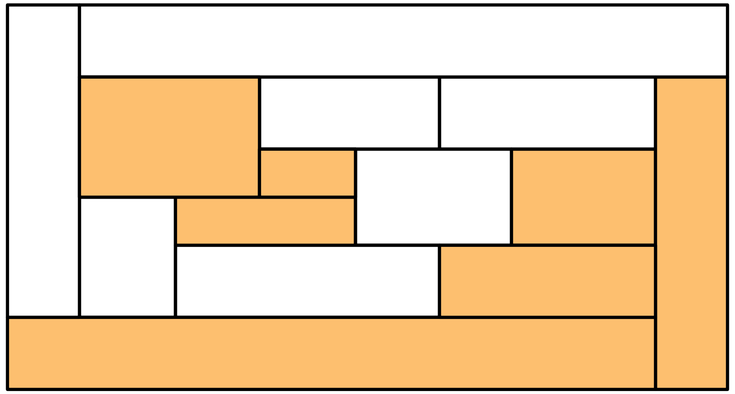

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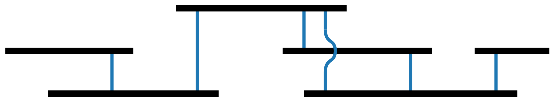

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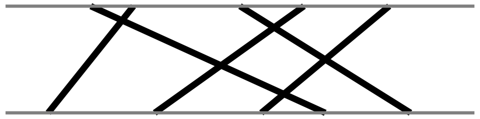

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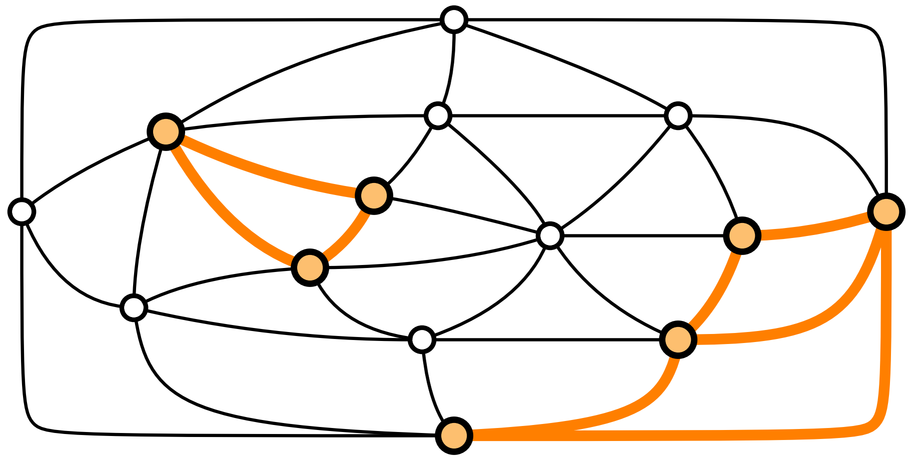

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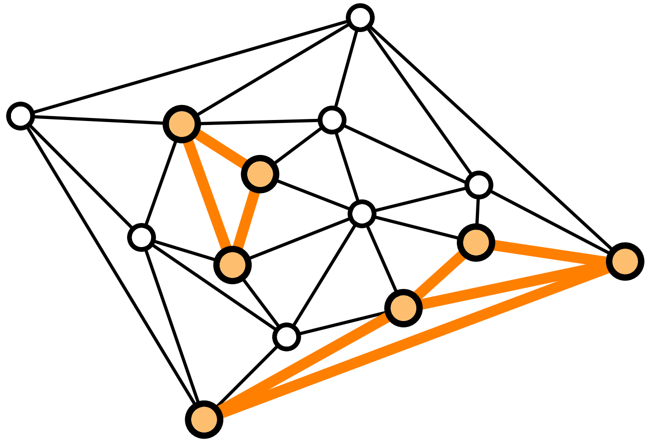

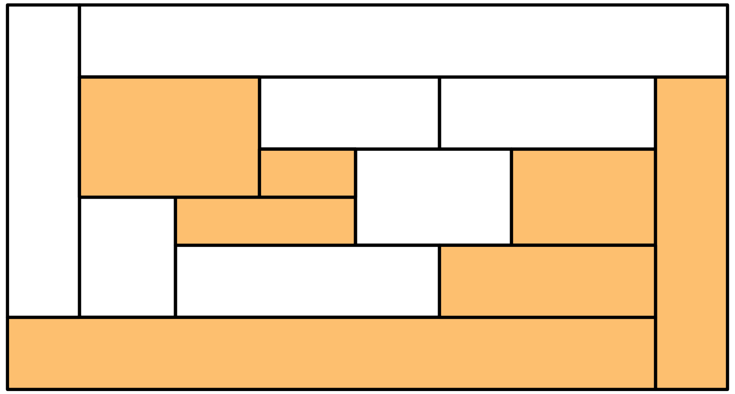

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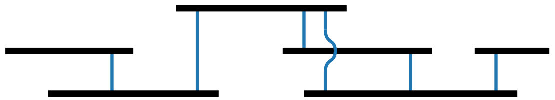

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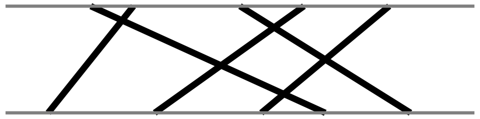

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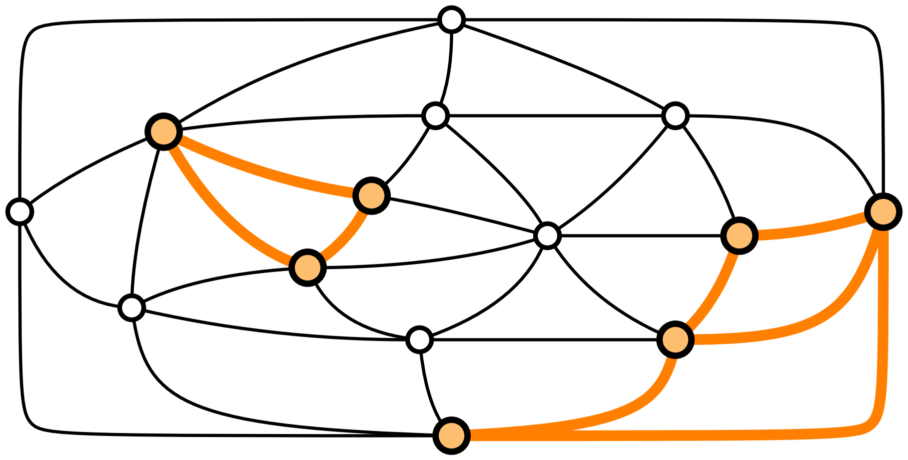

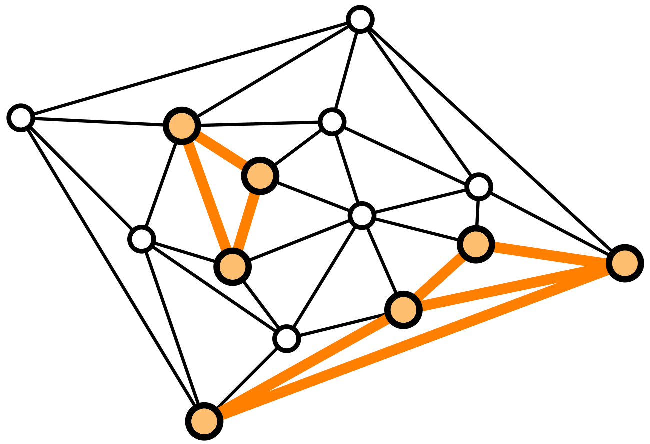

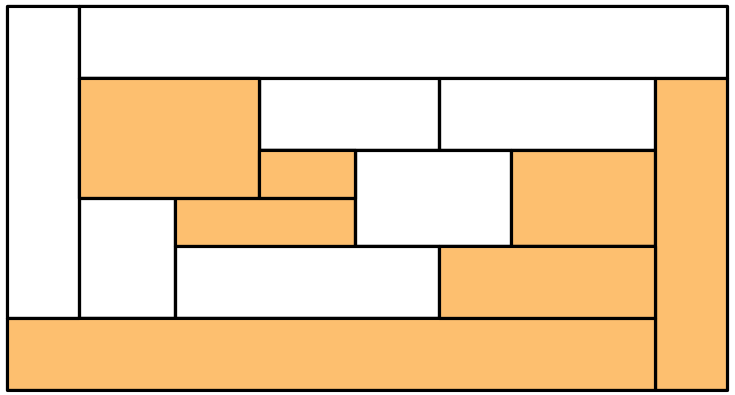

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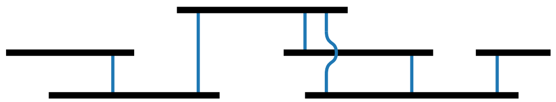

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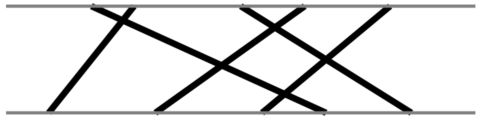

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NP-hard for:

- planar straight-line drawings

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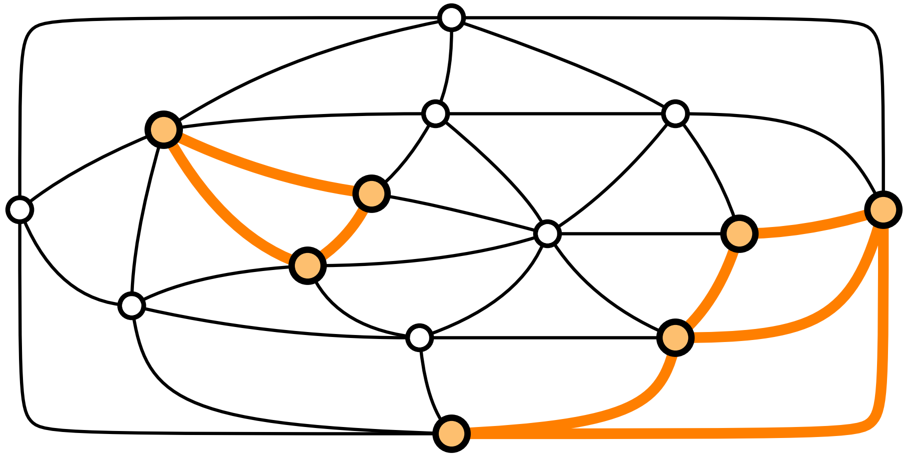

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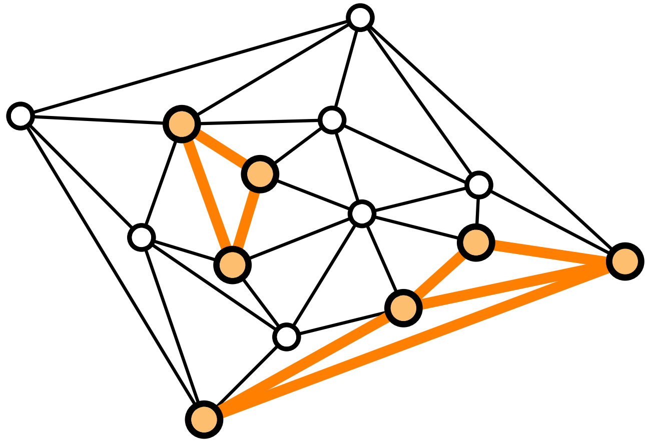

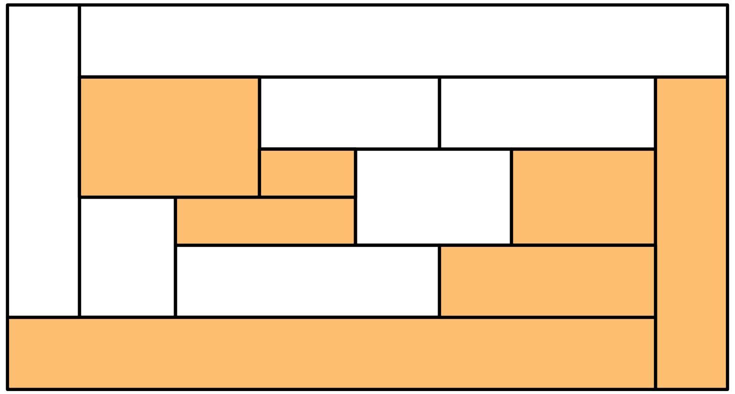

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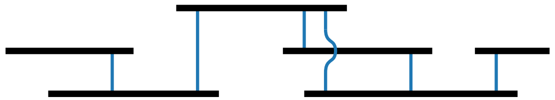

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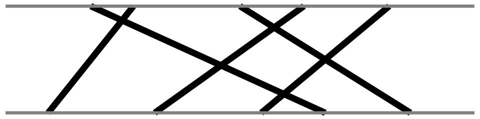

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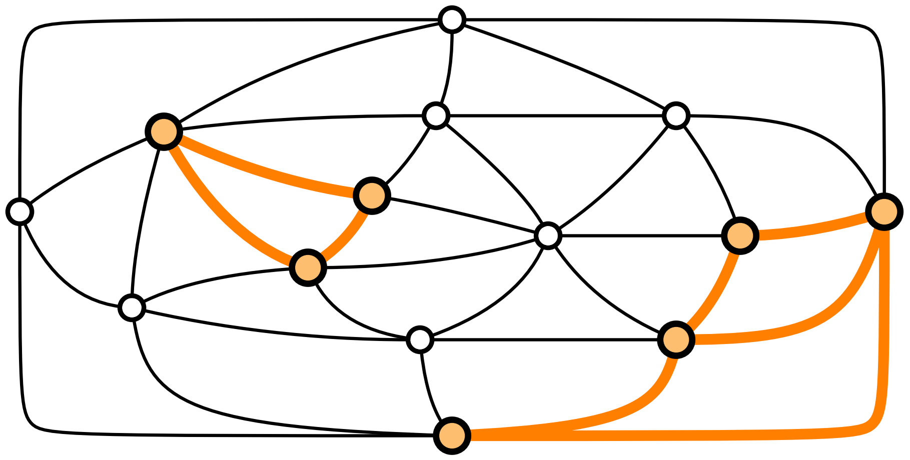

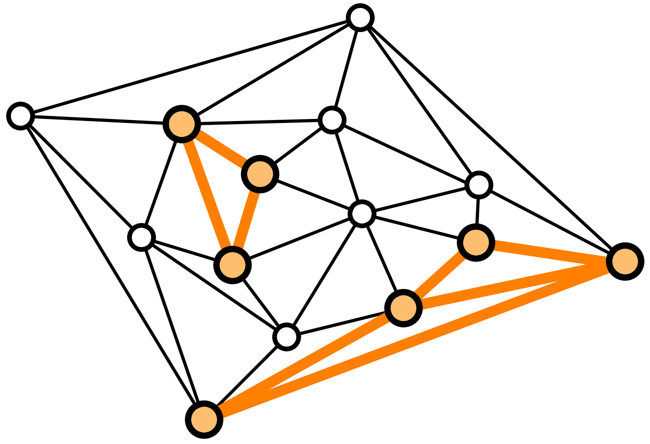

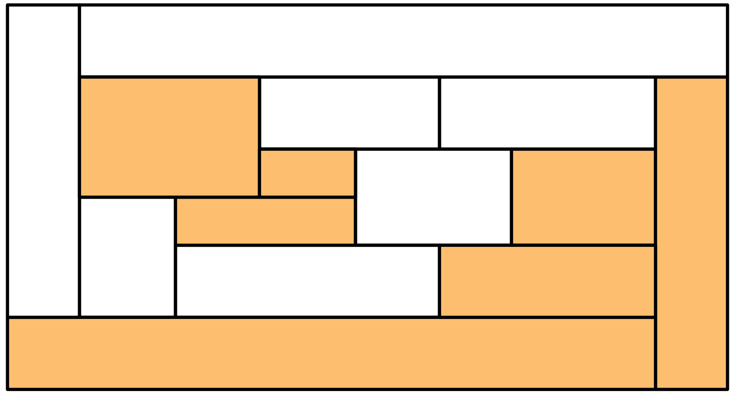

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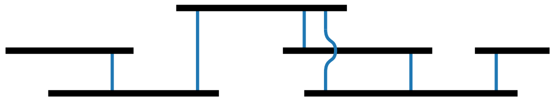

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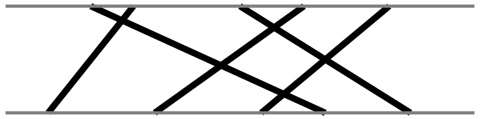

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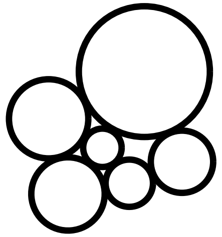

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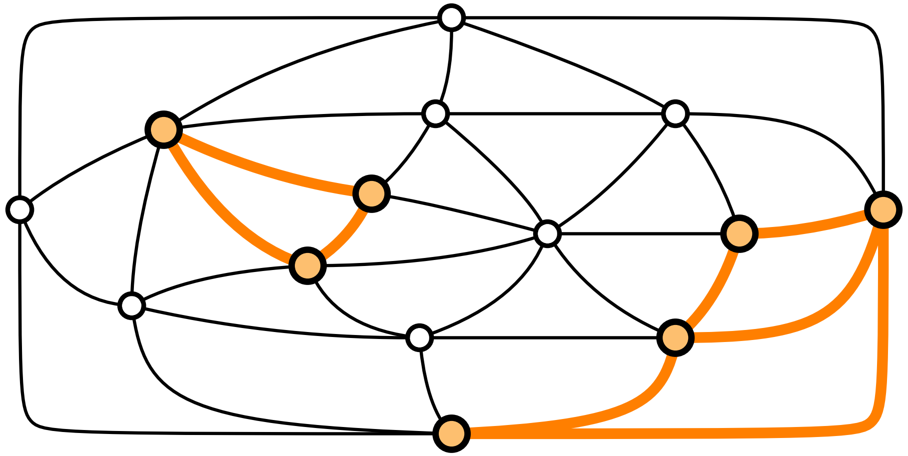

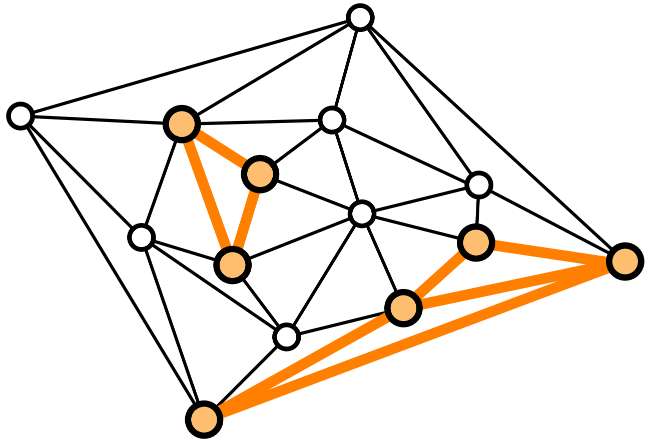

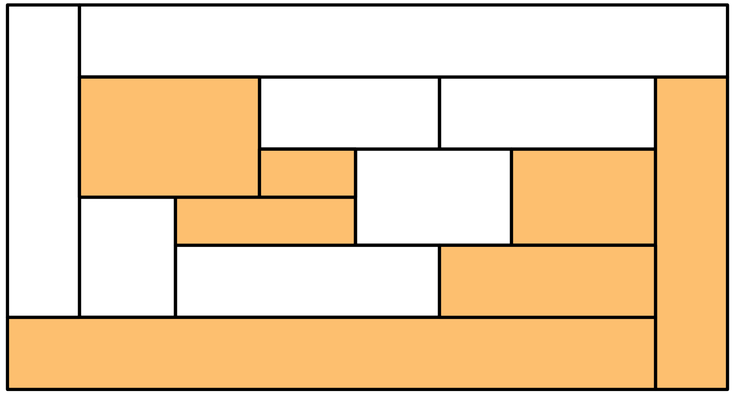

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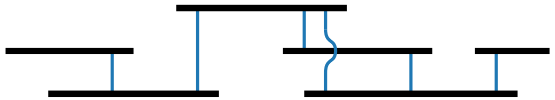

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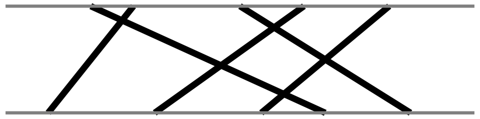

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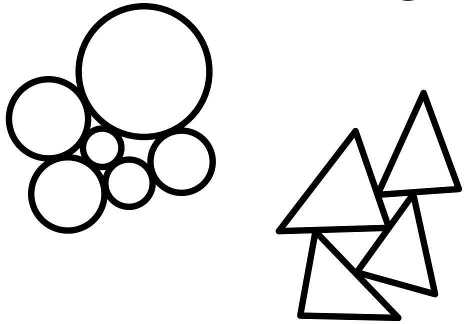

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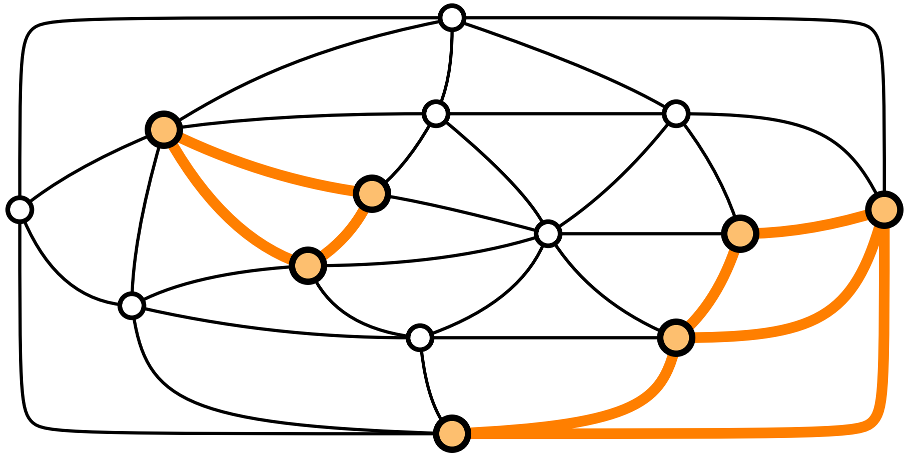

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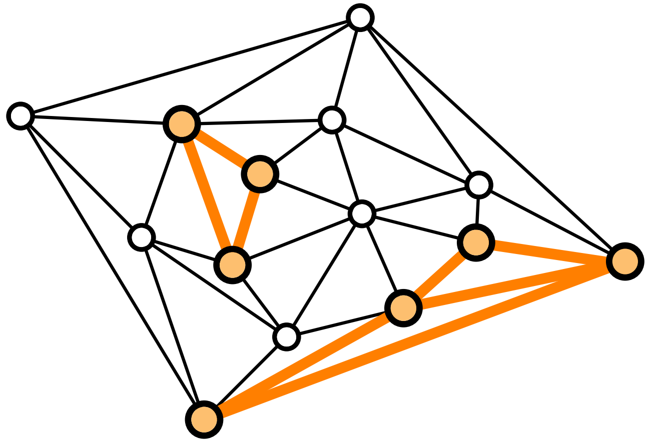
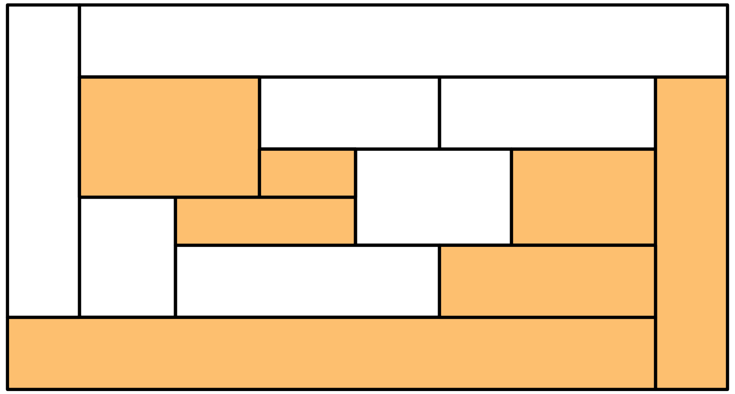
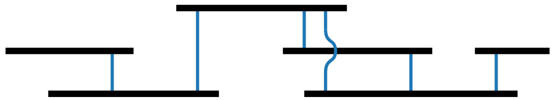


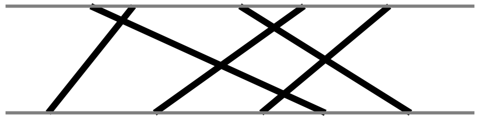


Polytime for:

- (unit) interval graphs



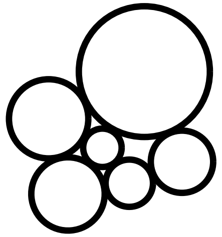
- permutation graphs

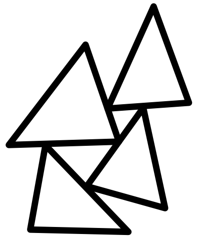


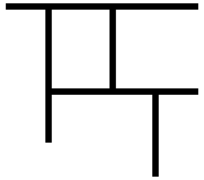
- circle graphs

NP-hard for:

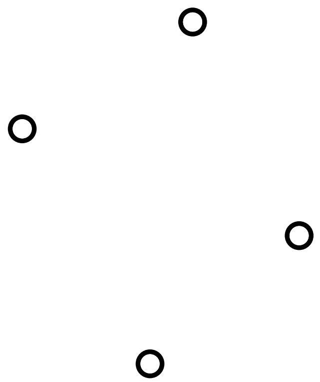
- planar straight-line drawings
- contacts of
  - disks
  - triangles
  - orthogonal segments







Bar Visibility Representation



Bar Visibility Representation

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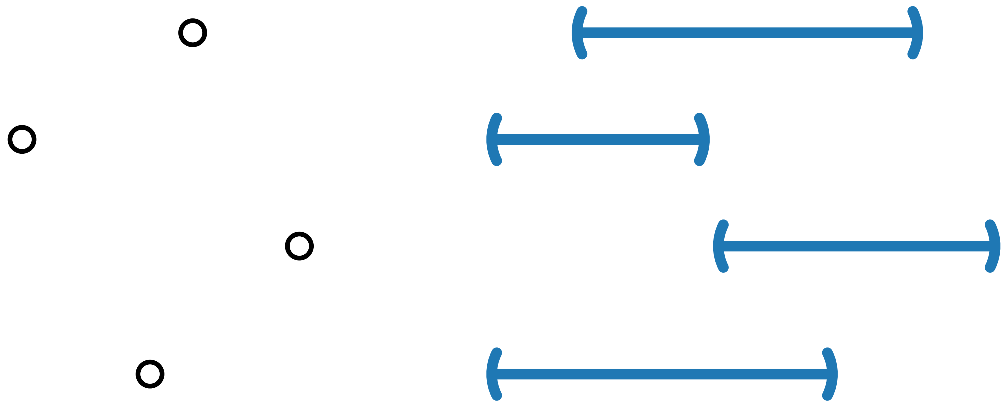

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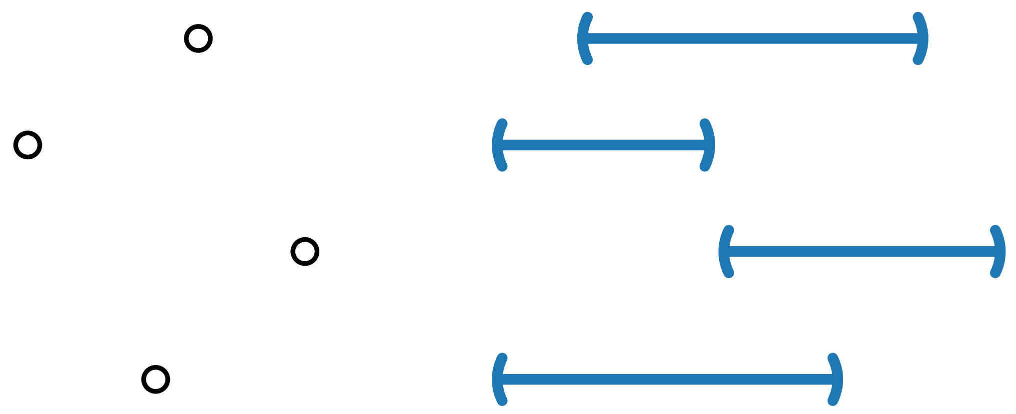

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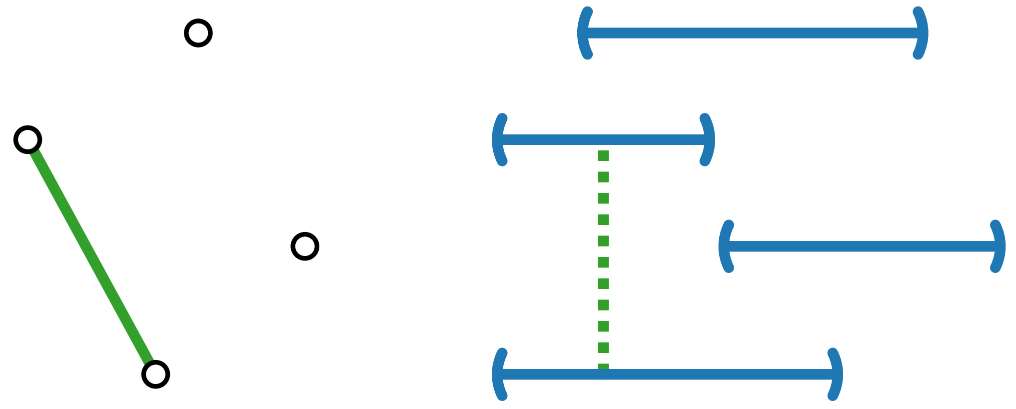

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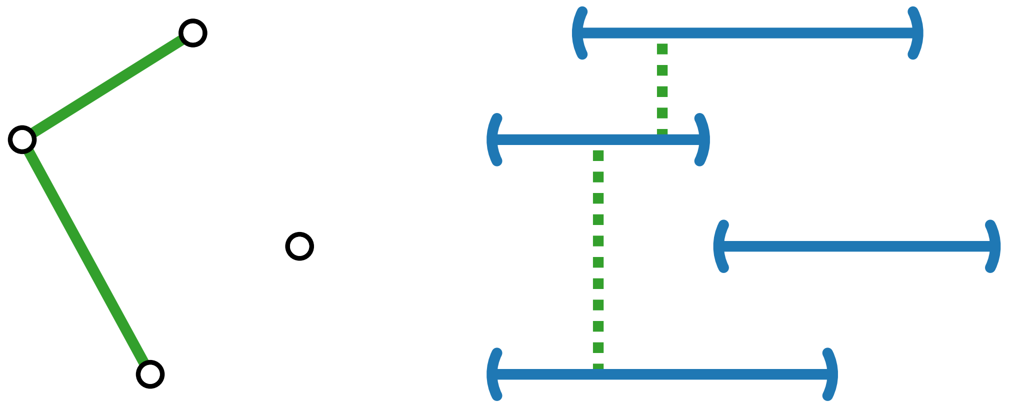

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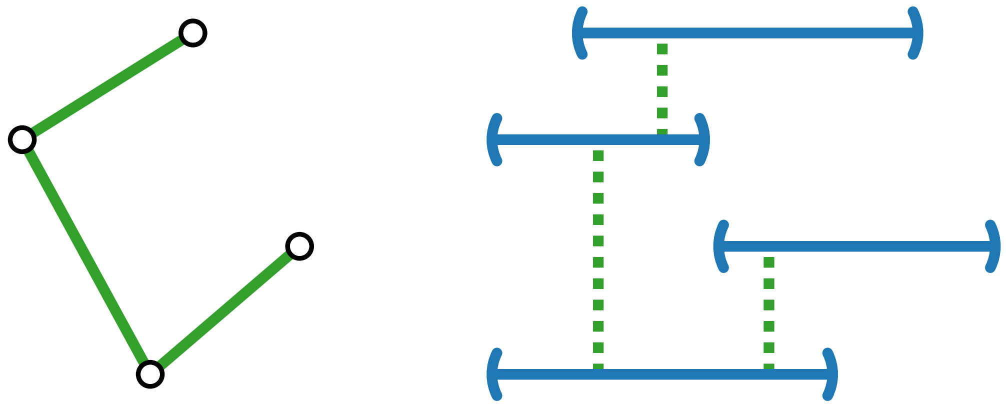

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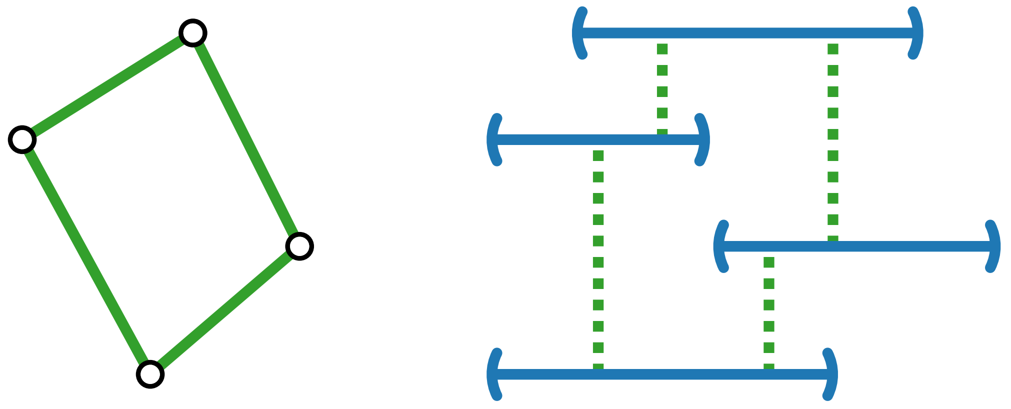

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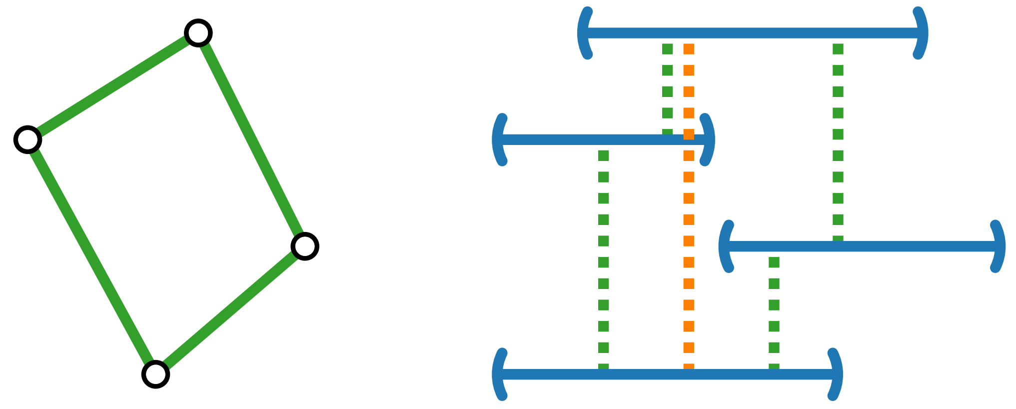

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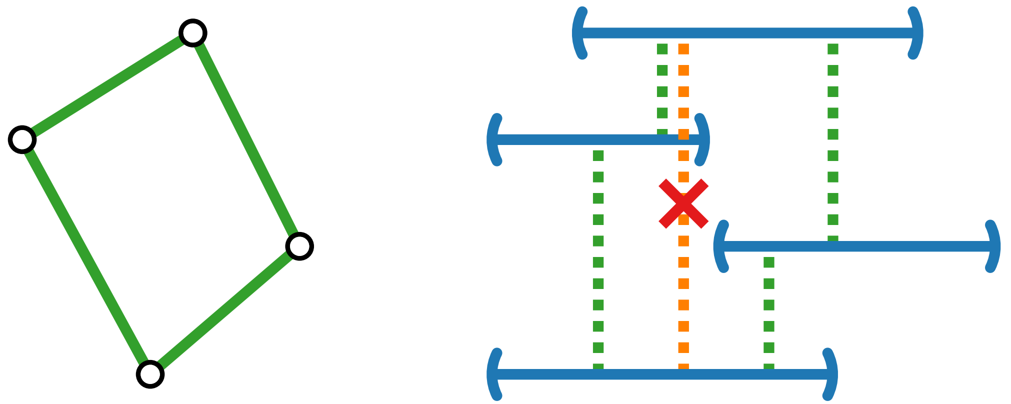

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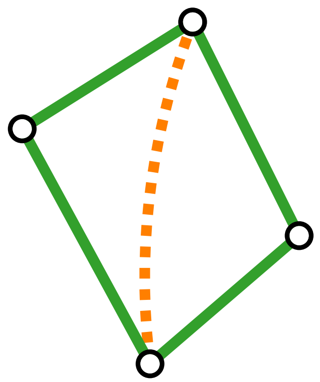

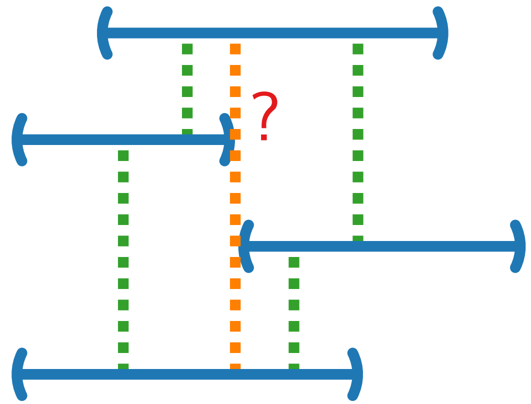

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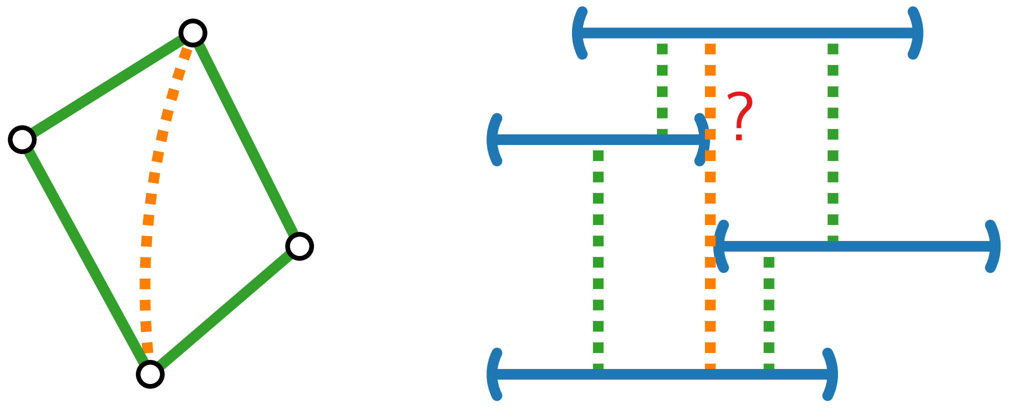

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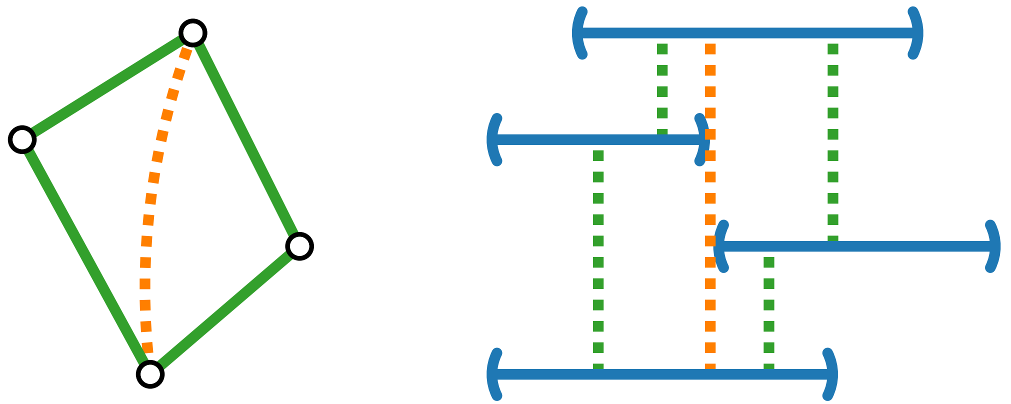

Models.

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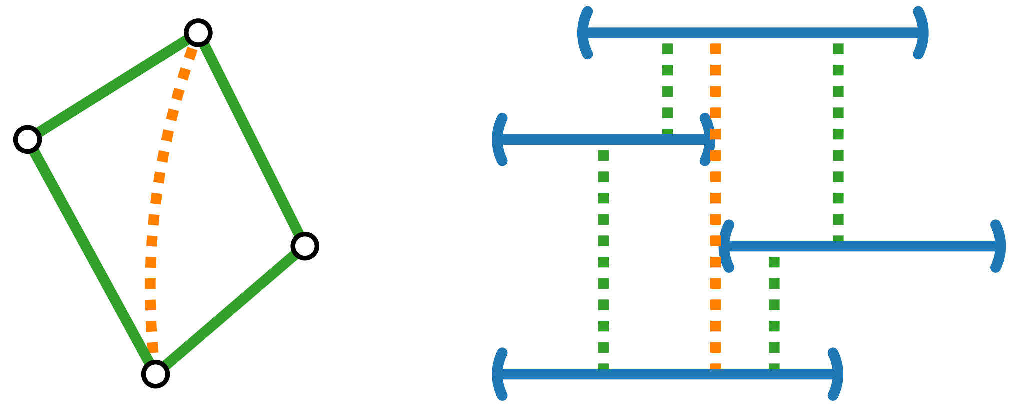

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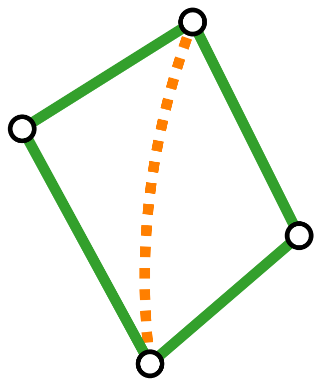

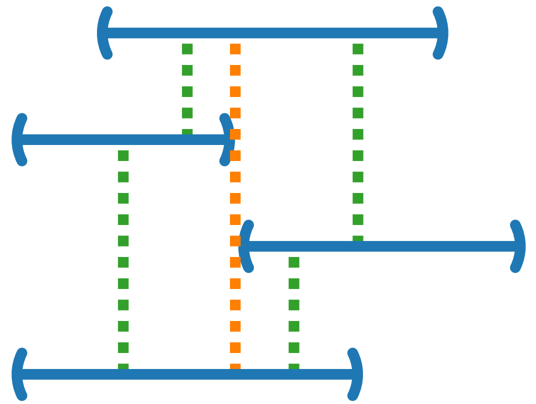

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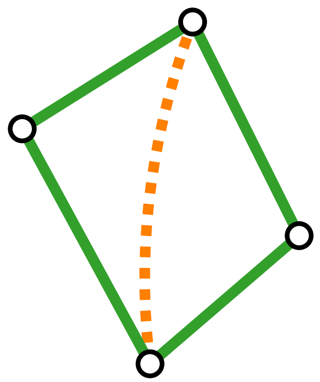

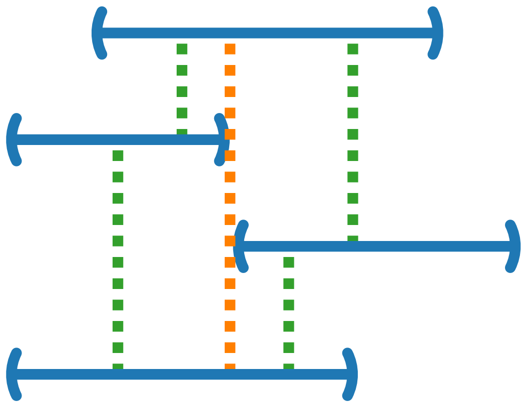

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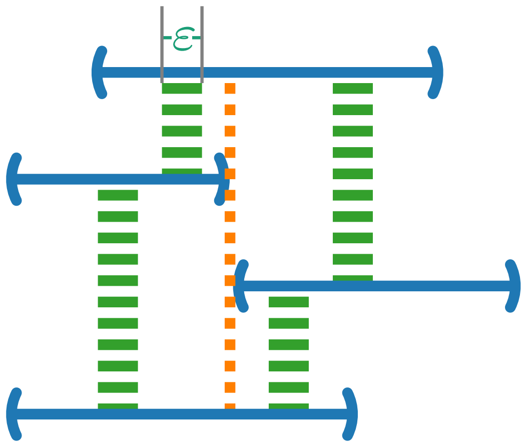

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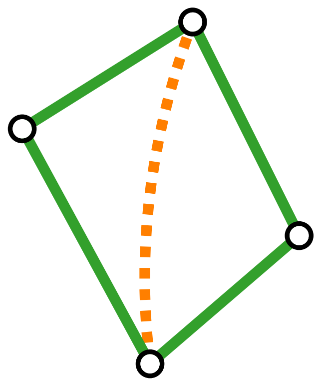

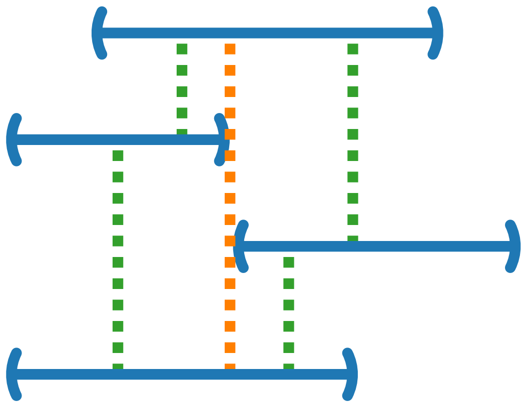

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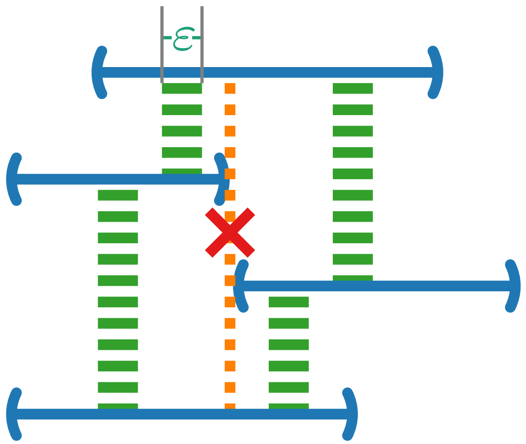

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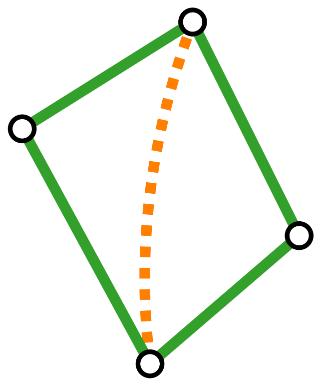

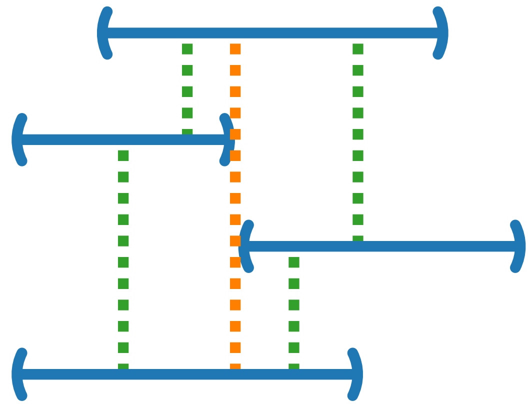

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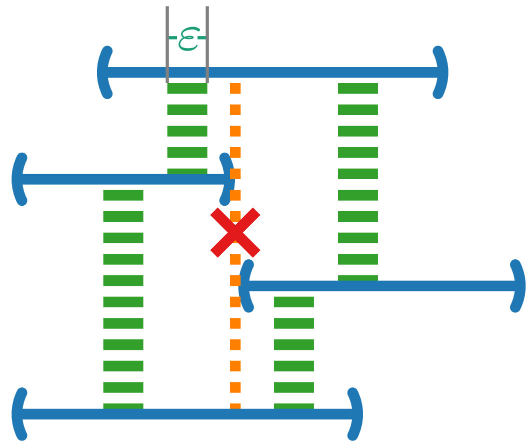

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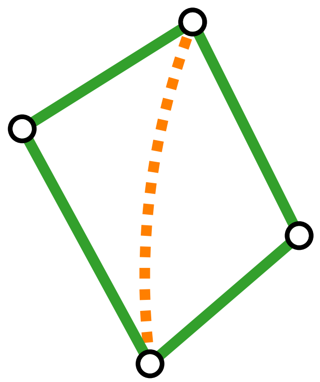

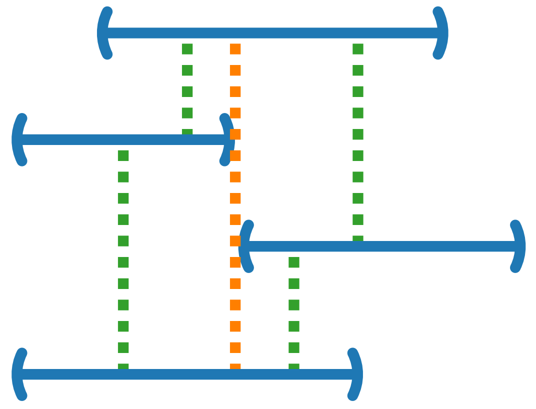

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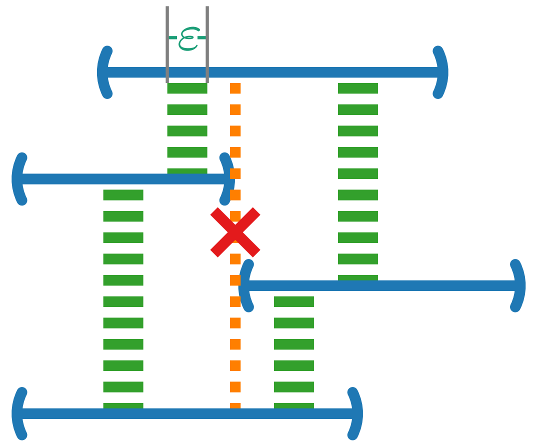
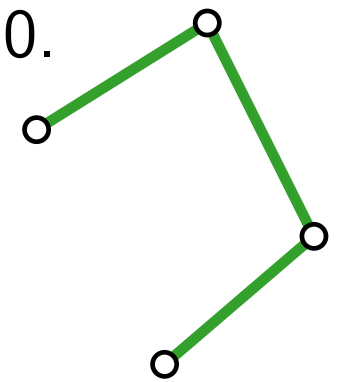
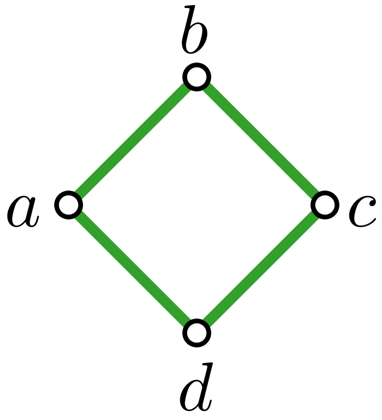
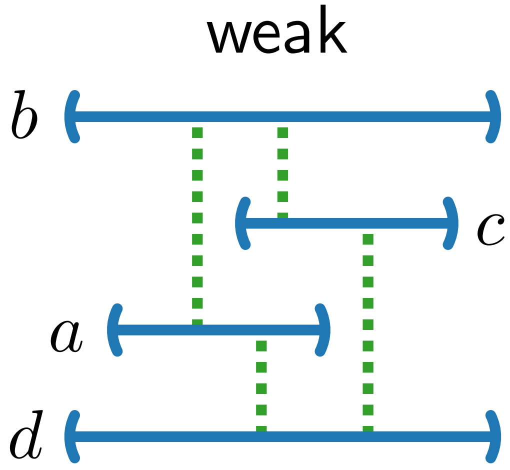




Problems





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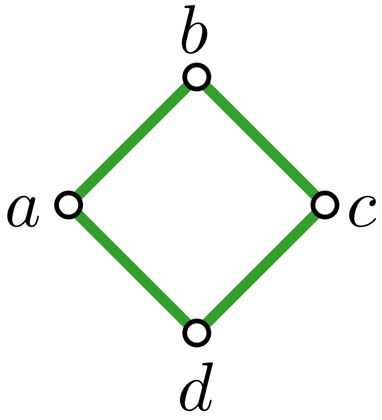

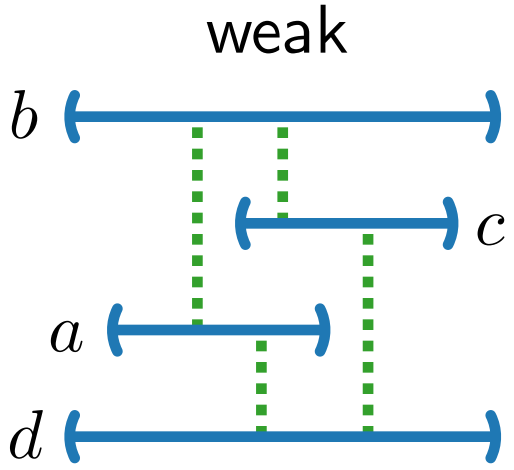

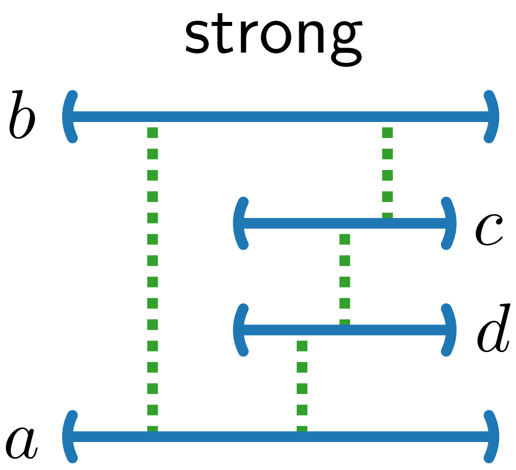

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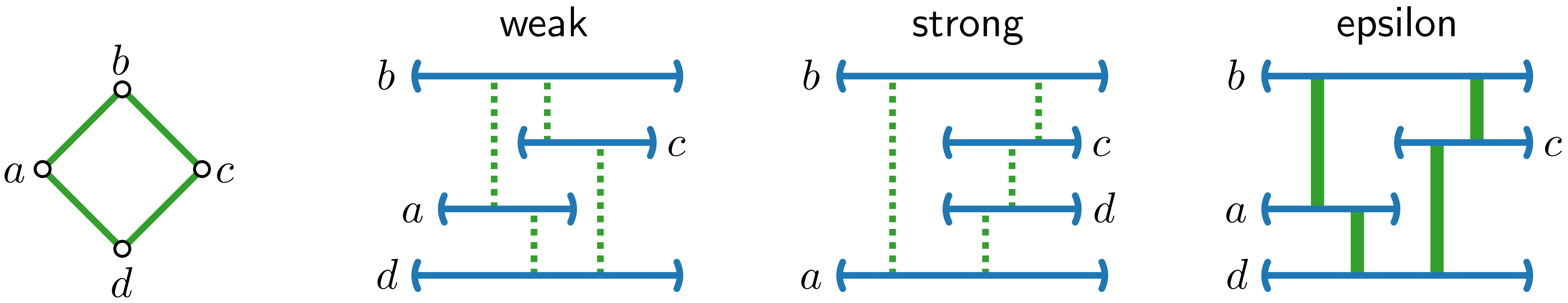

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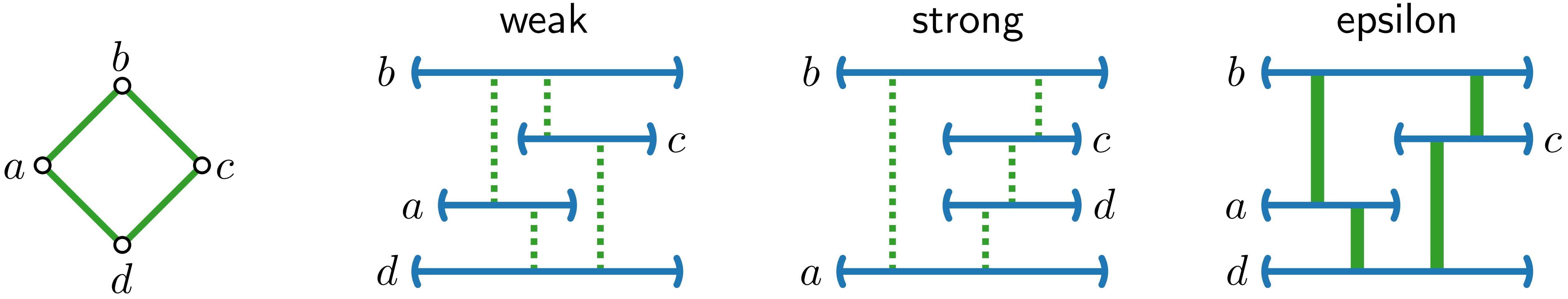
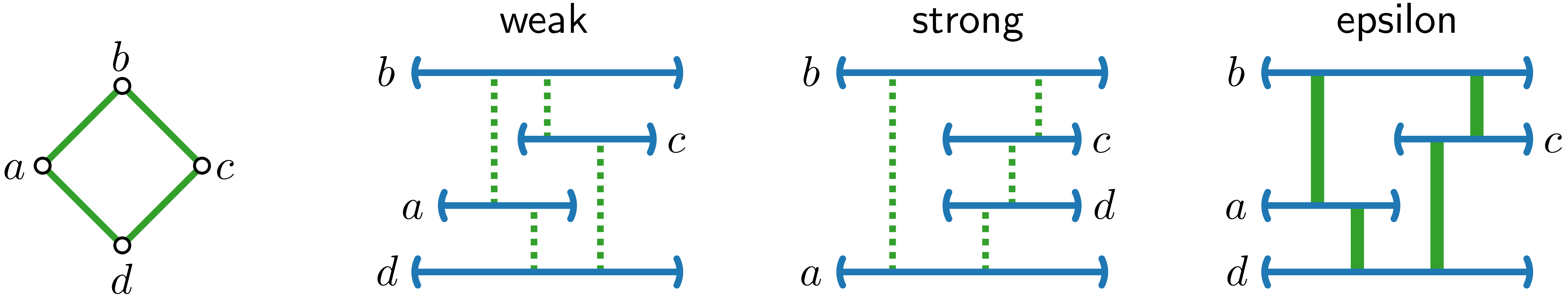


Recognition Problem.
Given a graph G, **decide** whether there exists a weak/strong/ε-bar visibility representation ψ of G.

Problems



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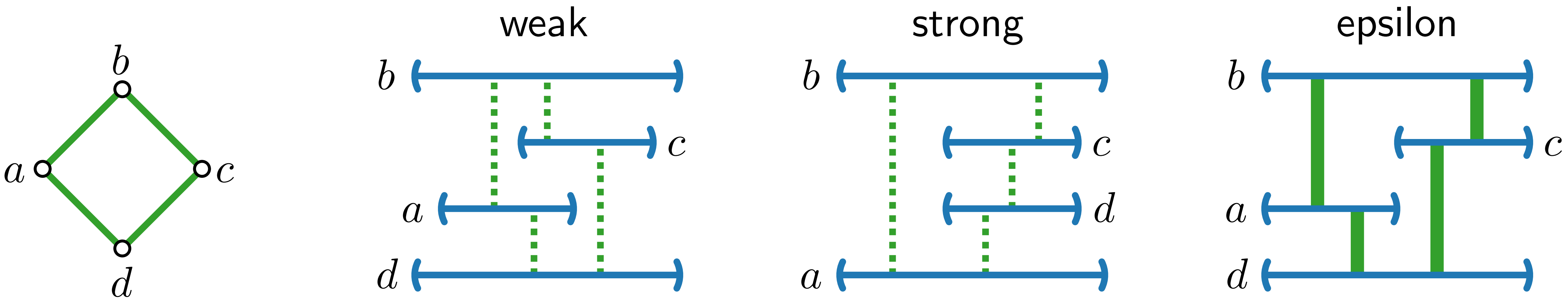

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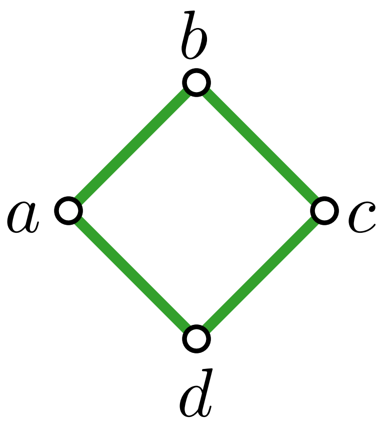
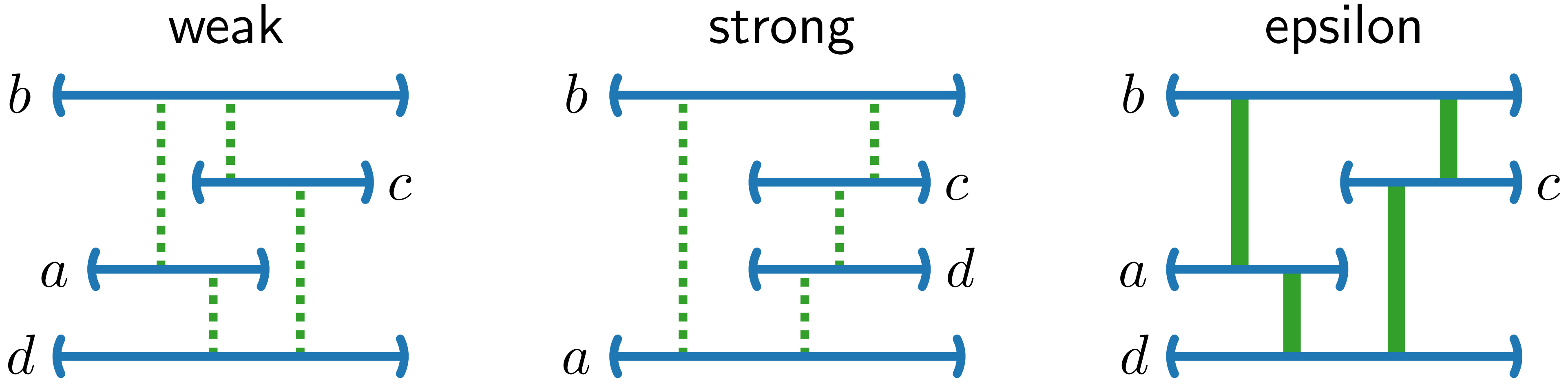
Partial Representation Extension Problem.
Given a graph G and a **set of bars ψ' of $V' \subseteq V(G)$**, **decide** whether there exists a weak/strong/ε-bar visibility representation ψ of G **where $\psi|_{V'} = \psi'$** (and **construct** ψ if a representation exists).

Background





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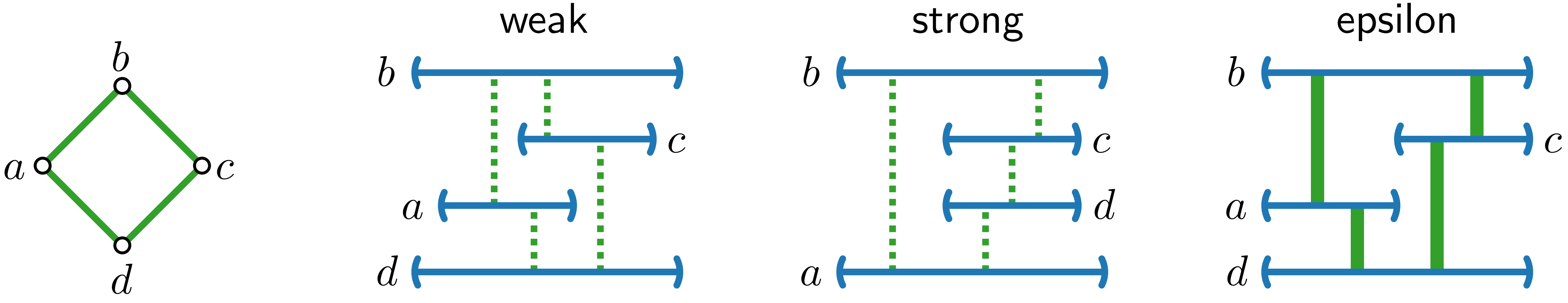

Weak Bar Visibility.

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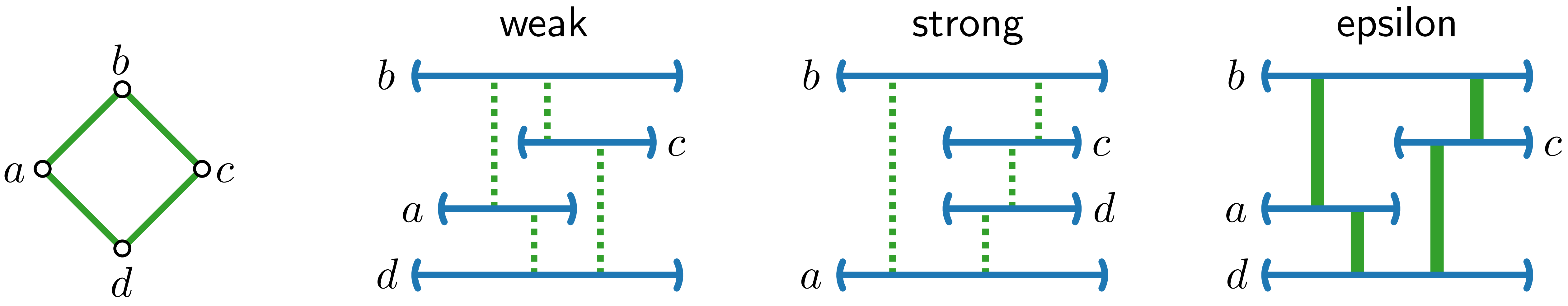

Weak Bar Visibility.

- Exactly all planar graphs [Tamassia & Tollis '86; Wismath '85]

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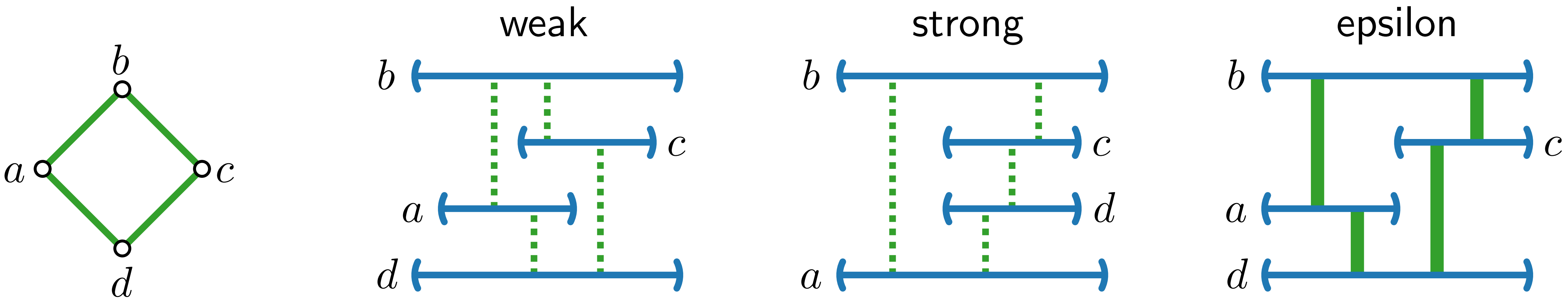

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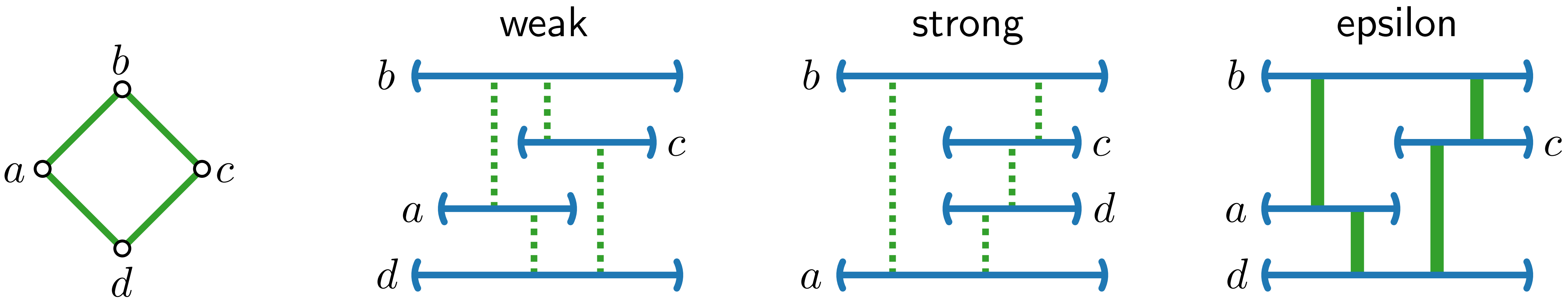

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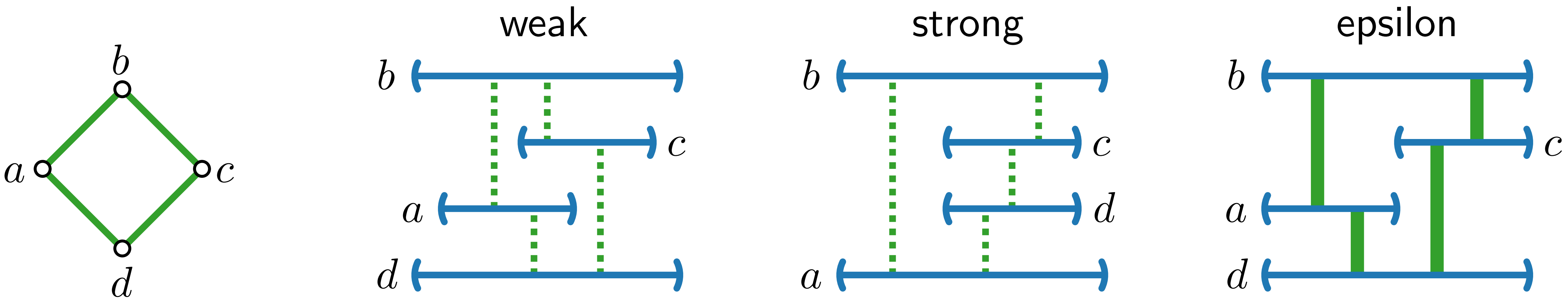

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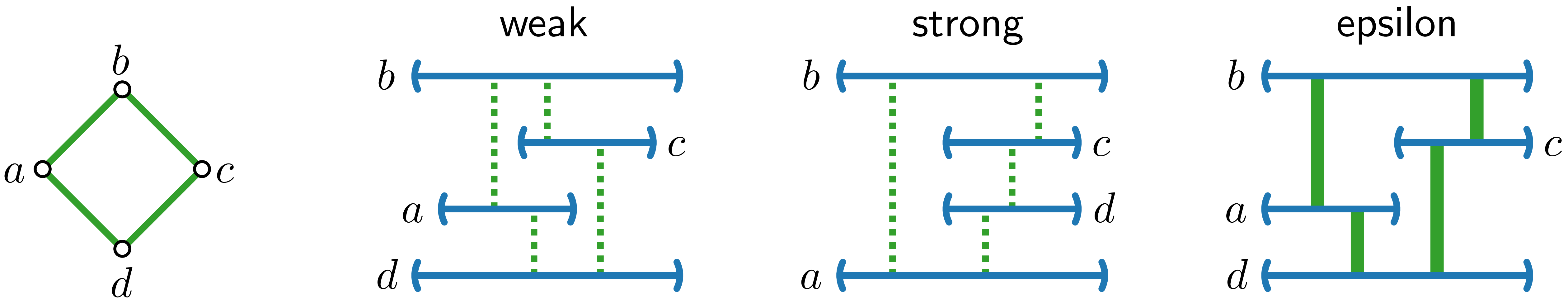

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Strong Bar Visibility.

- NP-complete to recognize [Andreae '92]

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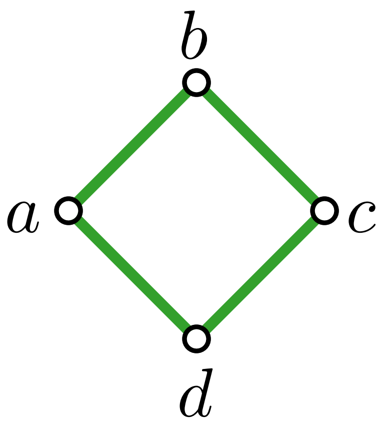
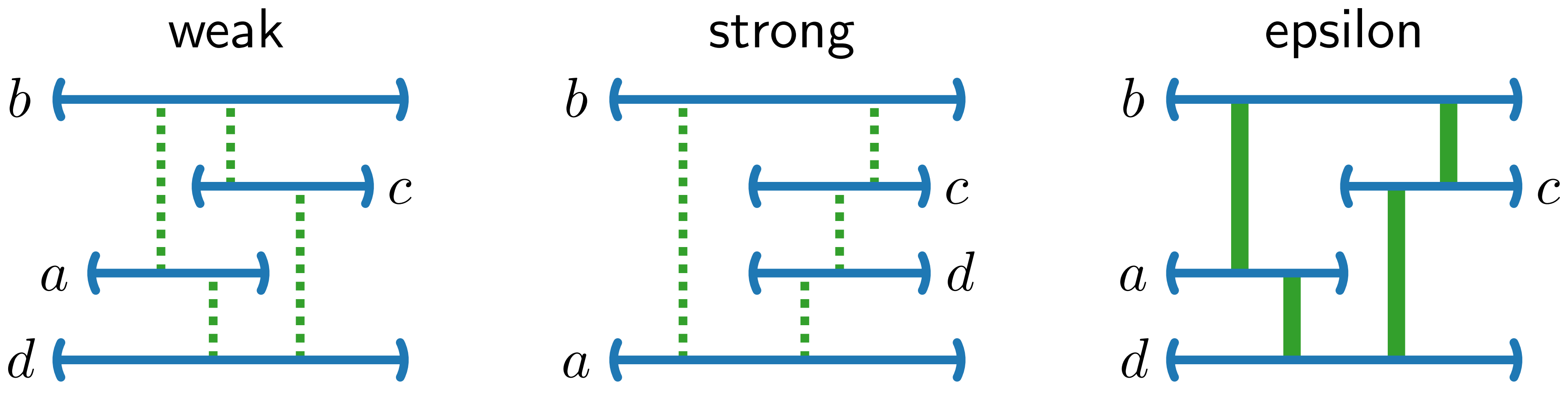
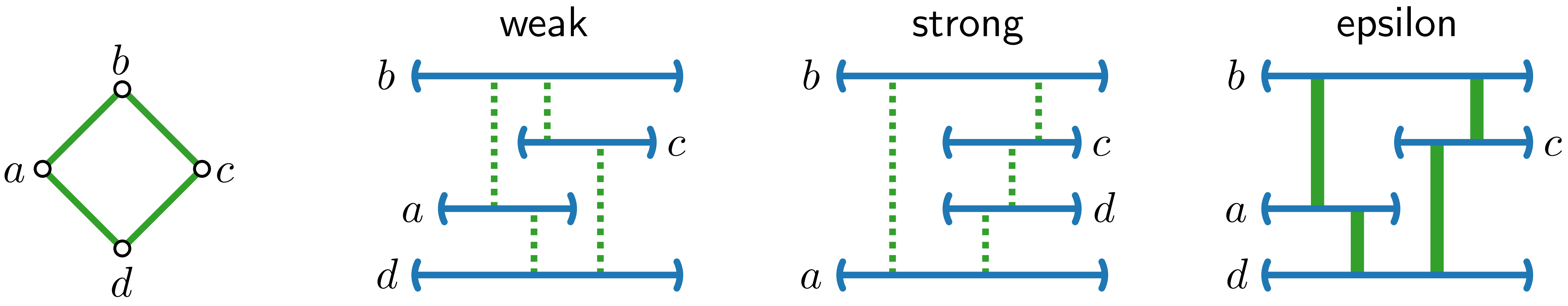




ε-Bar Visibility.

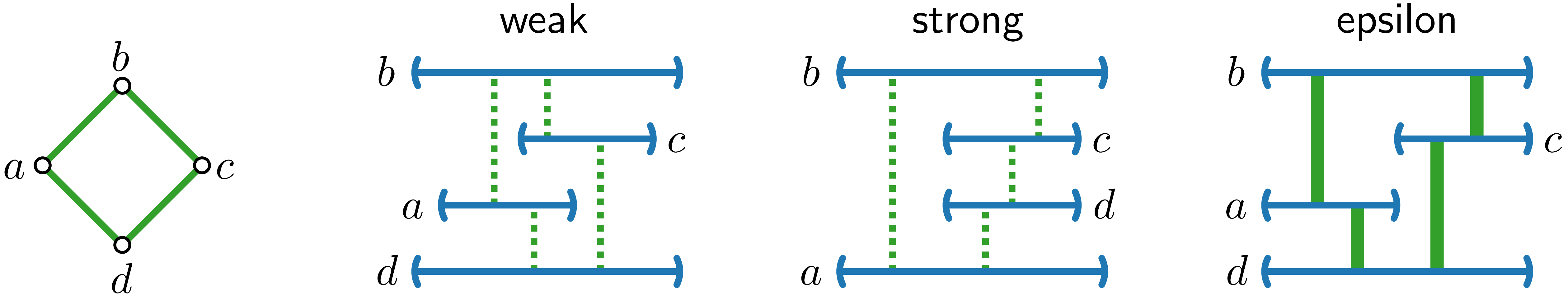
Background



ε-Bar Visibility.

- Exactly all planar graphs that can be embedded with all **cut vertices** on the outer face [T&T '86, Wismath '85]

Background



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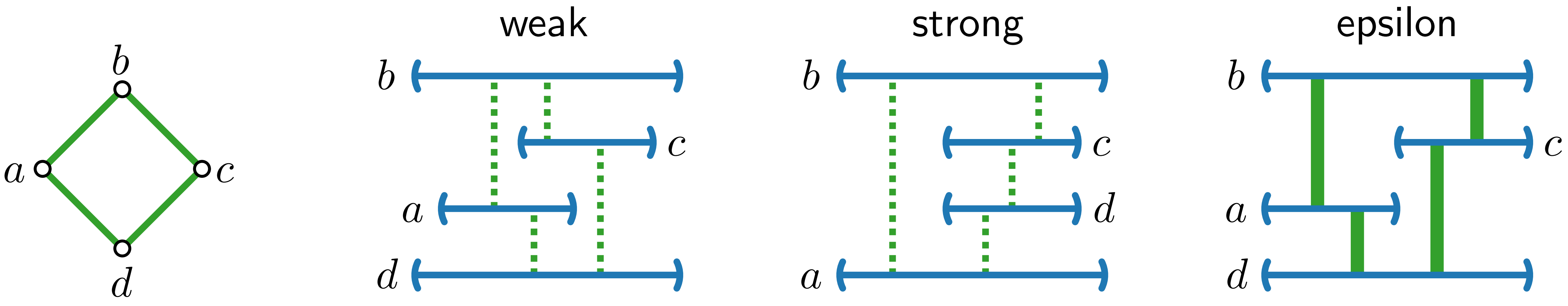

ε-Bar Visibility.

- Exactly all planar graphs that can be embedded with all **cut vertices** on the outer face [T&T '86, Wismath '85]
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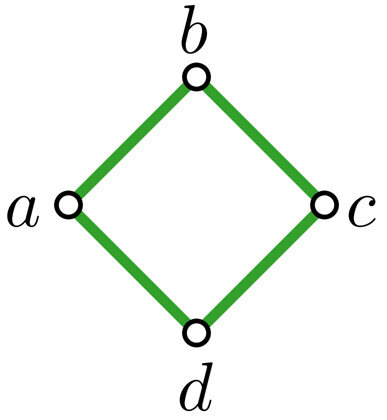

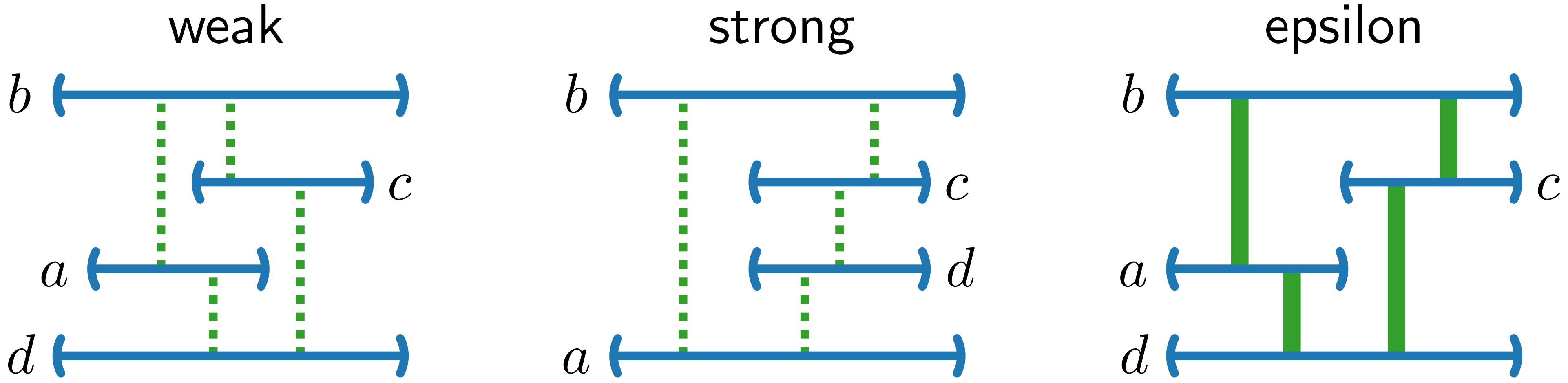

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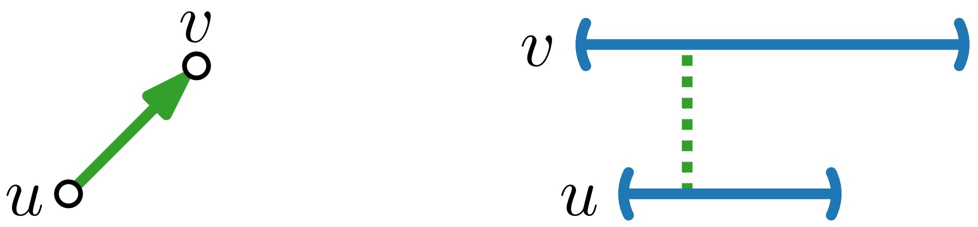

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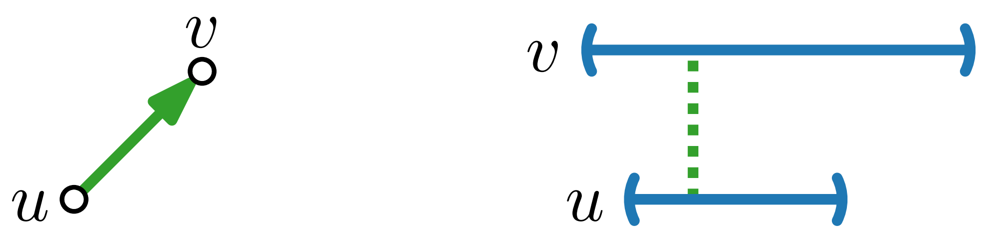

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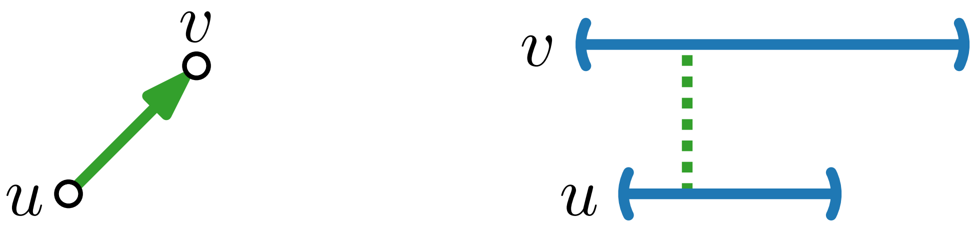

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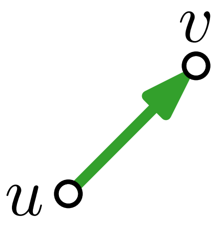

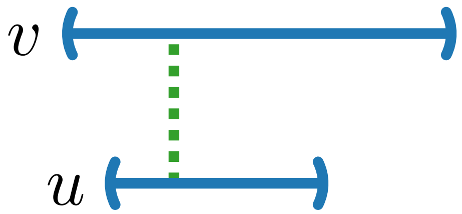

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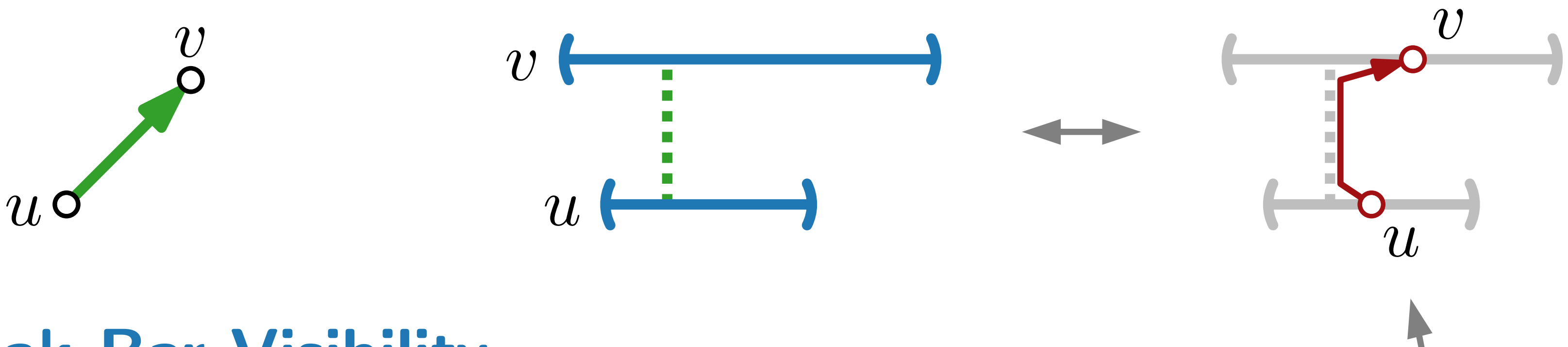

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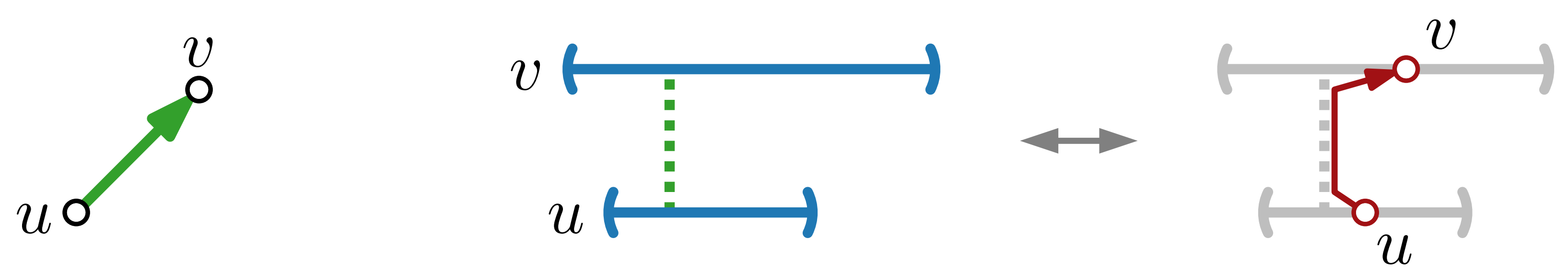

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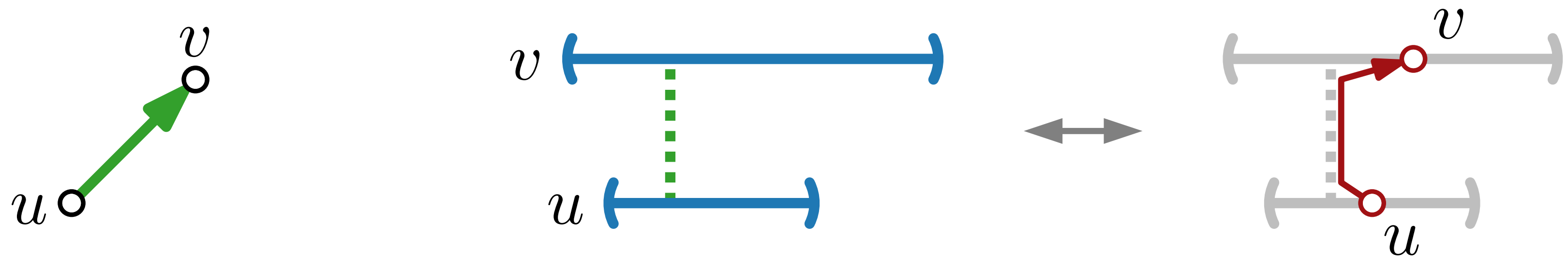

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Next, we consider ε-bar visibility representations of specific directed graphs ($\rightarrow$ st-graphs)

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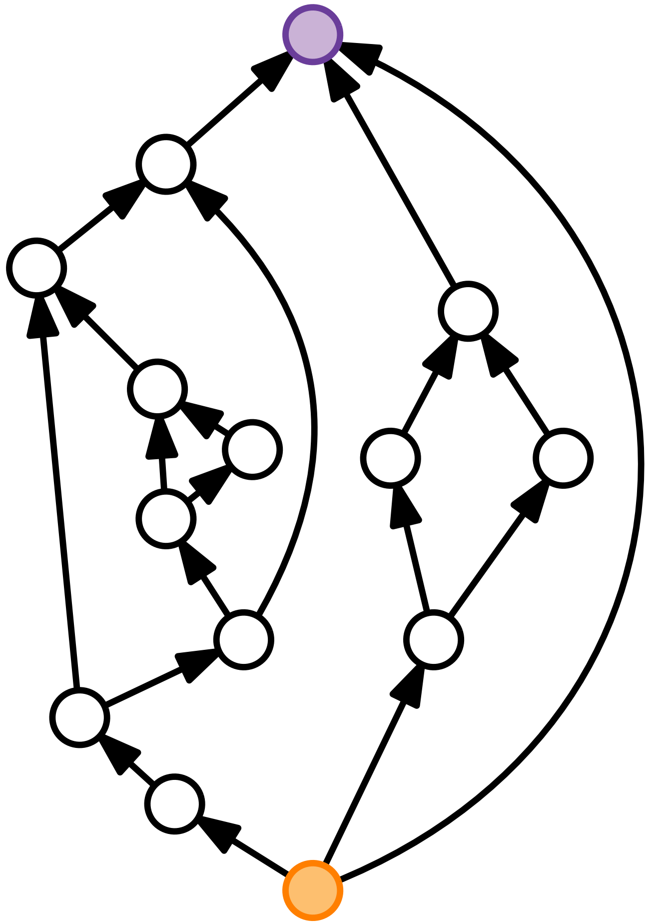

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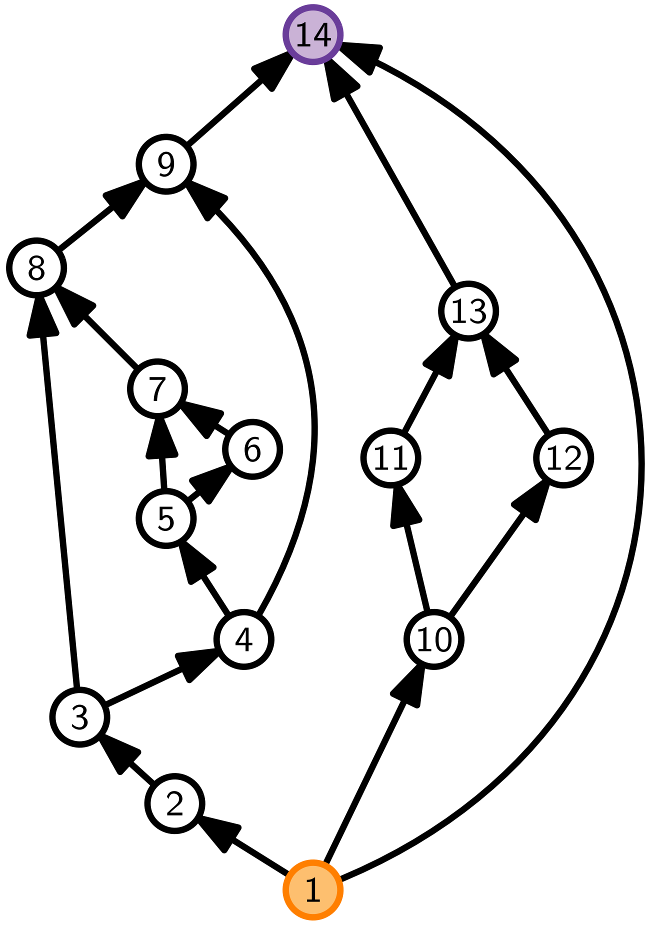

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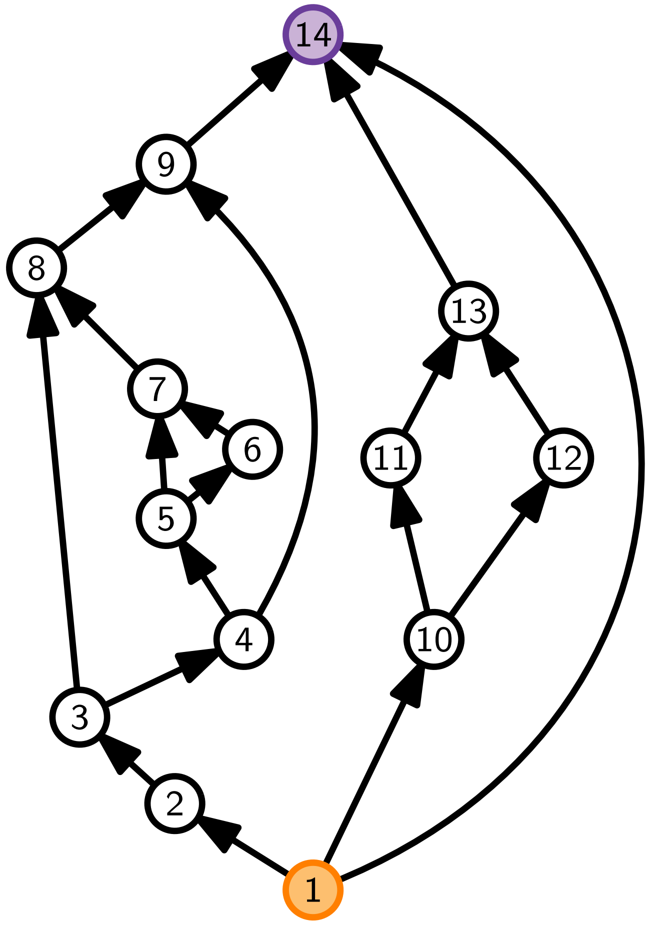

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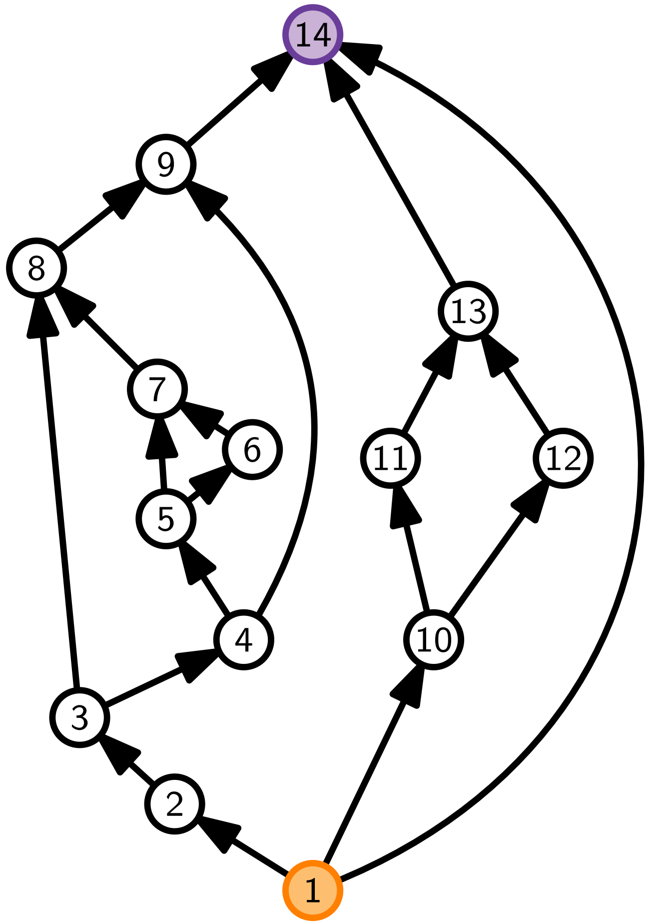

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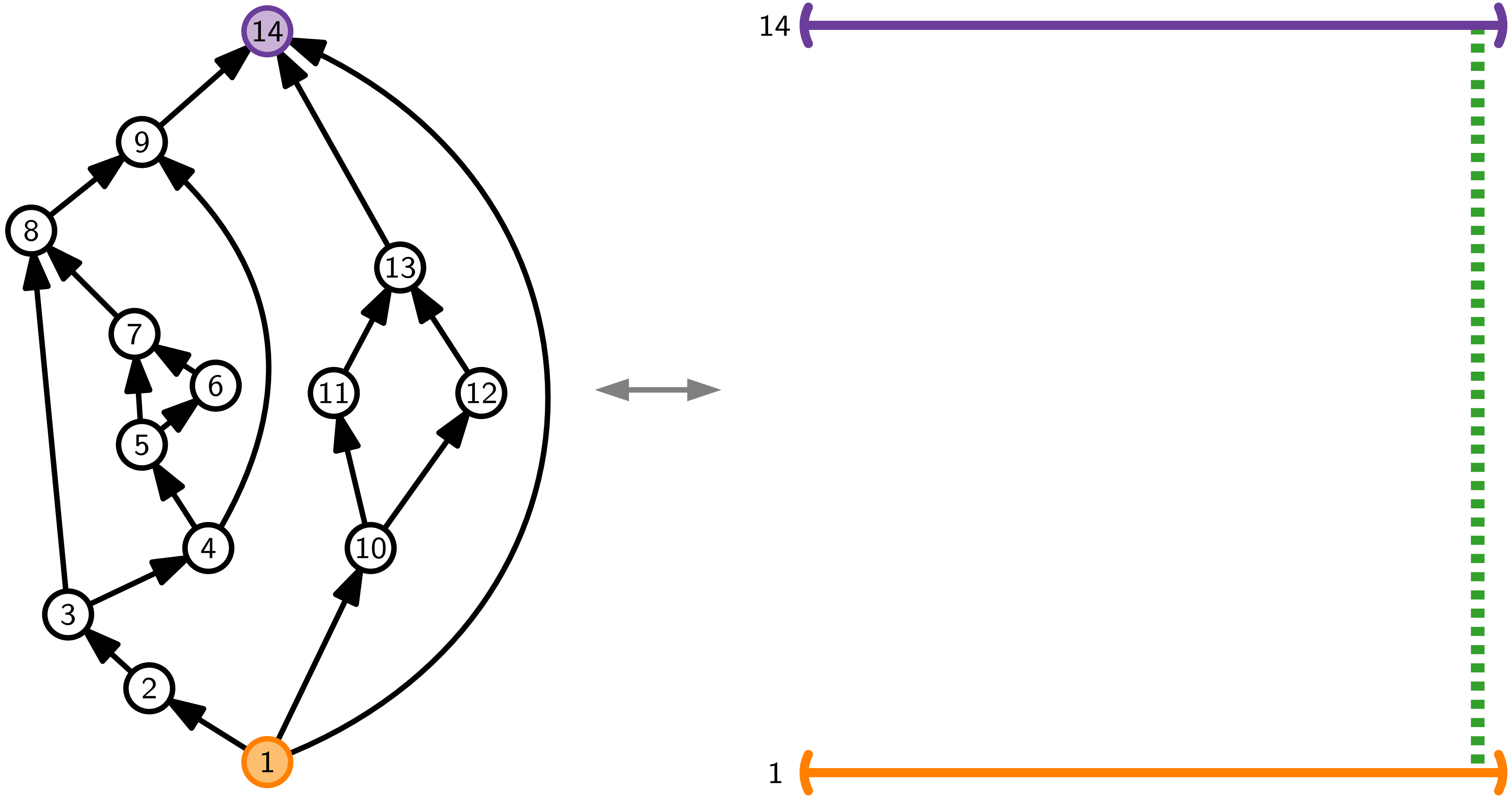

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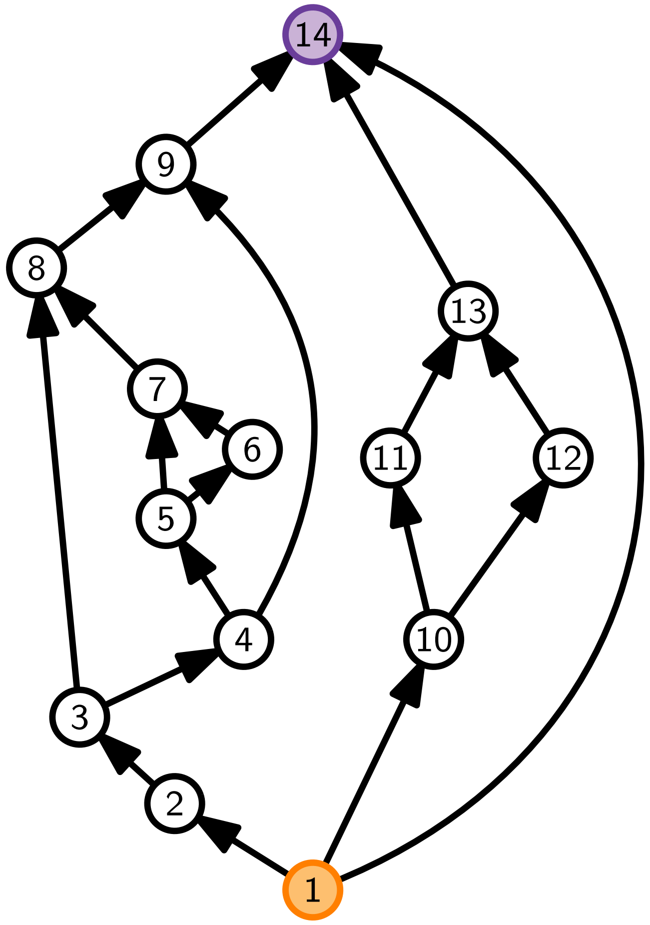

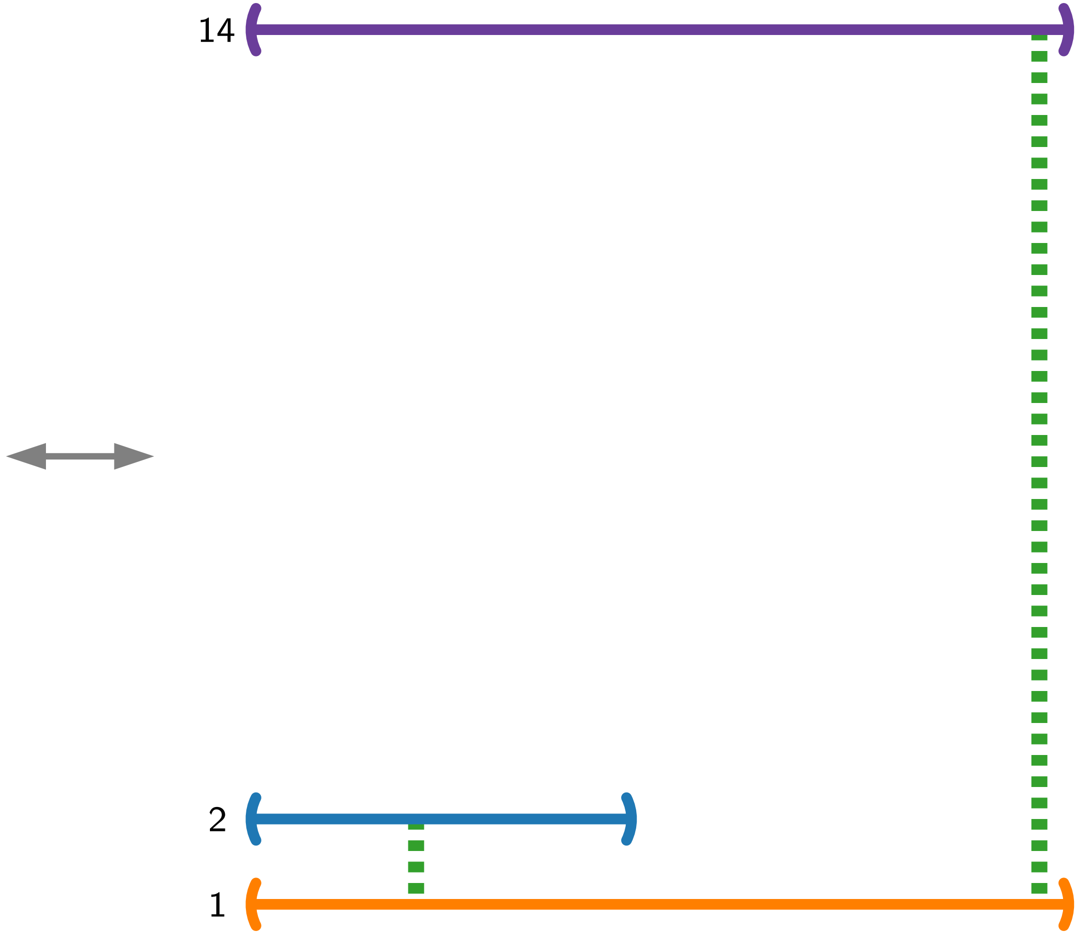

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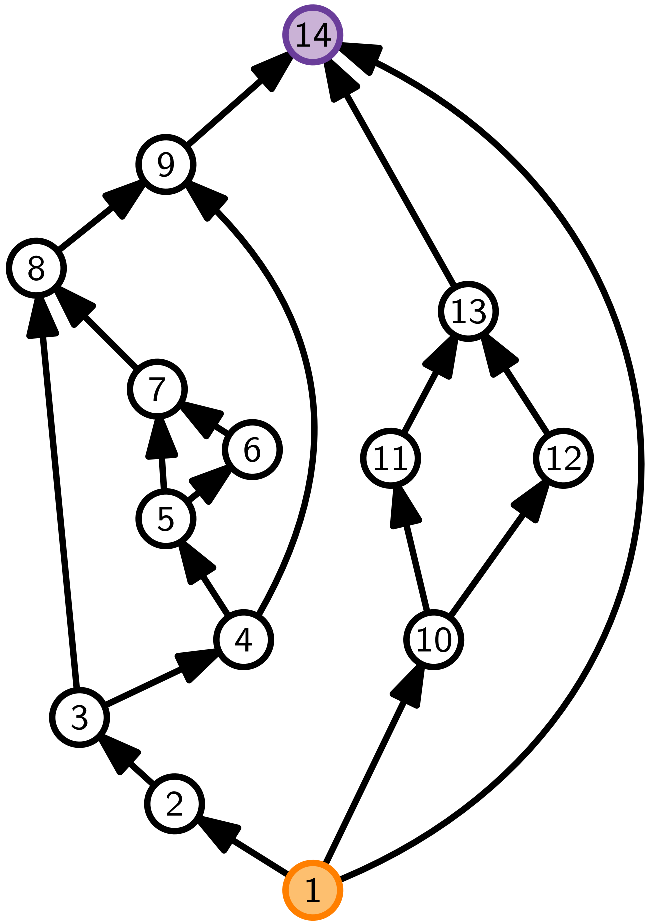

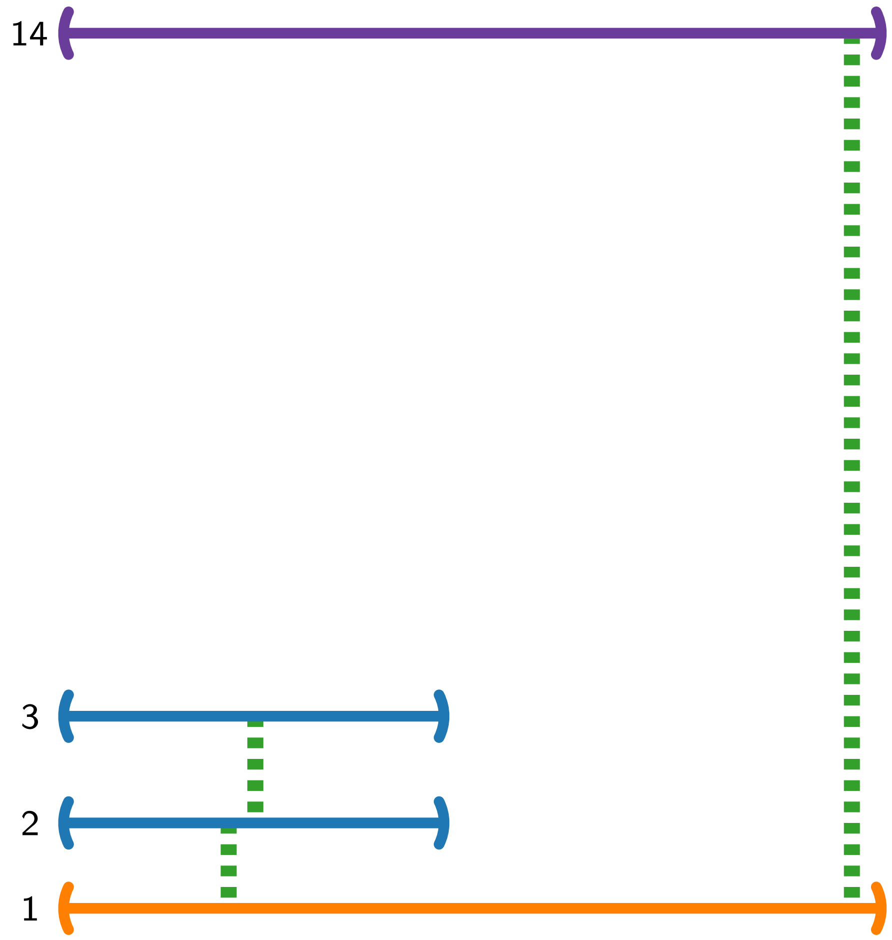

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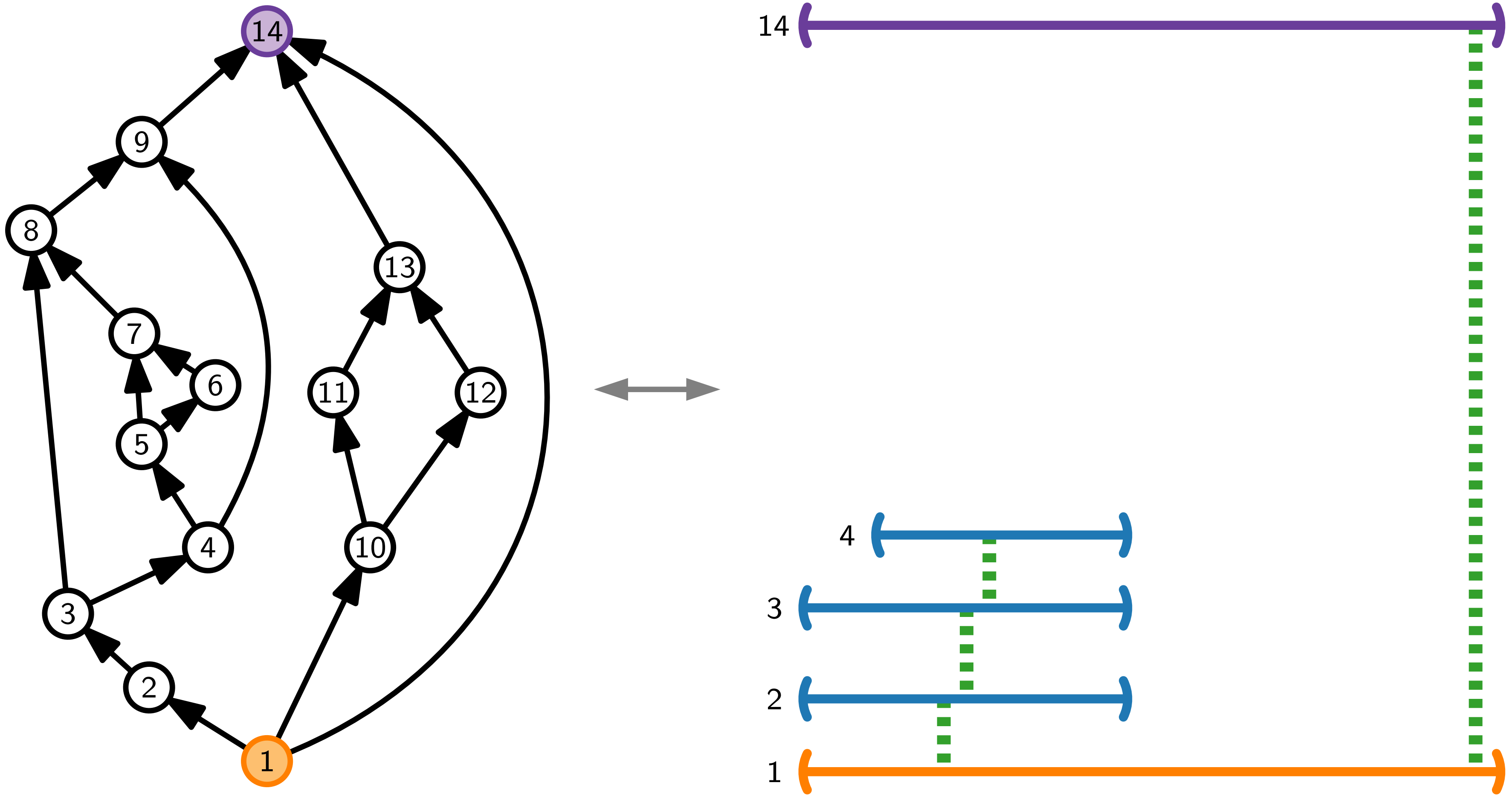

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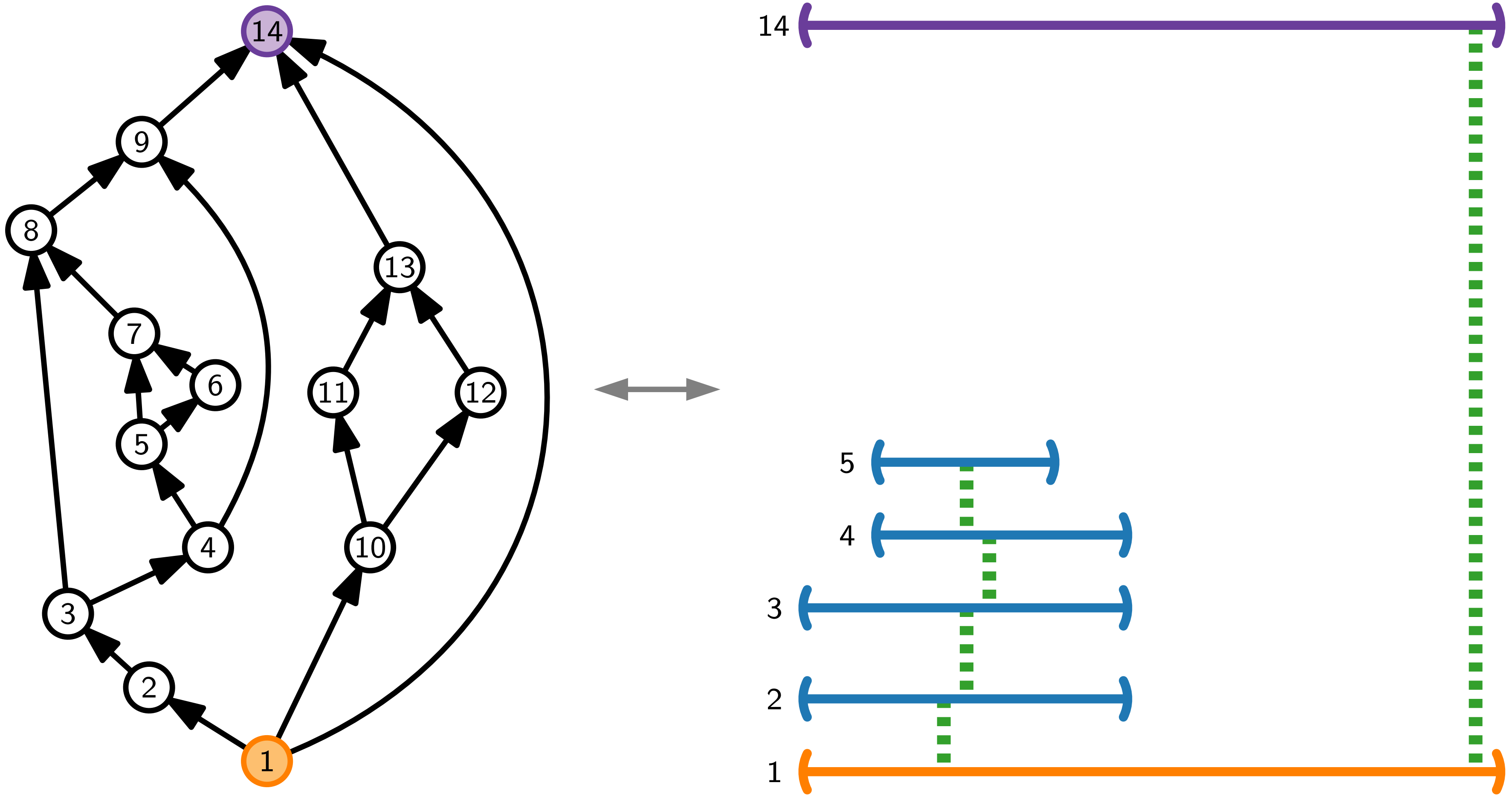

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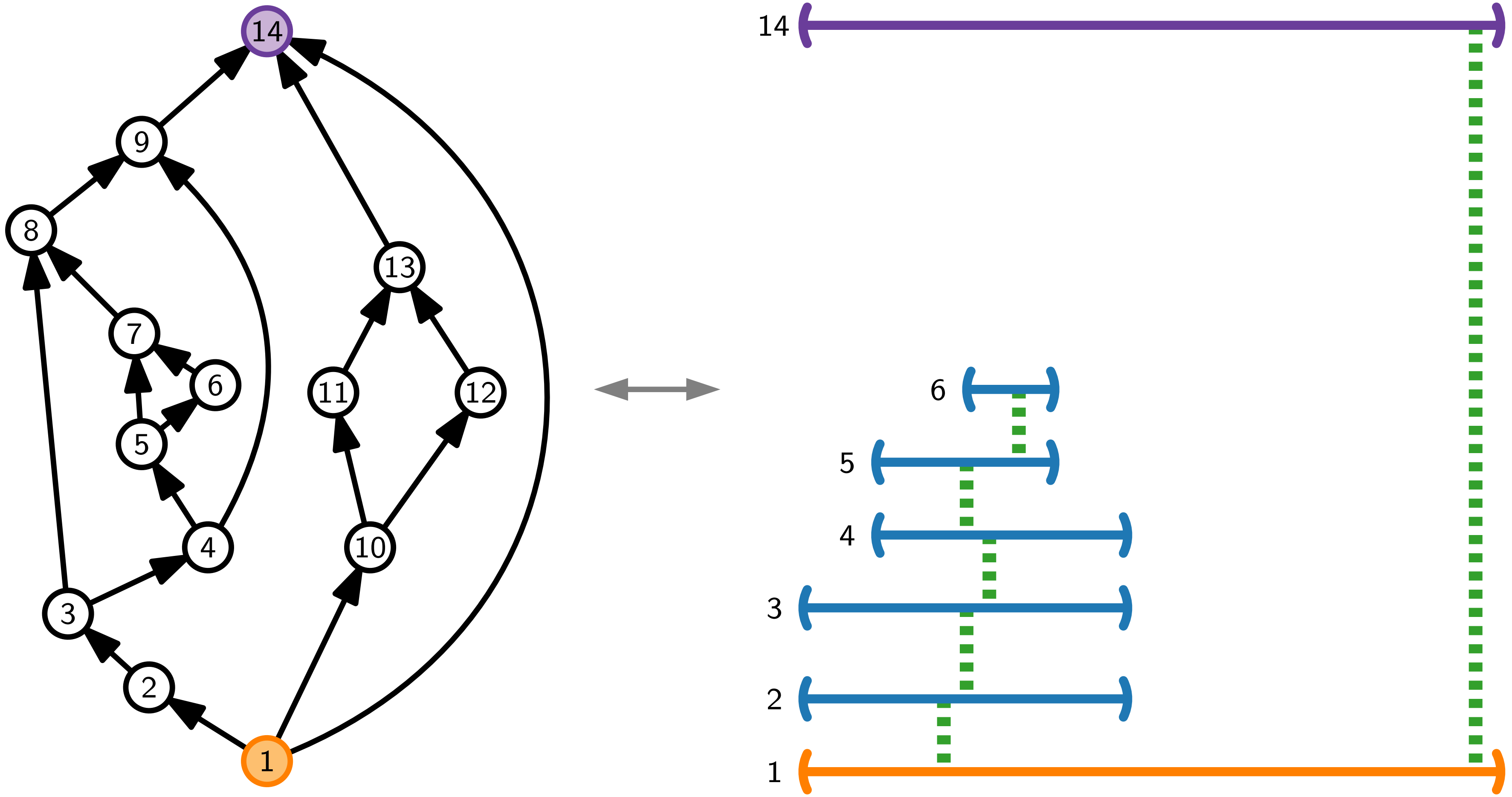

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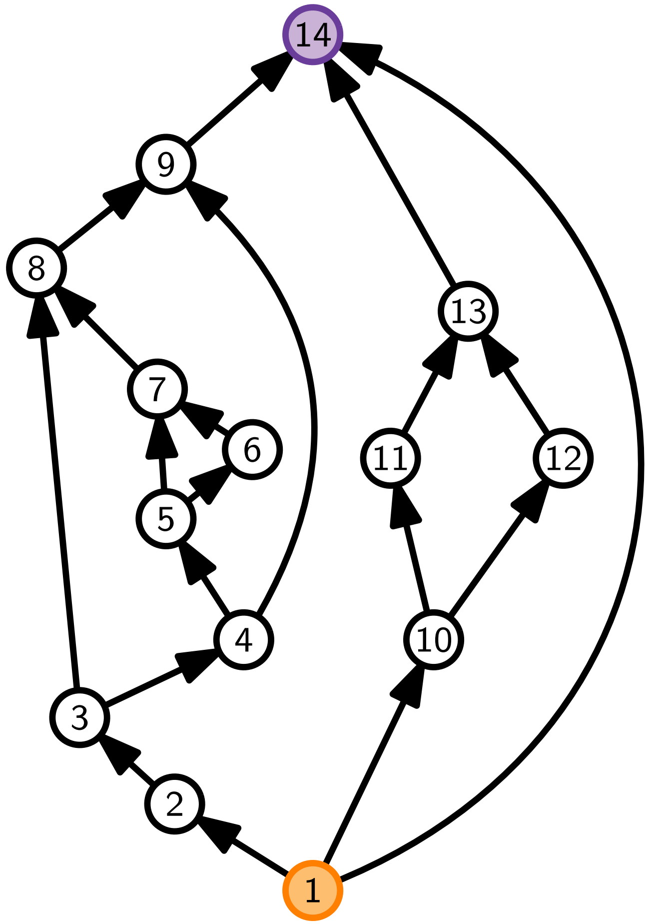

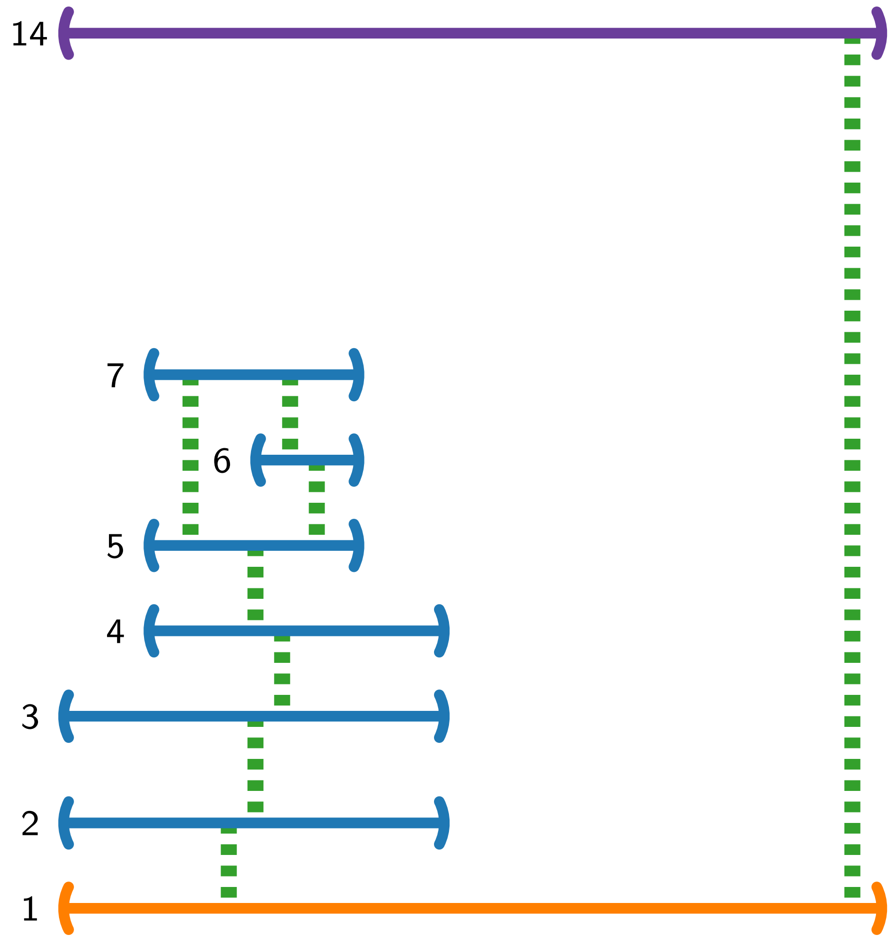

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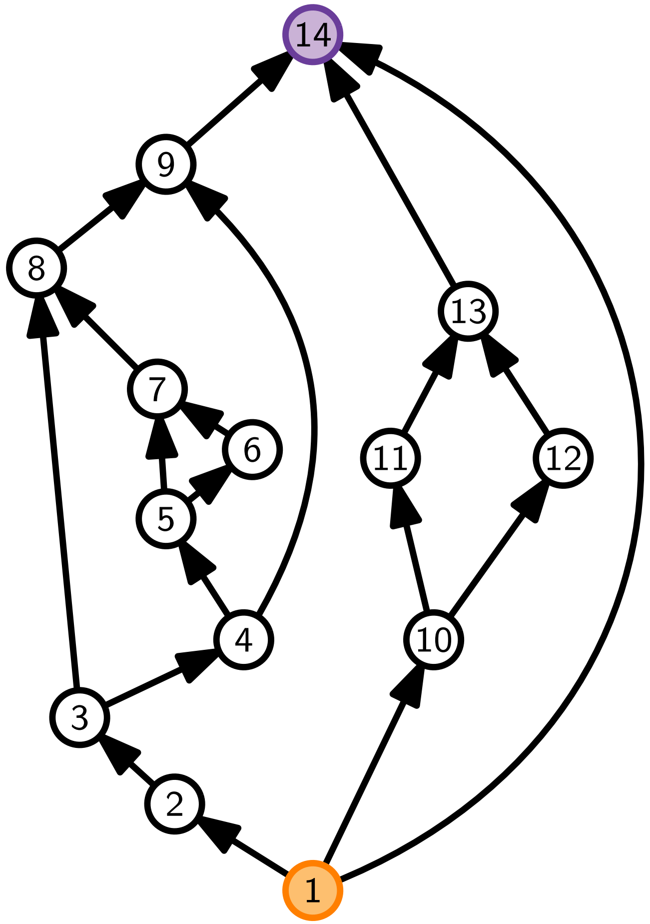

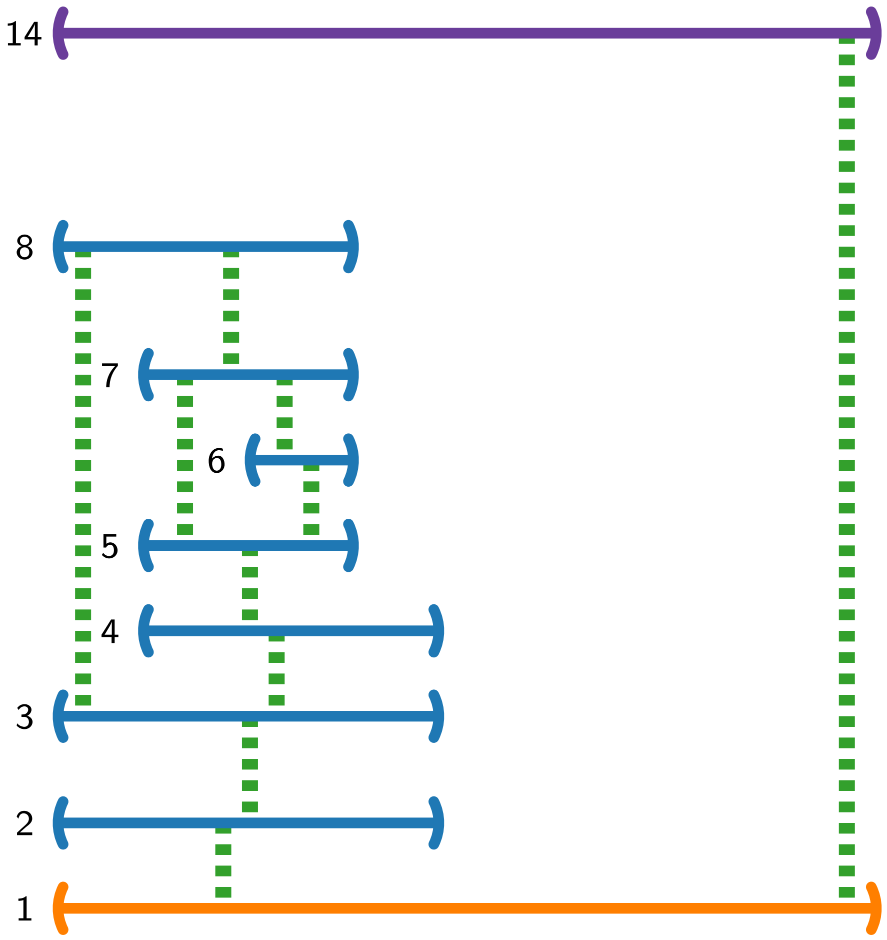

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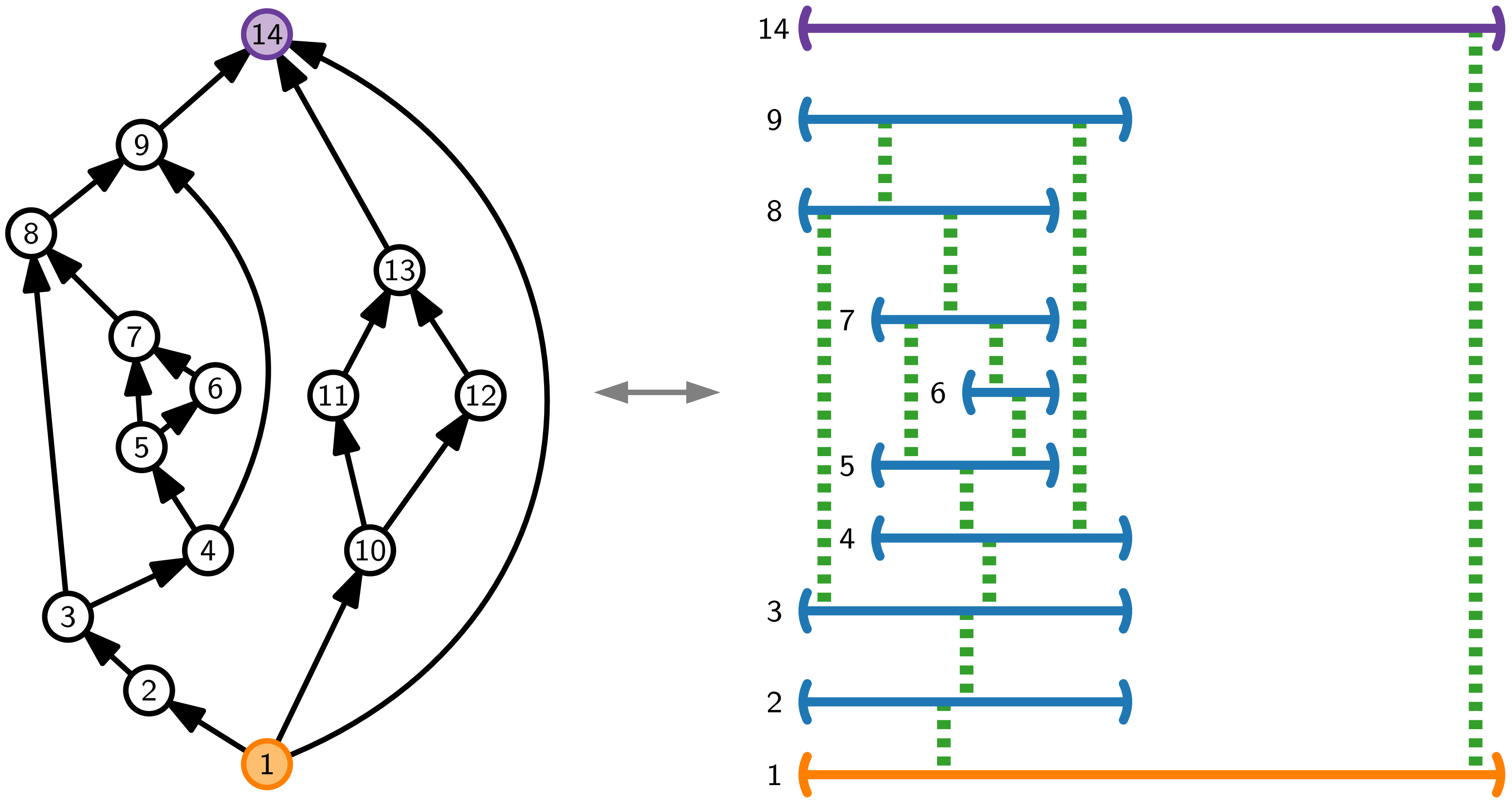

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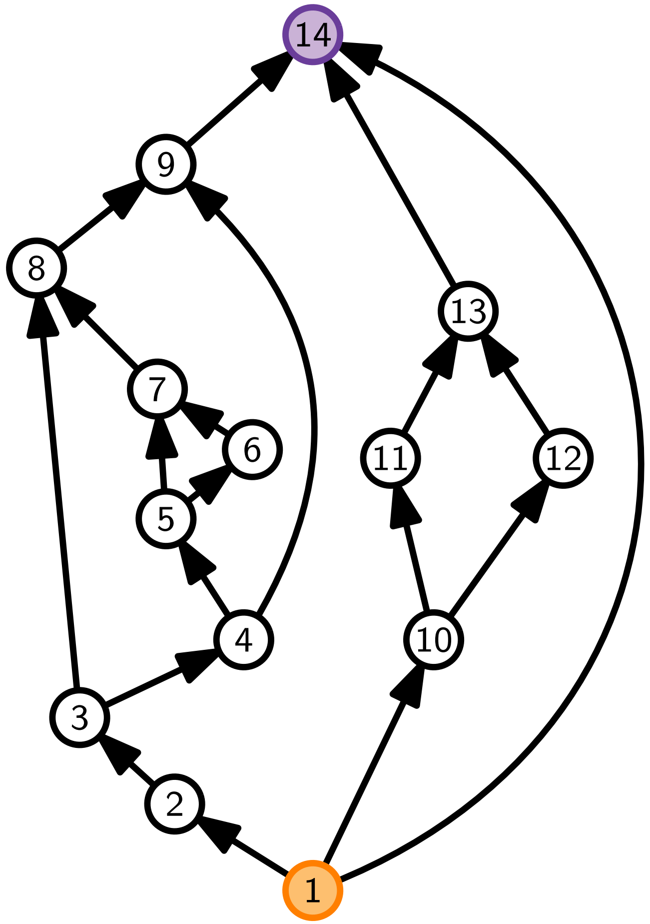

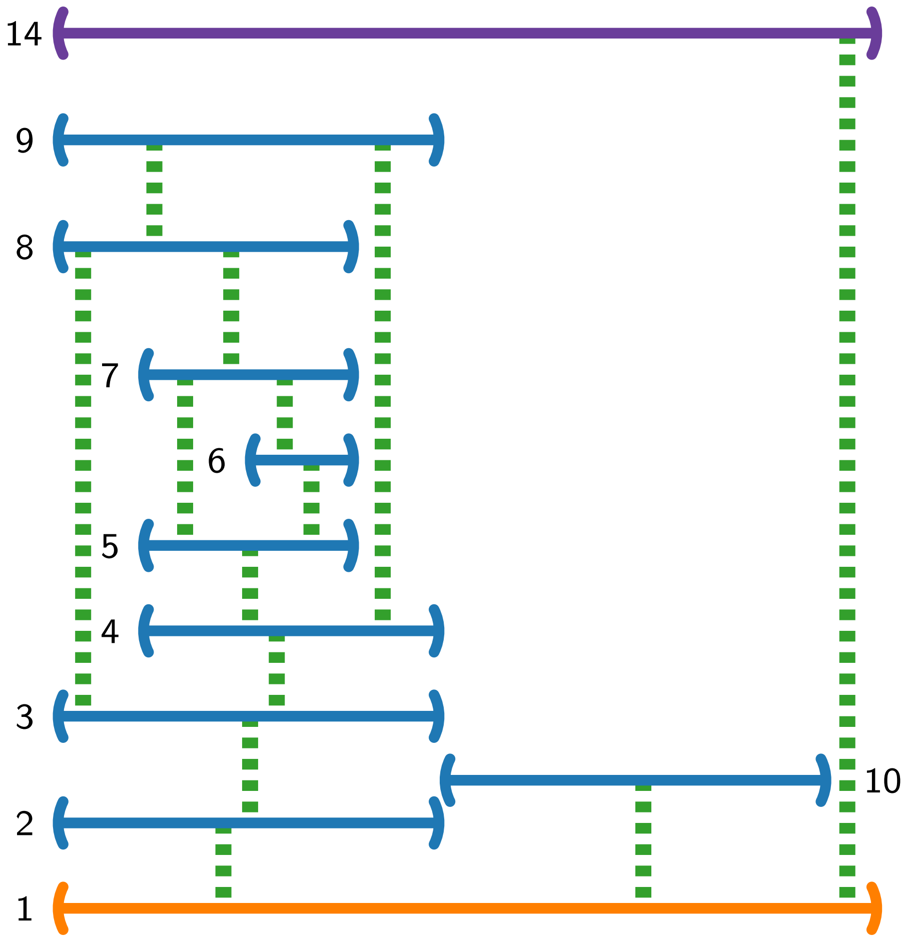

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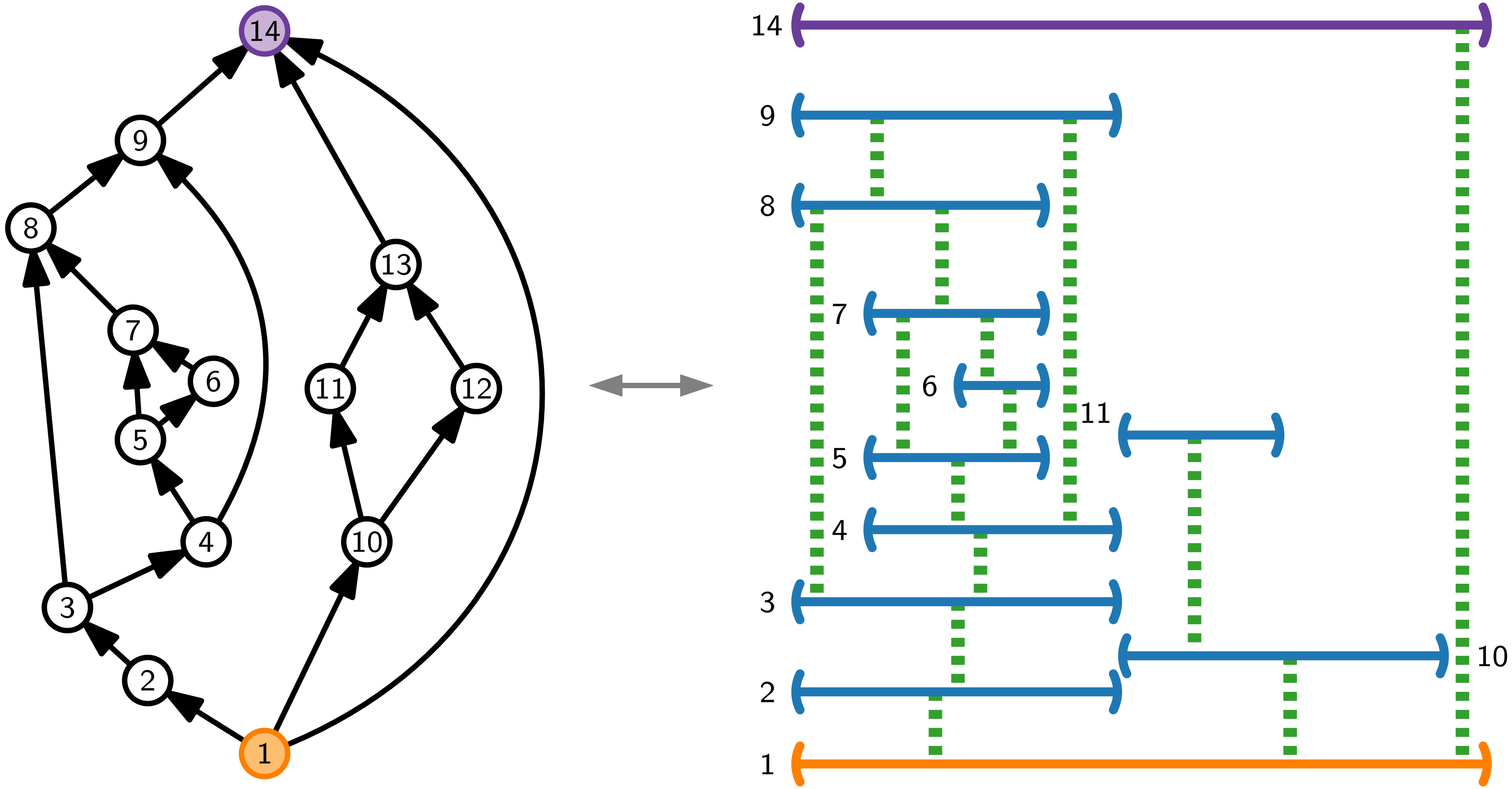

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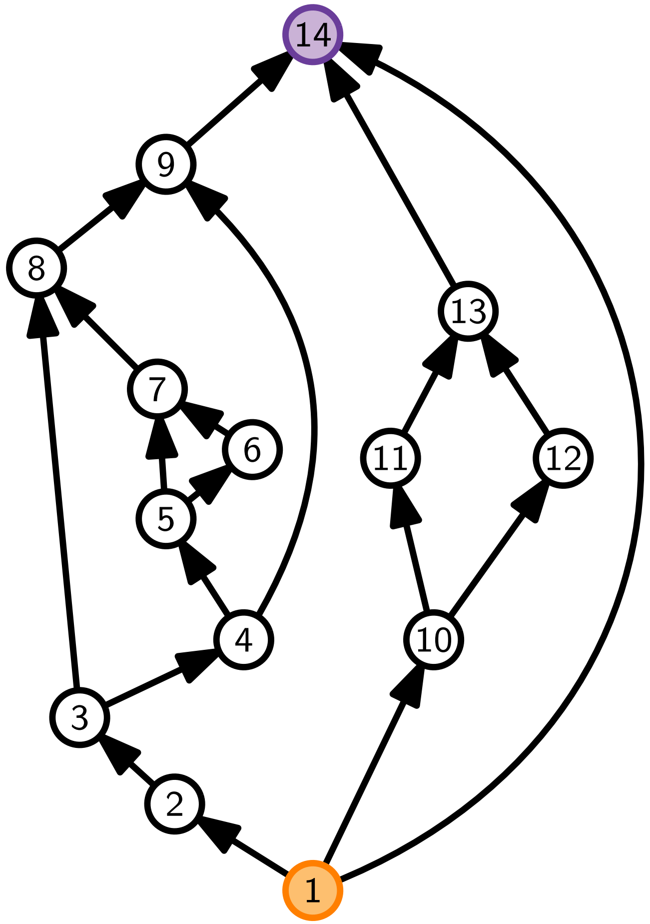

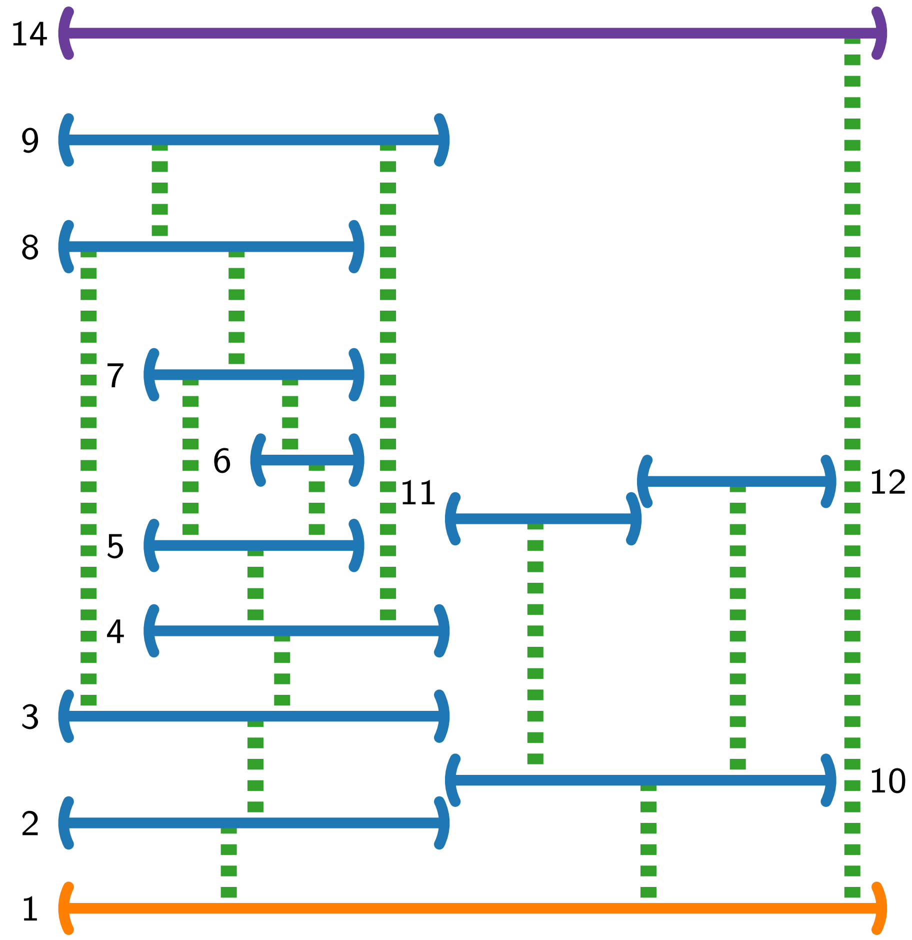

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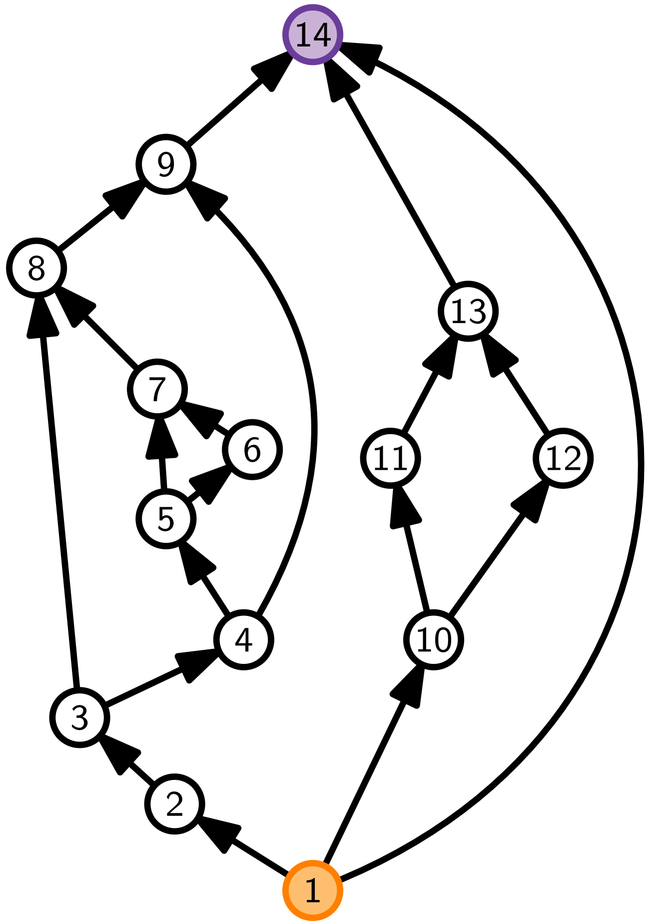

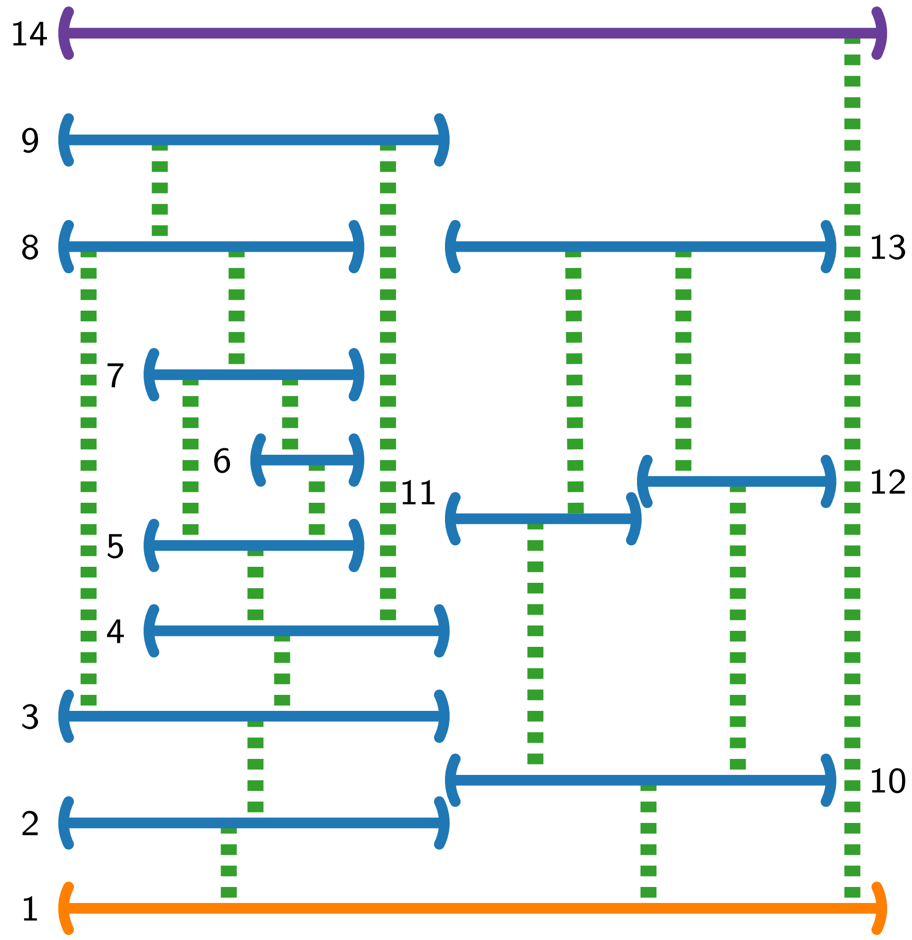

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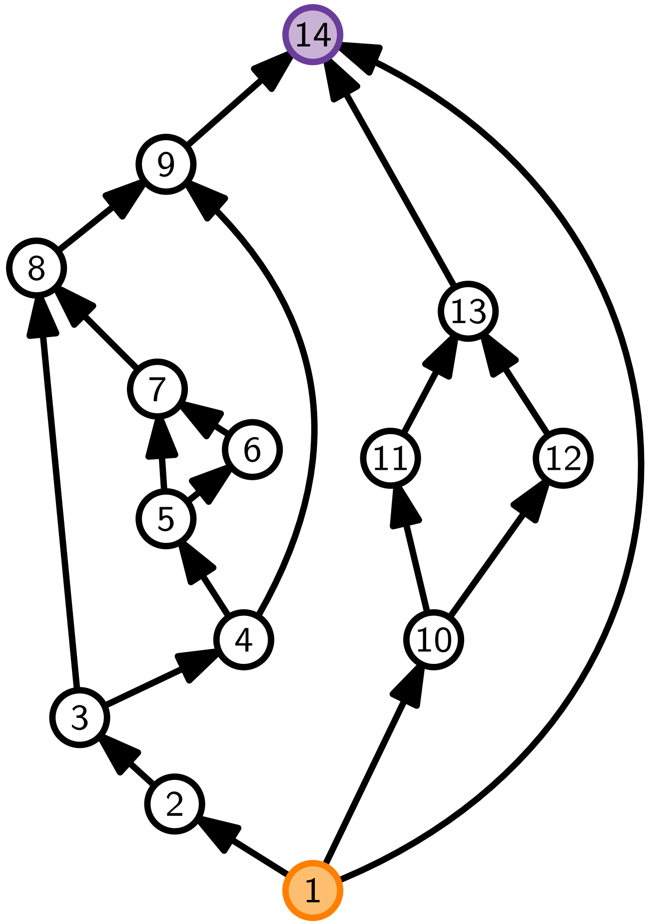

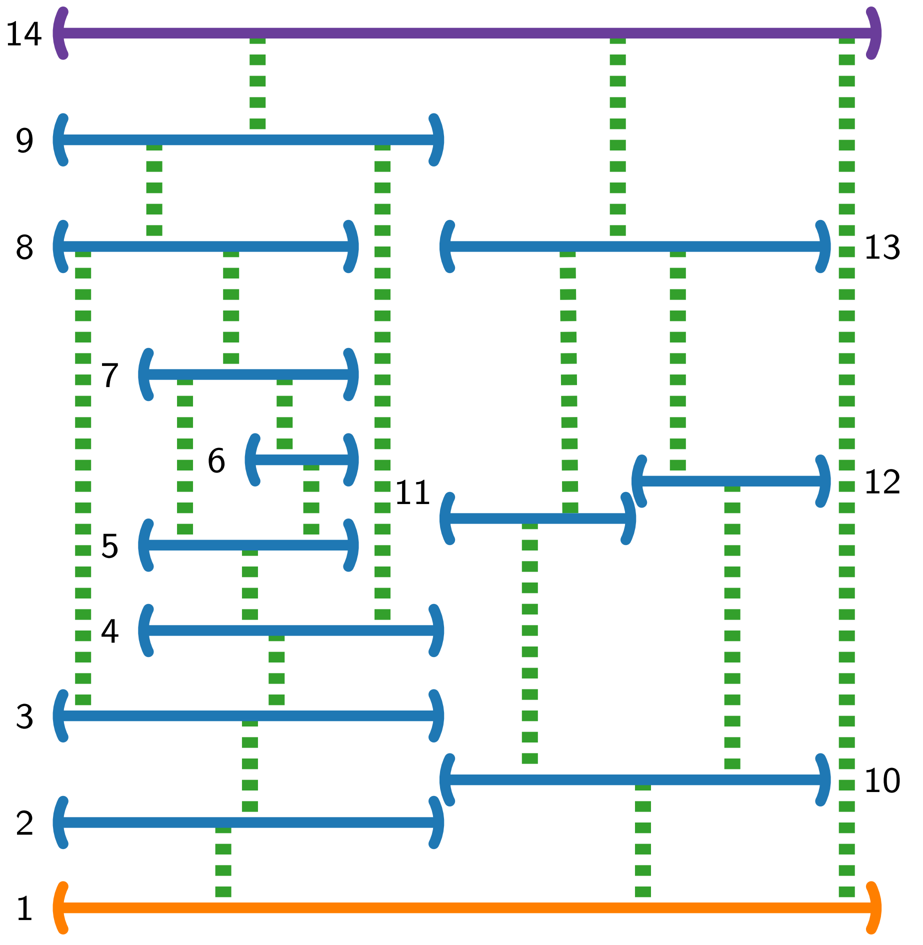

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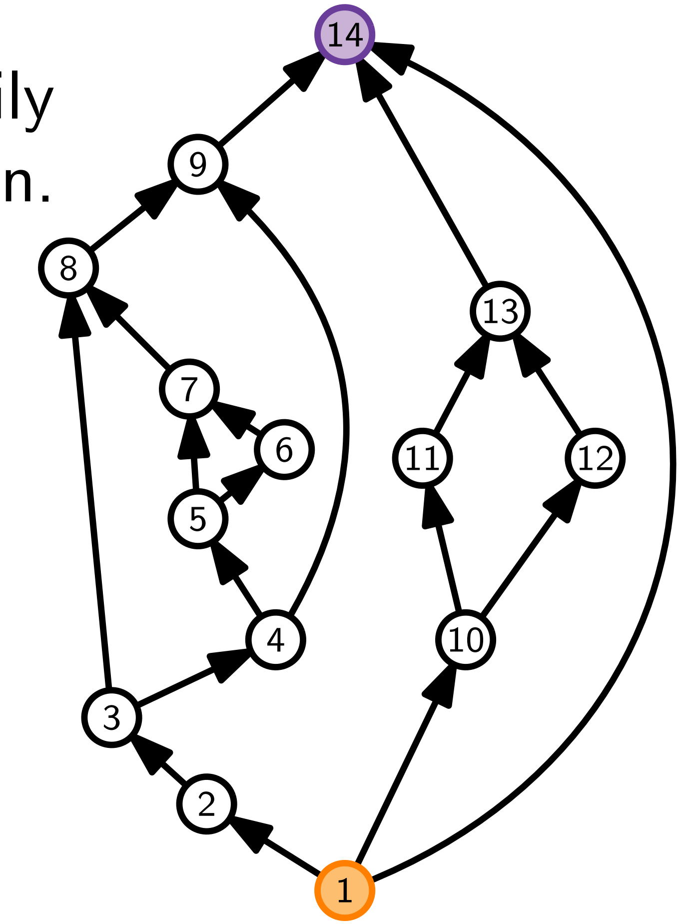

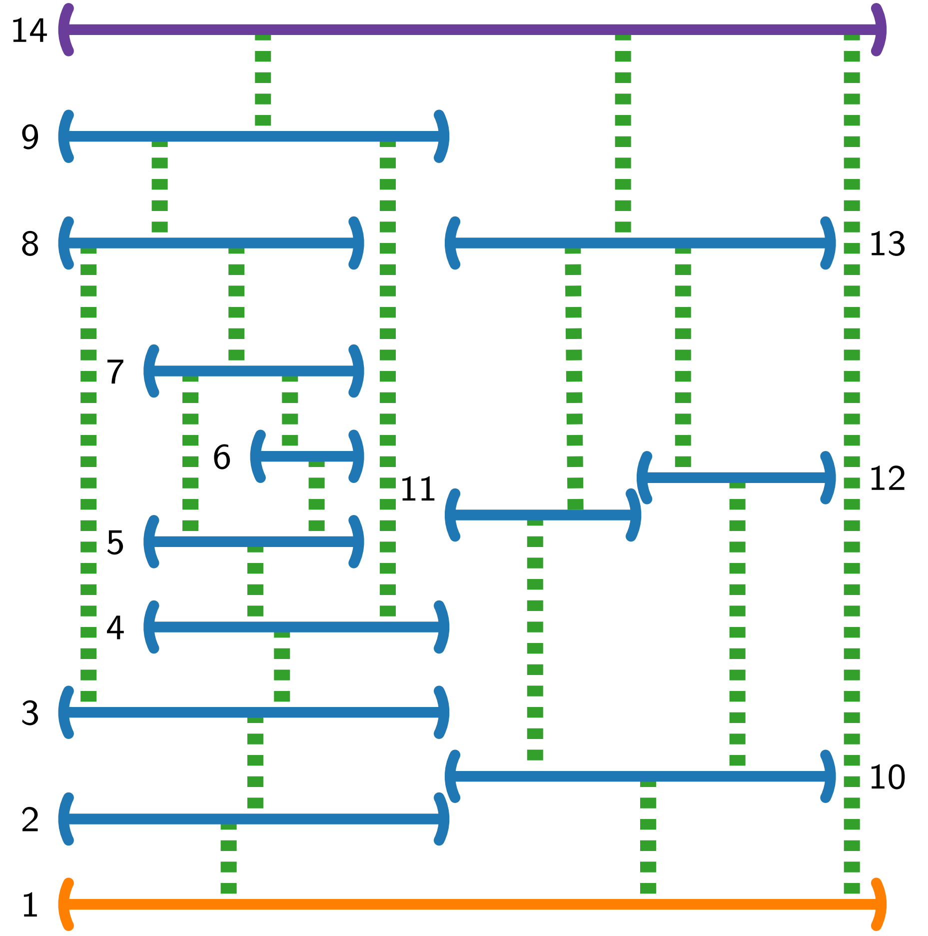

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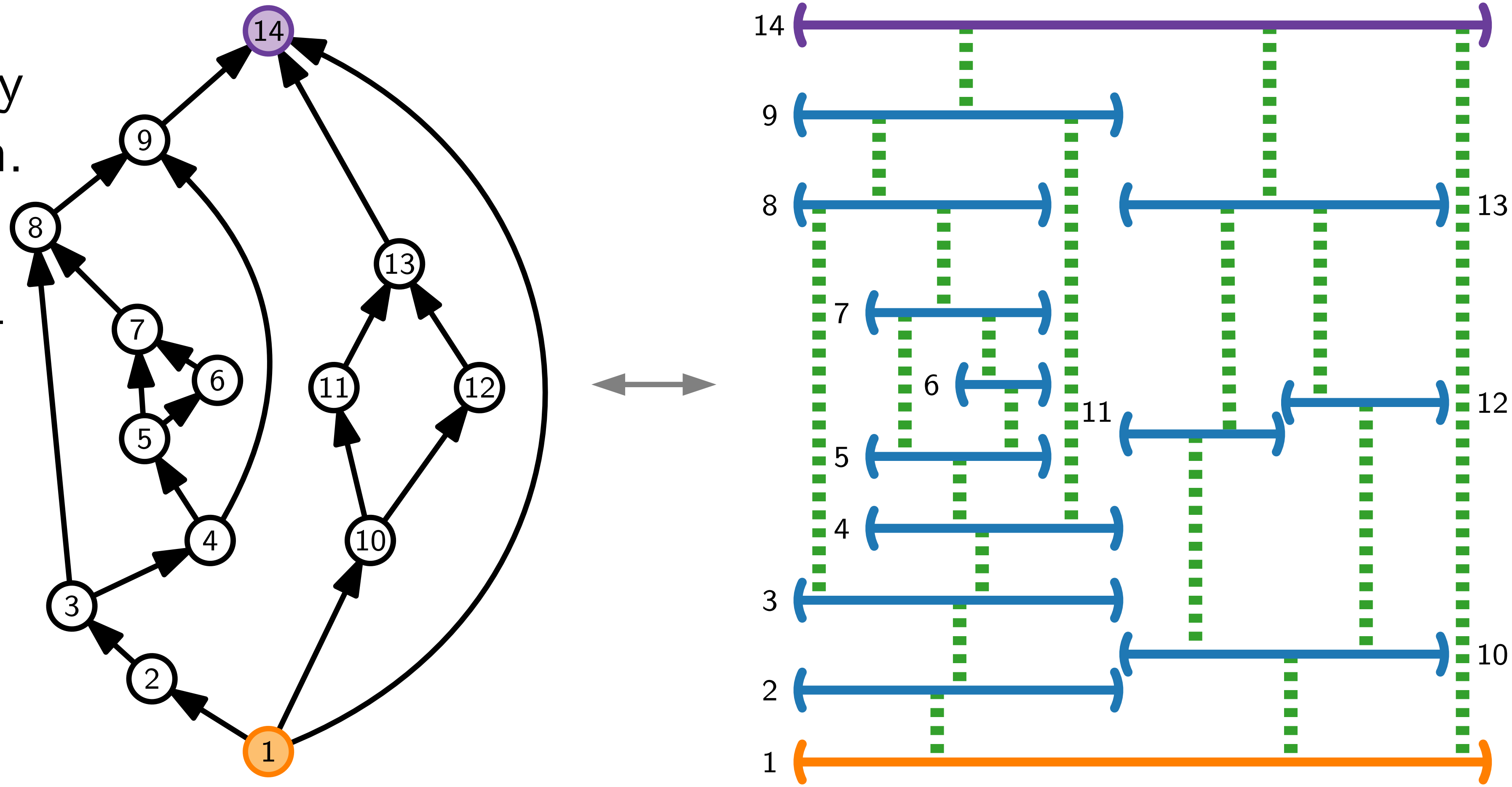

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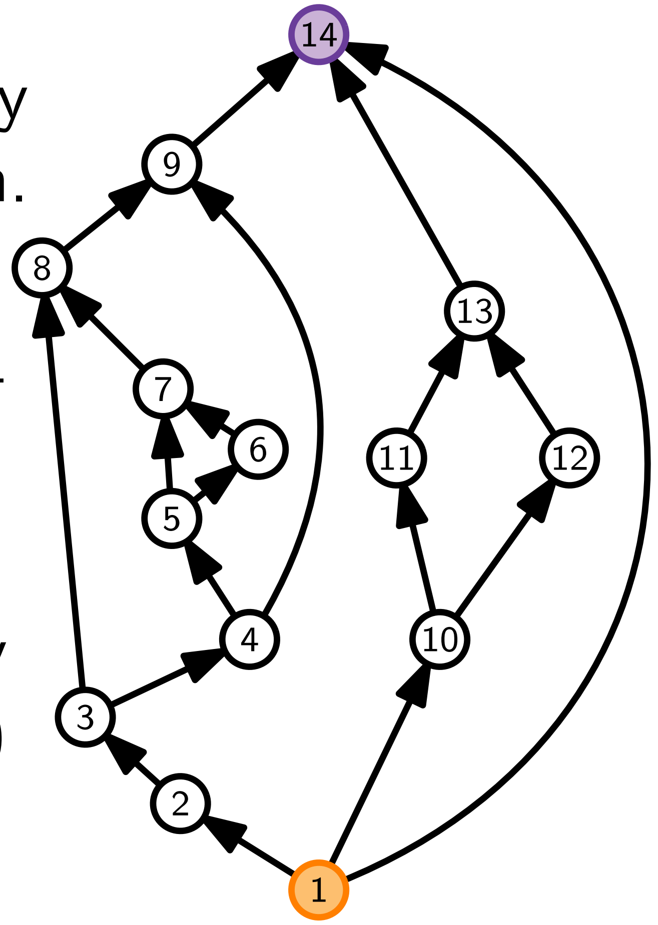

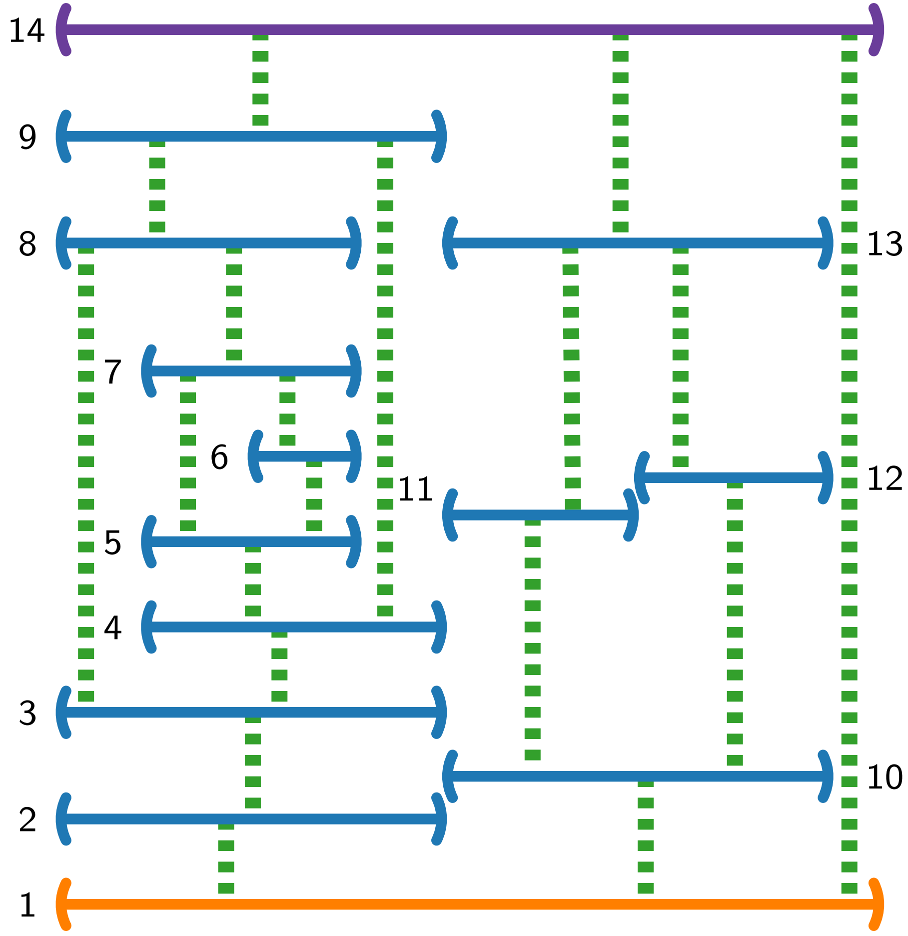

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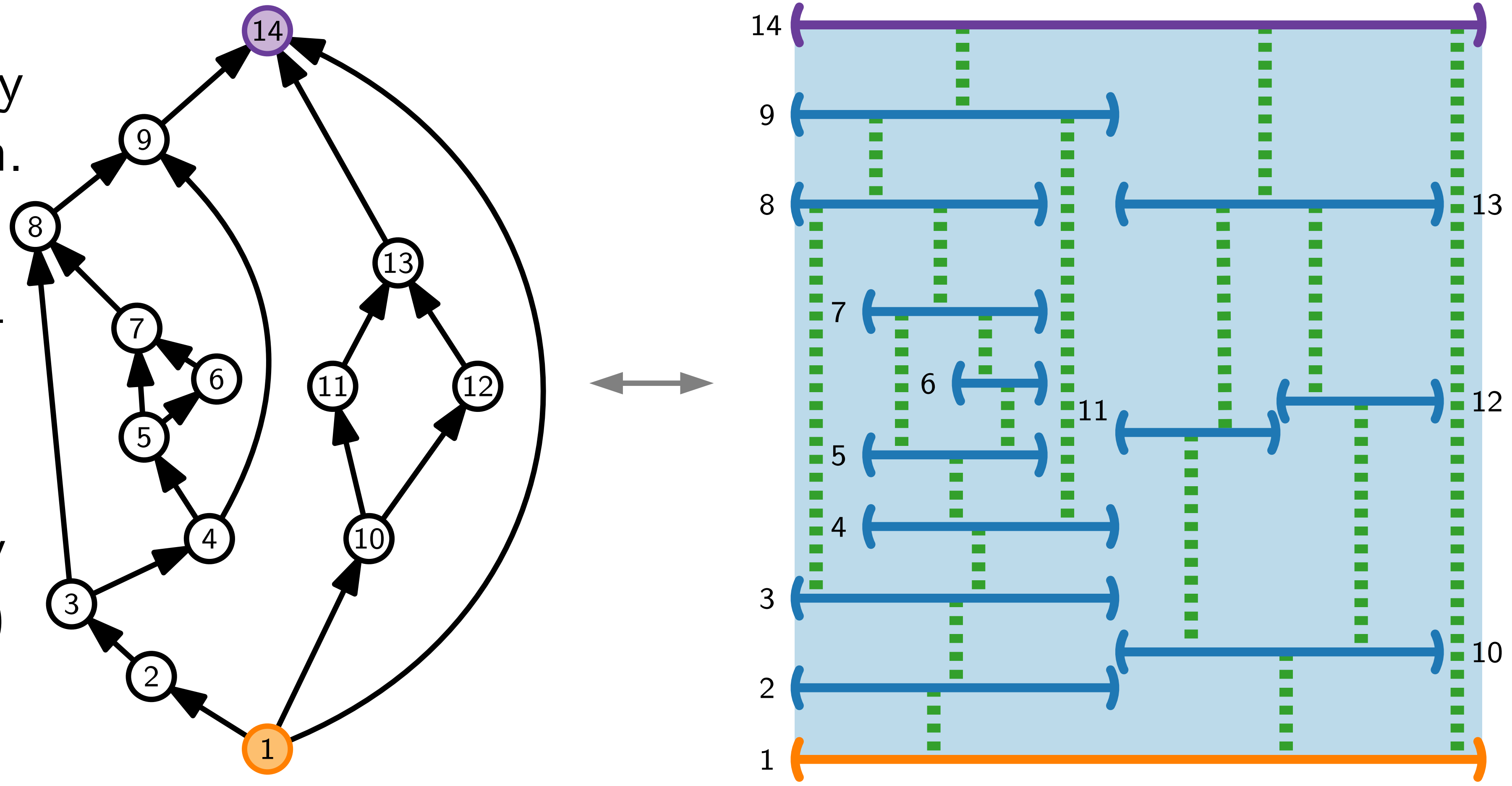

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[Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18]

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- Easier version: $\mathcal{O}(n^2)$

Theorem 2.
ε-bar visibility representation extension is NP-complete.

- Reduction from Planar Monotone 3-SAT

Theorem 3.
ε-bar visibility representation extension is NP-complete even for (series-parallel) st-graphs when restricted to the *integer grid* (or if any fixed $\varepsilon > 0$ is specified).

- Reduction from 3-Partition

SPQR-Tree

- An **SPQR-tree** T is a decomposition of a planar graph G by **separation pairs**.

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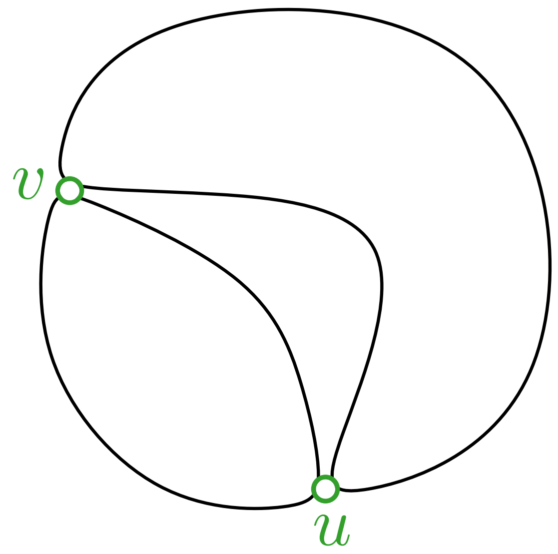

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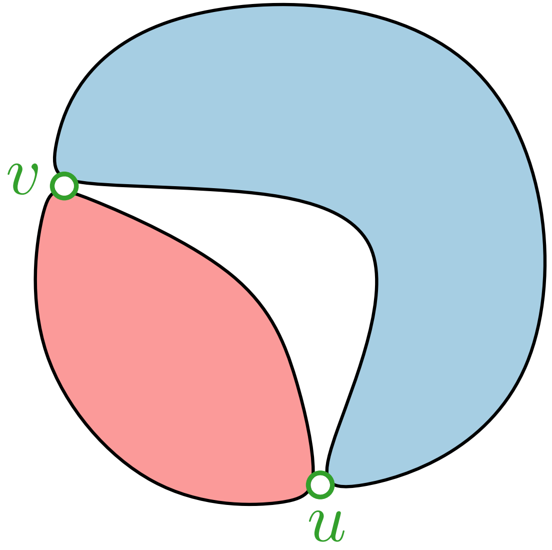

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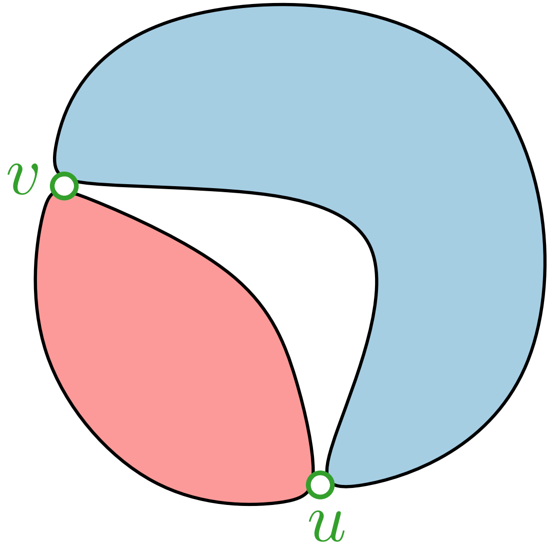

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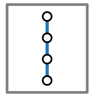

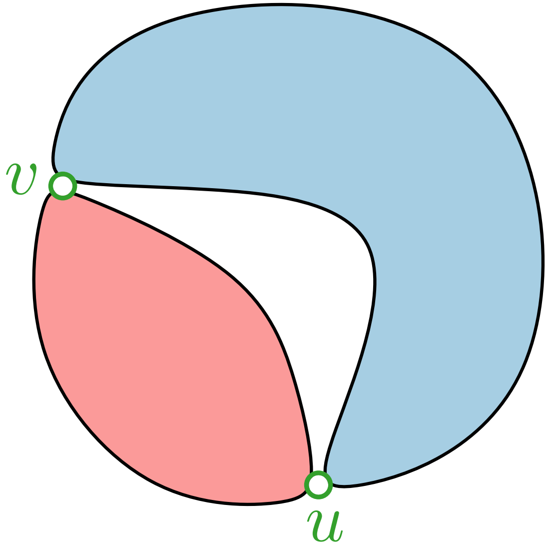

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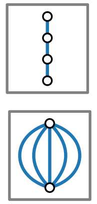

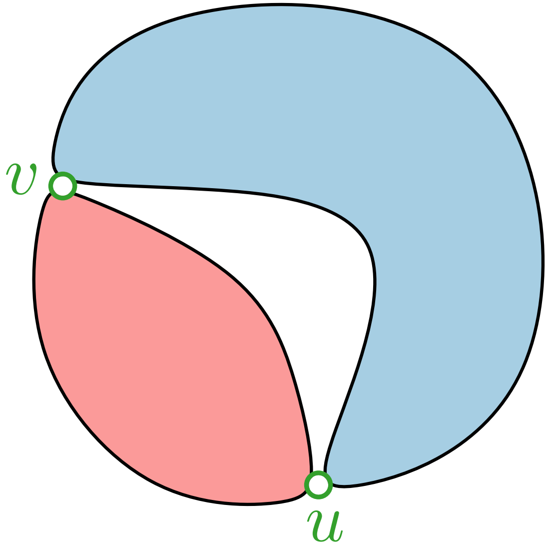

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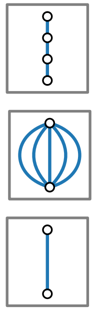

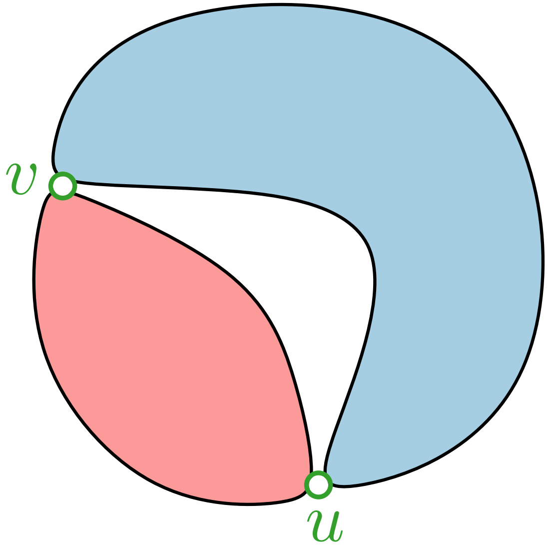

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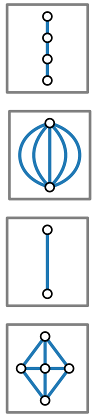

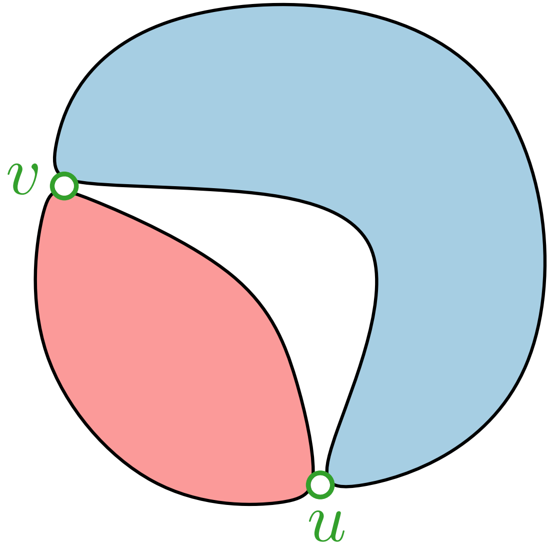

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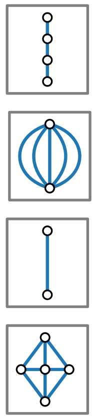

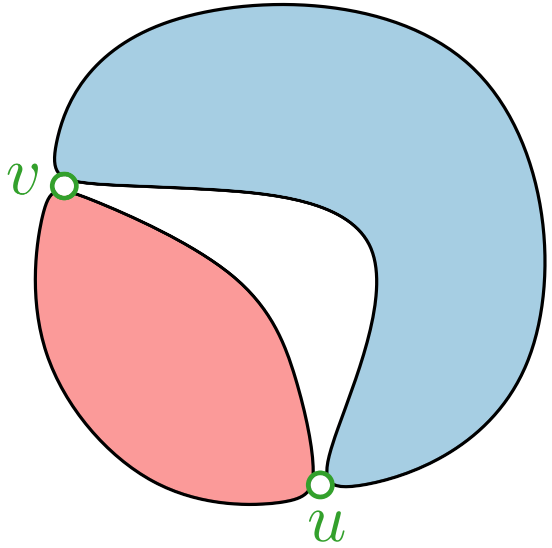

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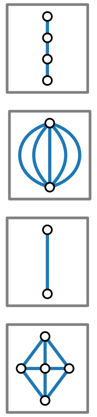

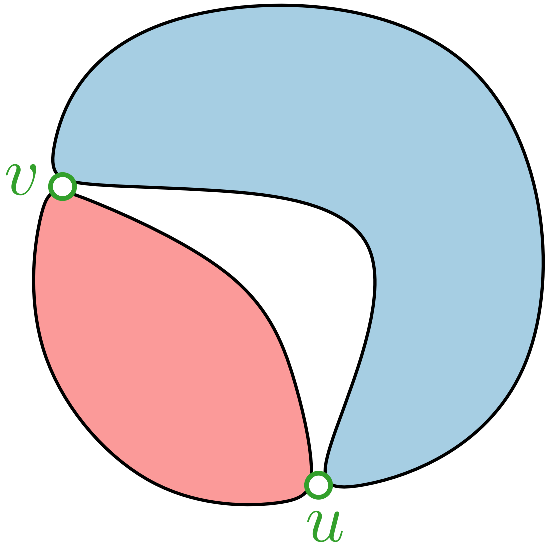

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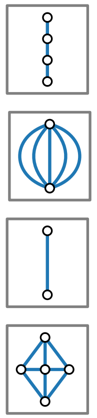

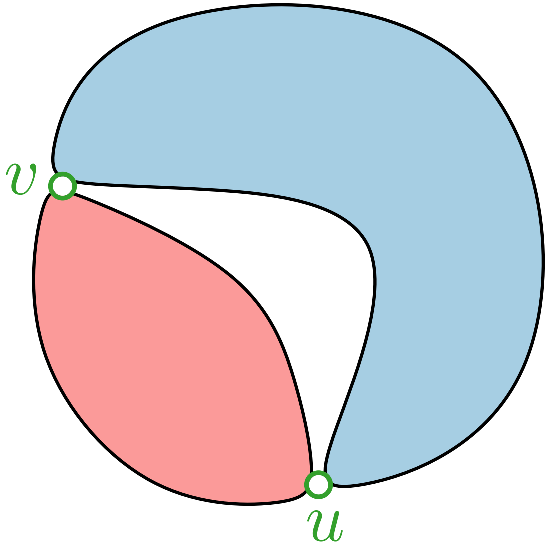

- A decomposition tree of a series-parallel graph is an SPQR-tree without **R**-nodes.
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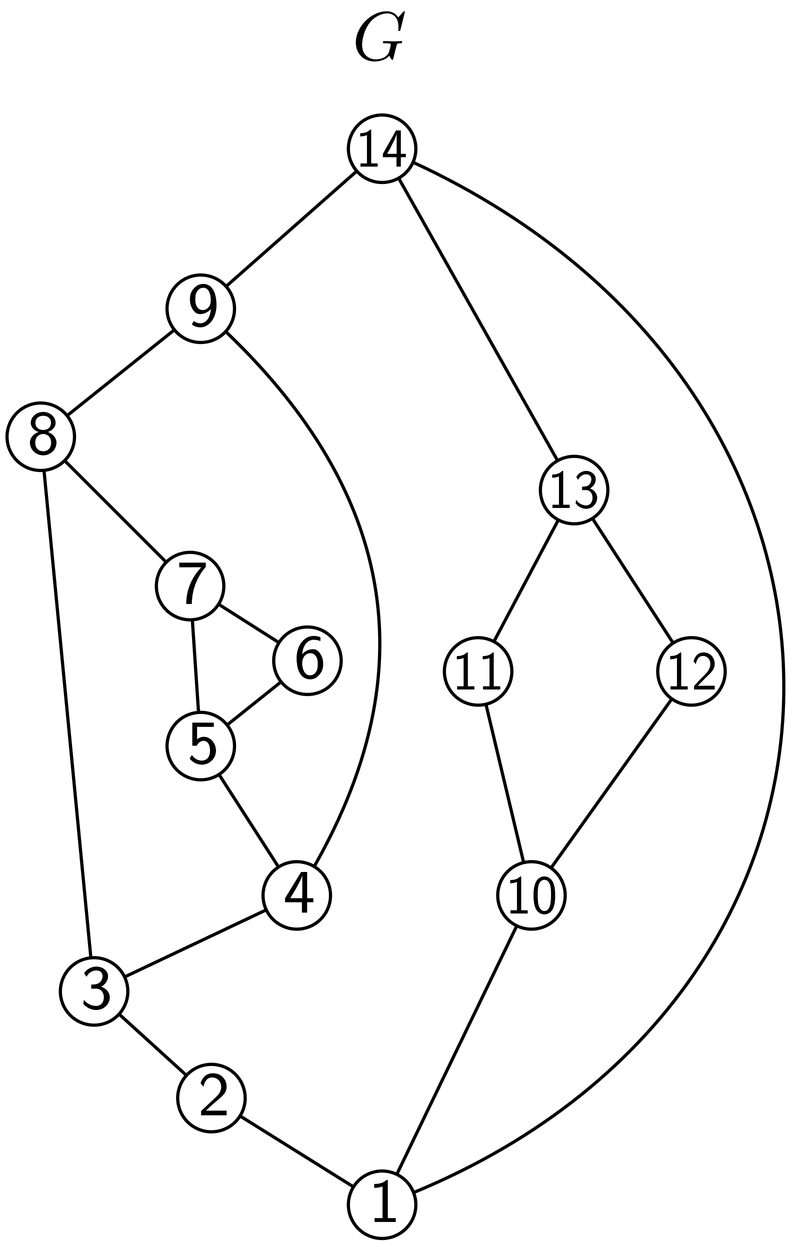
- T can be computed in time linear in the size of G. [Gutwenger, Mutzel '01]

SPQR-Tree – Example



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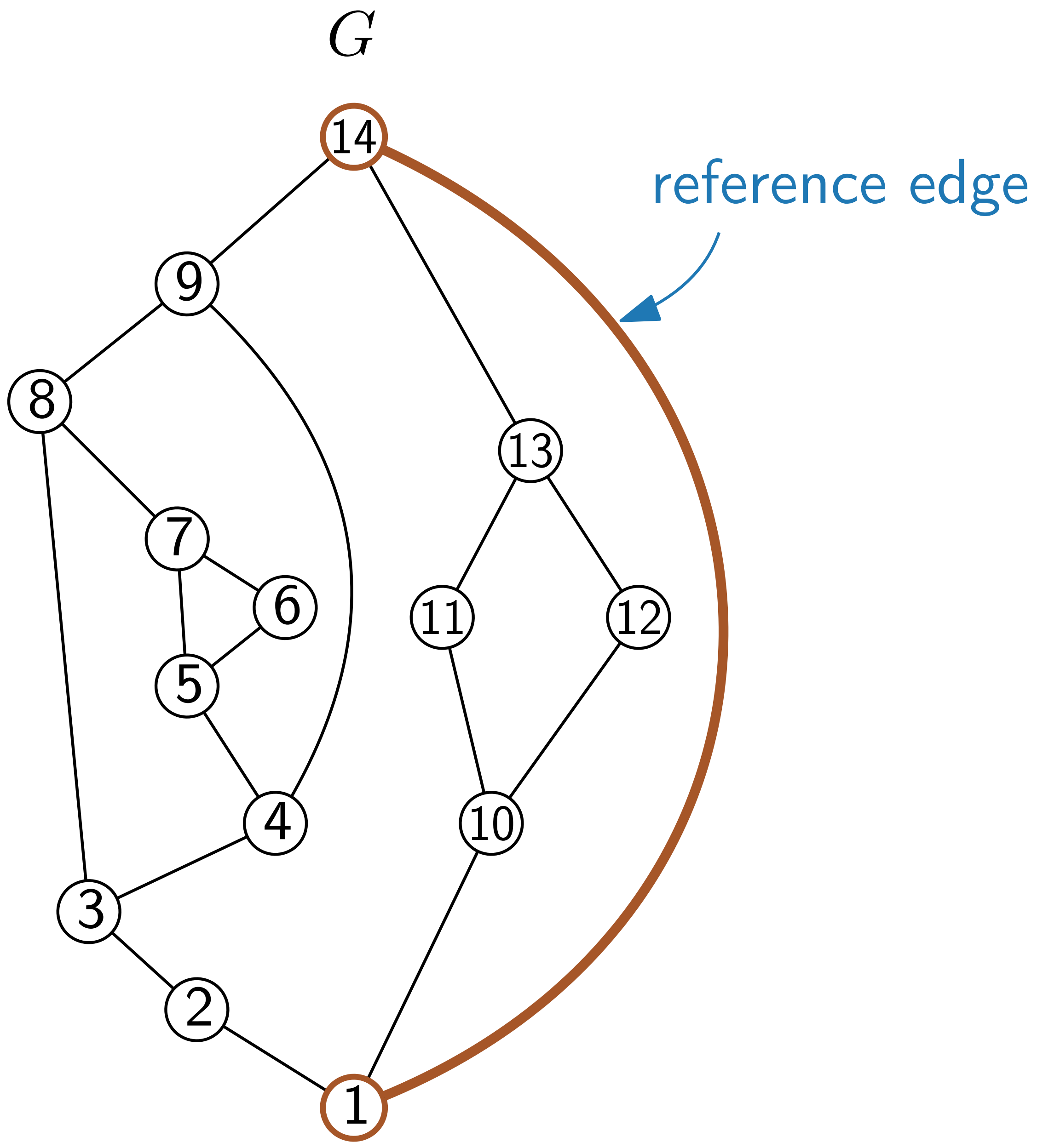

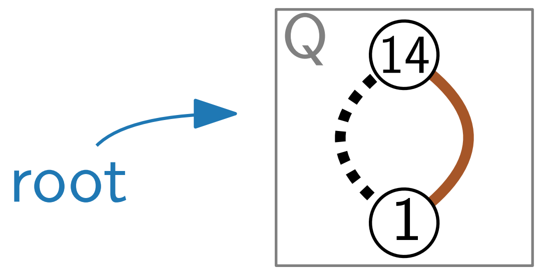

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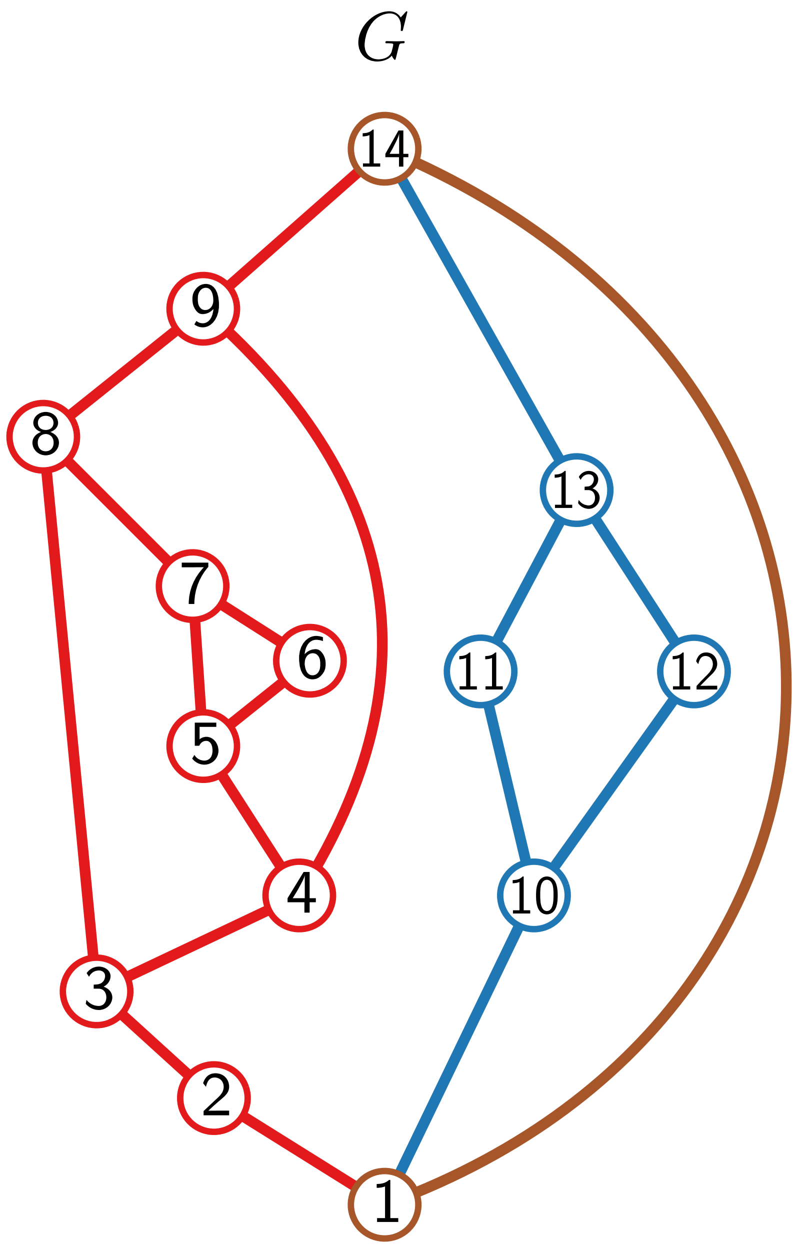

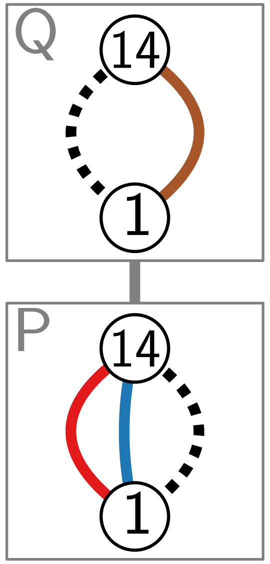

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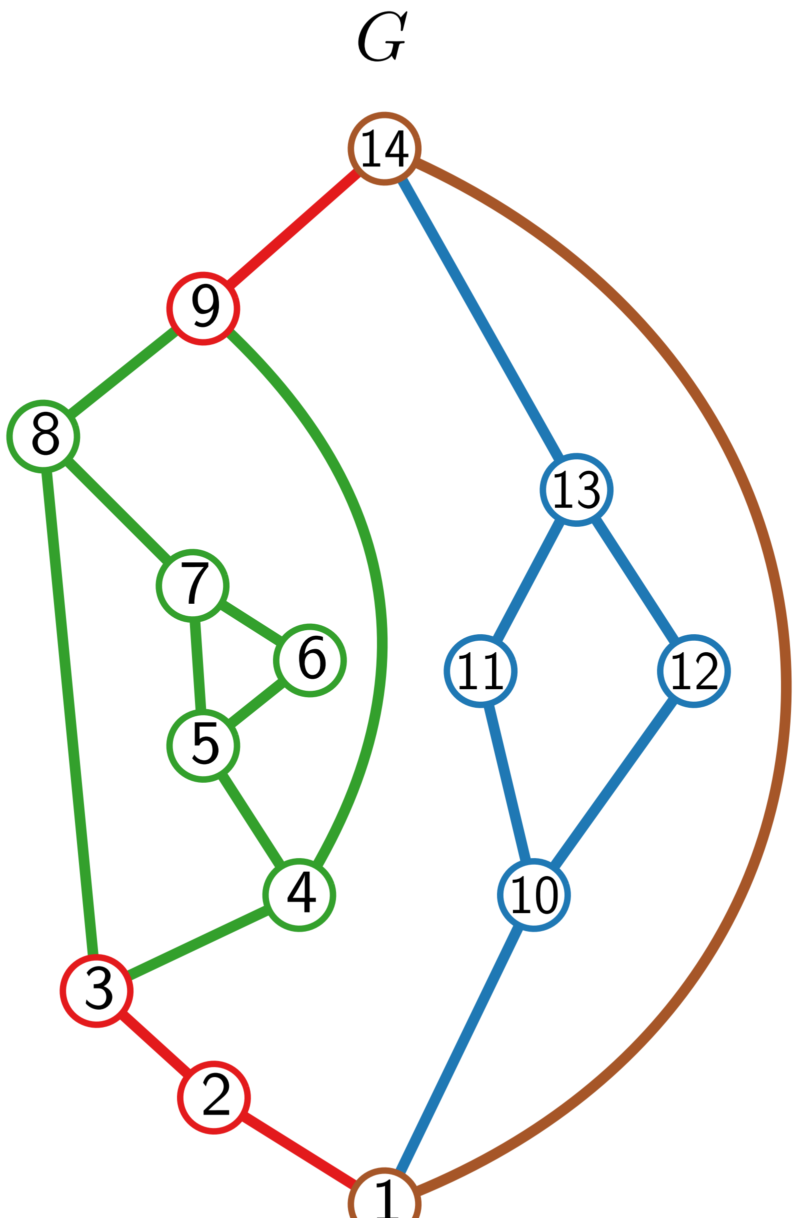

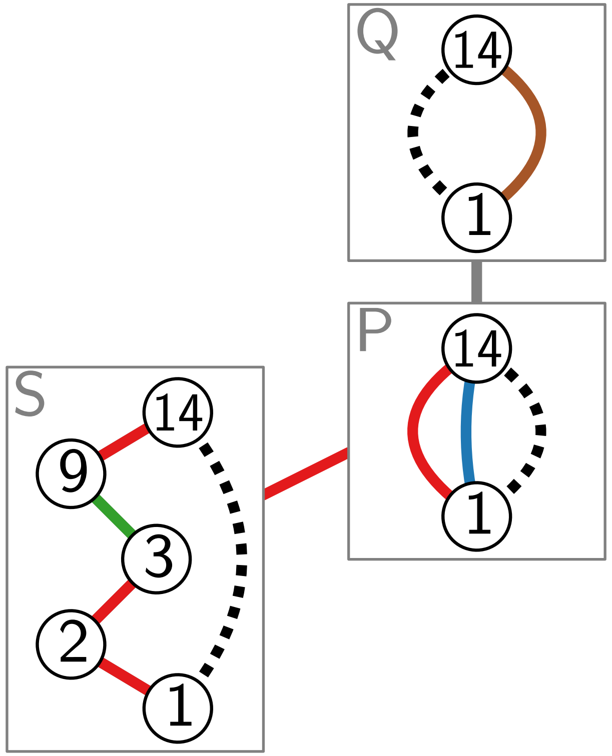

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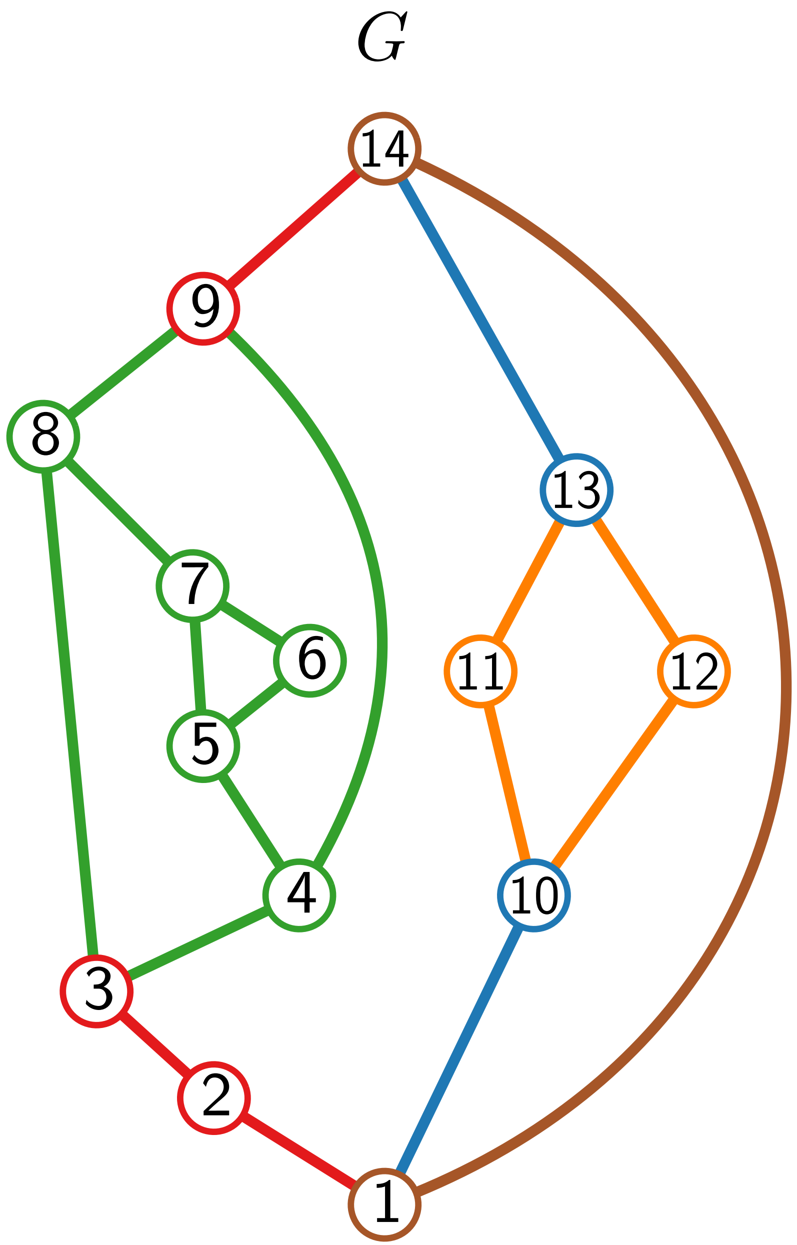

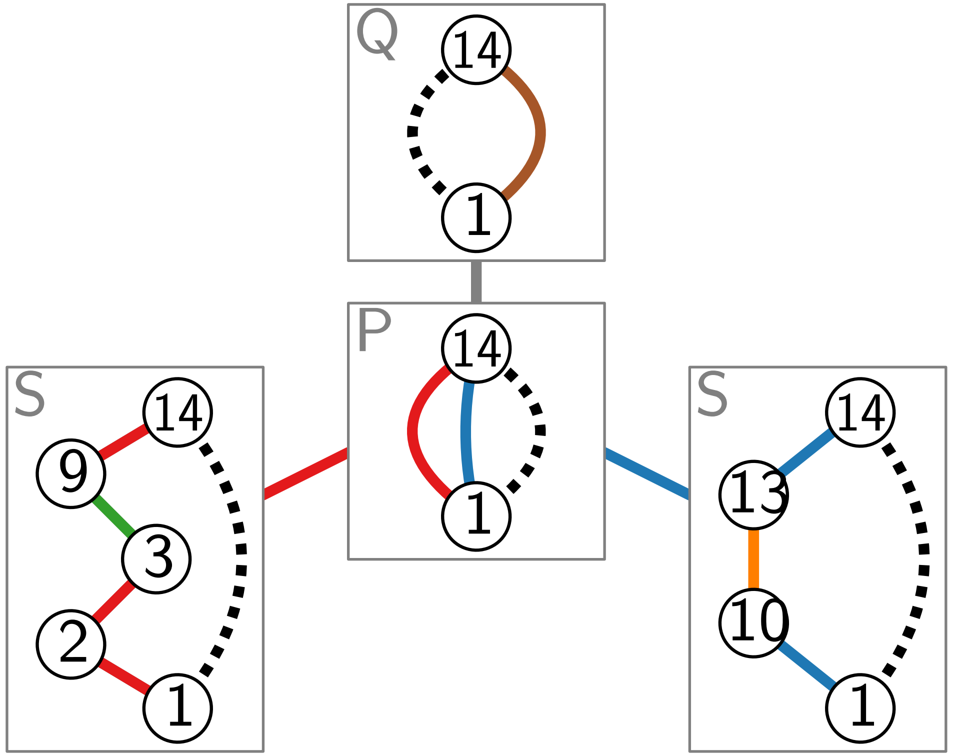

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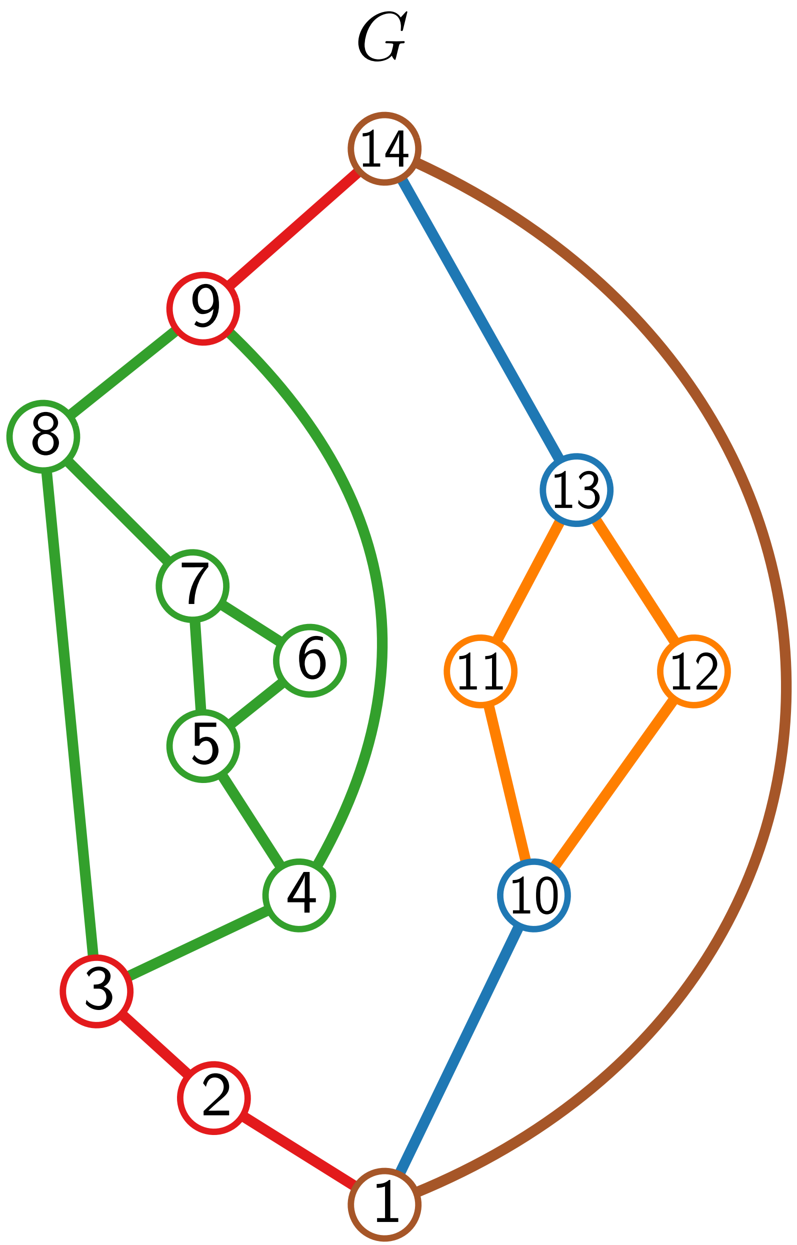

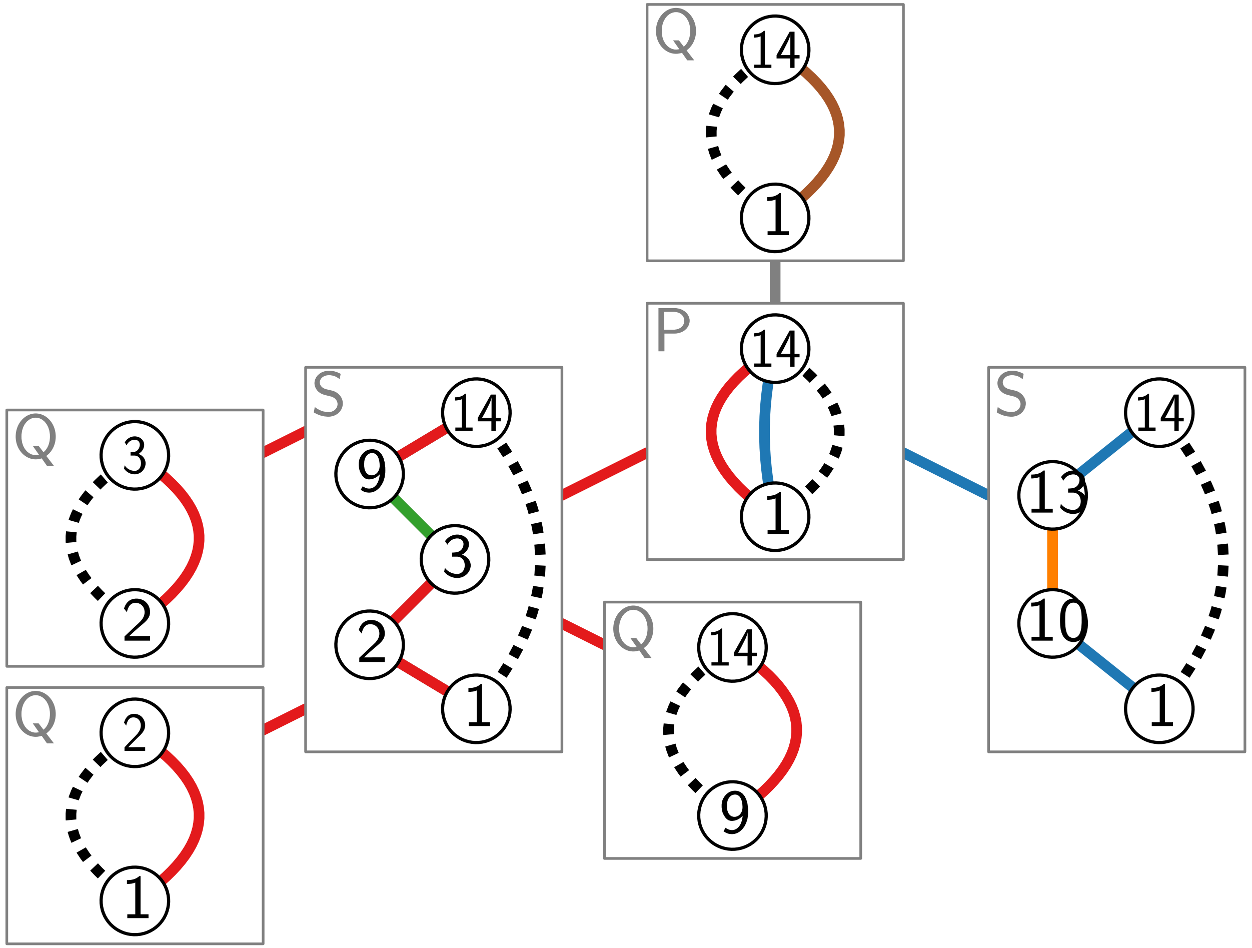

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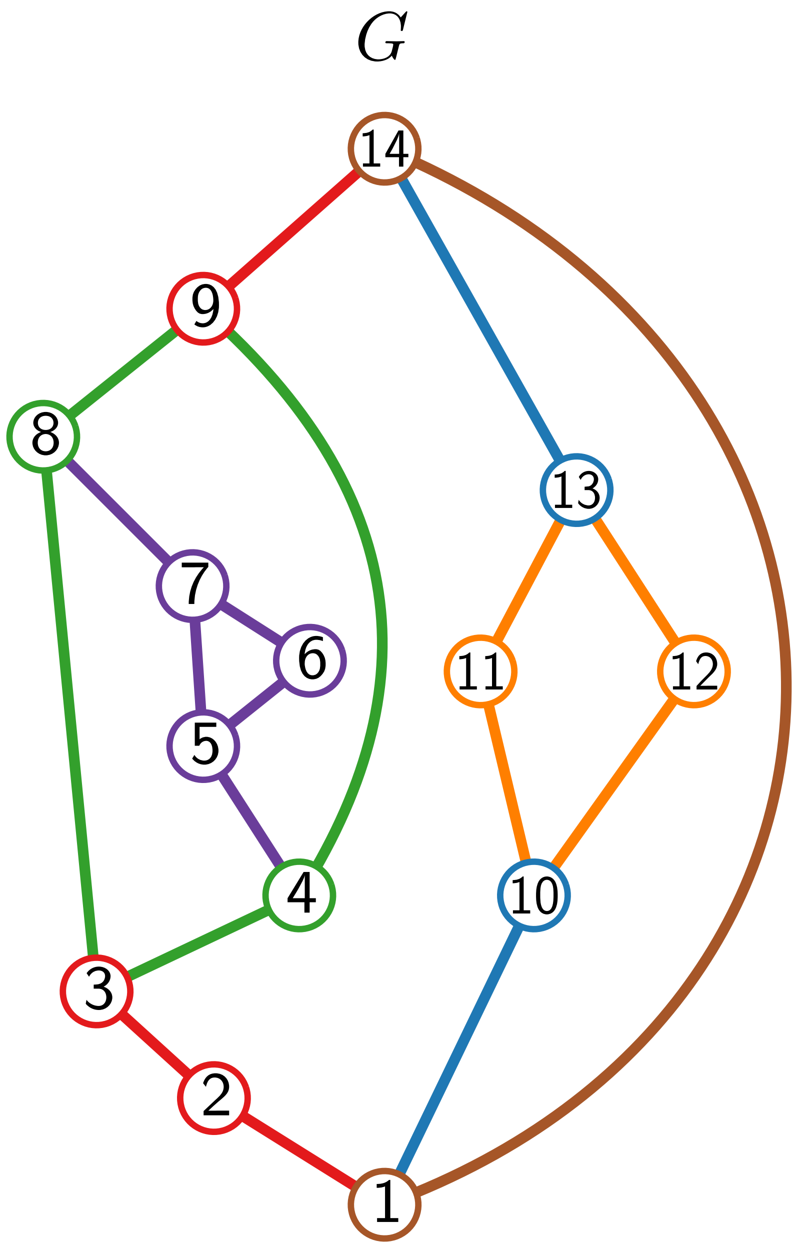

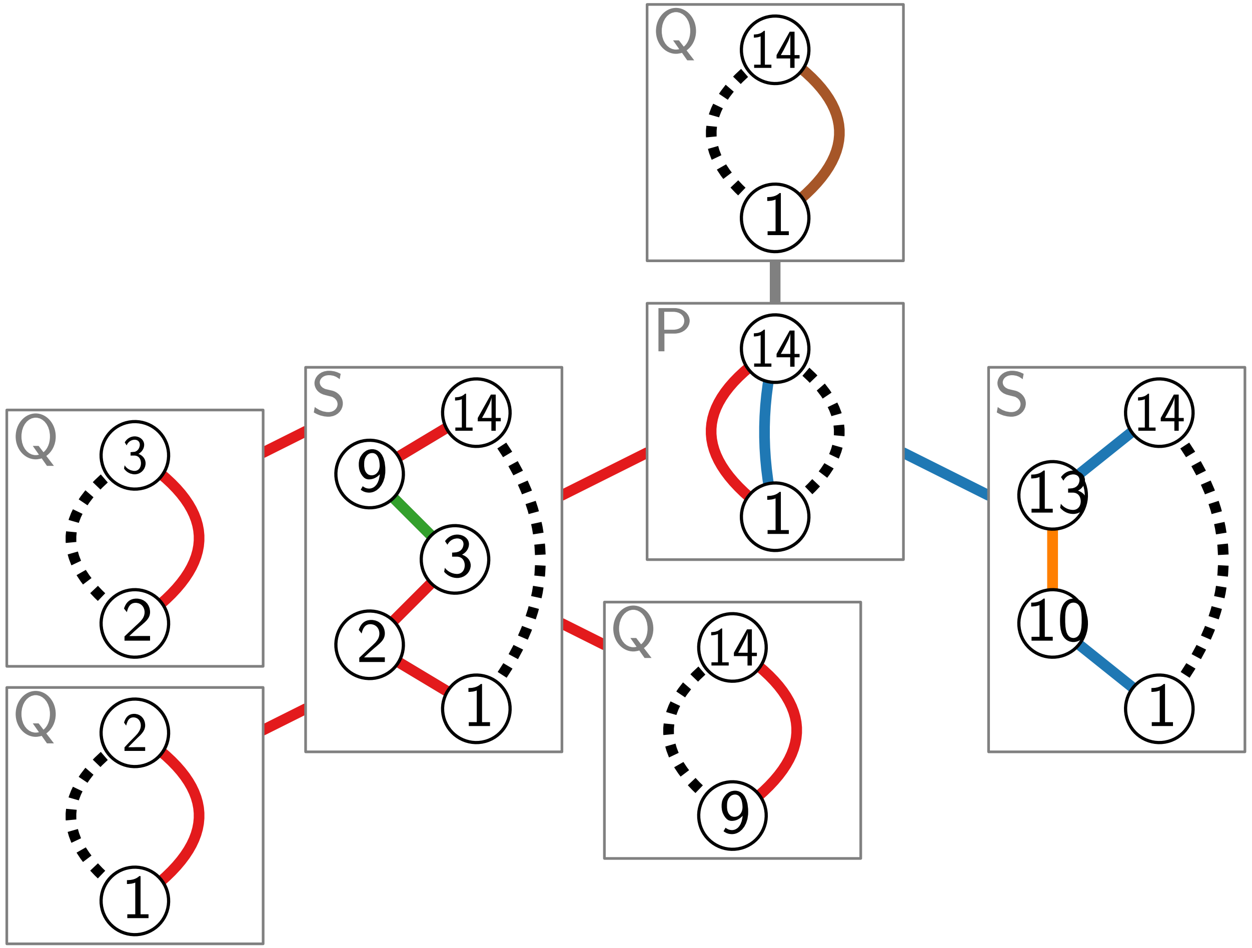

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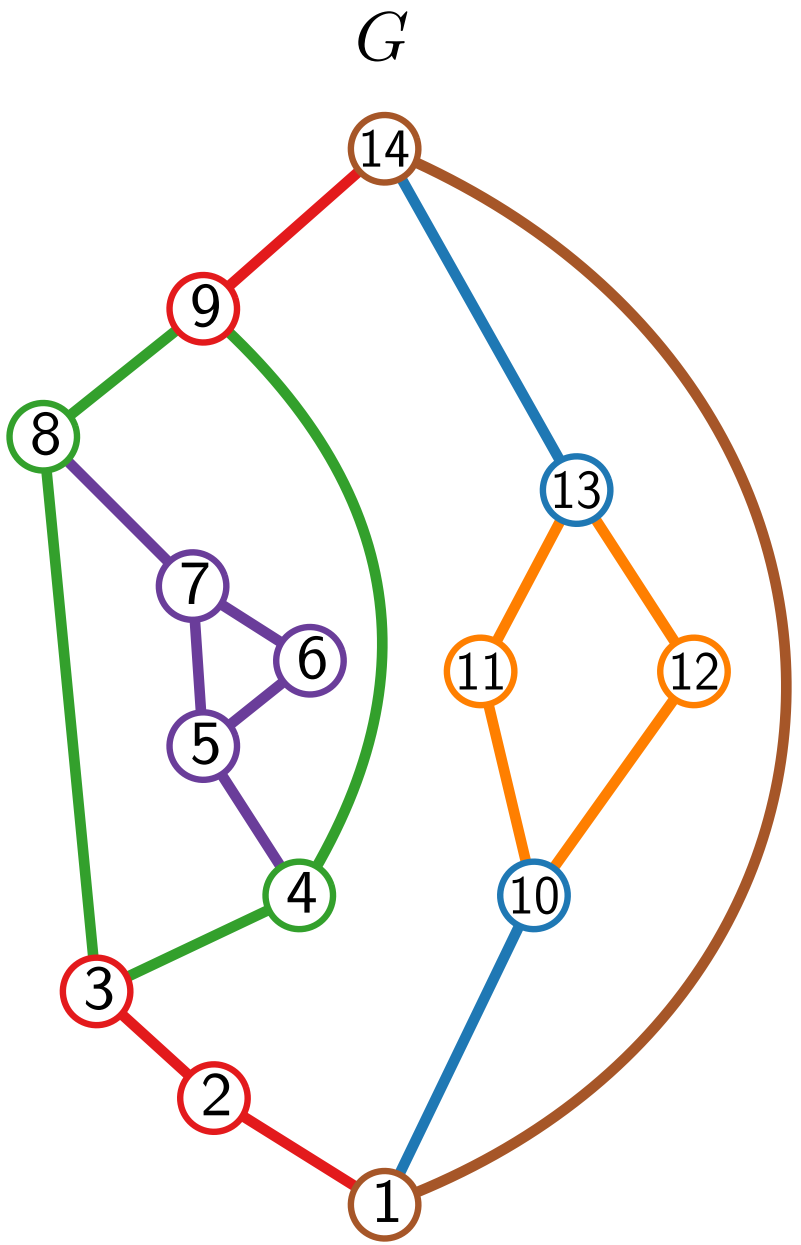

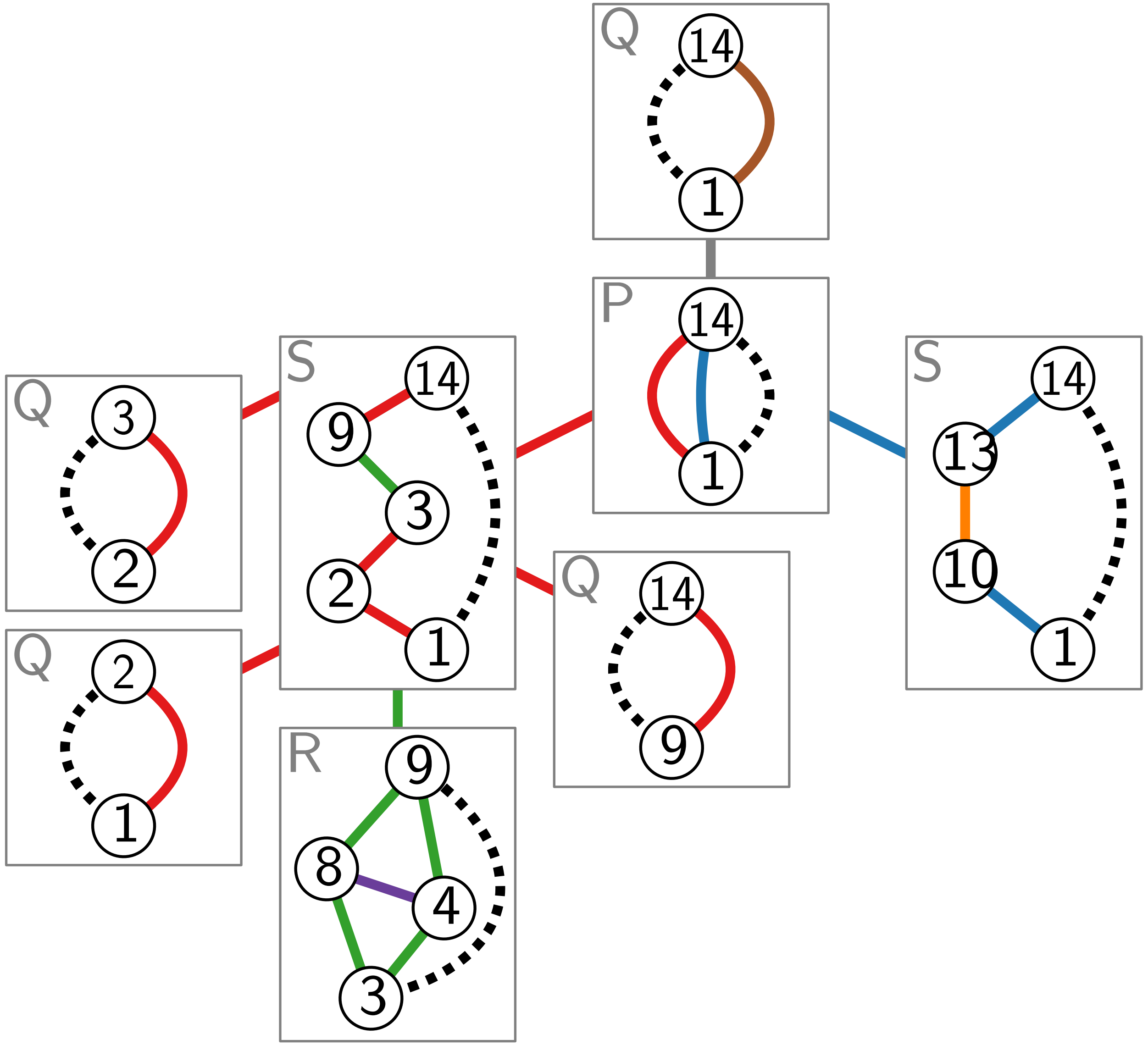

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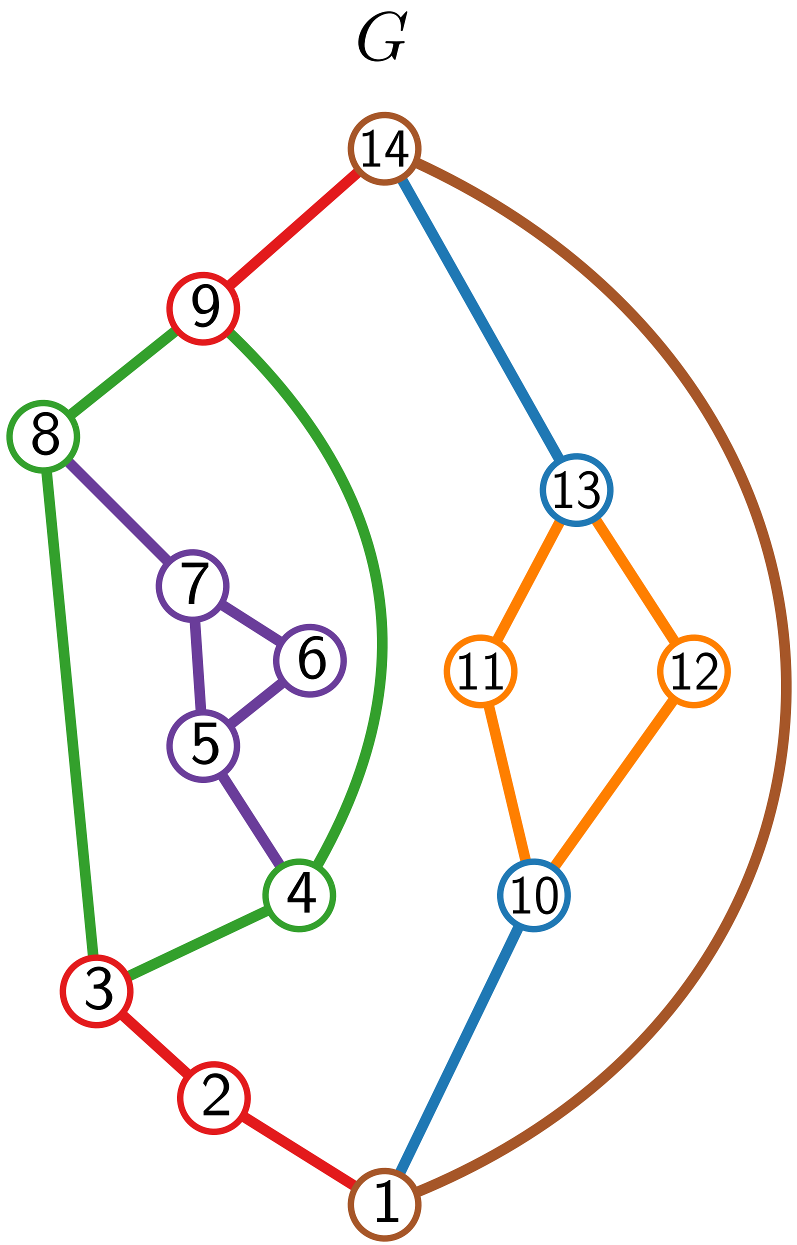

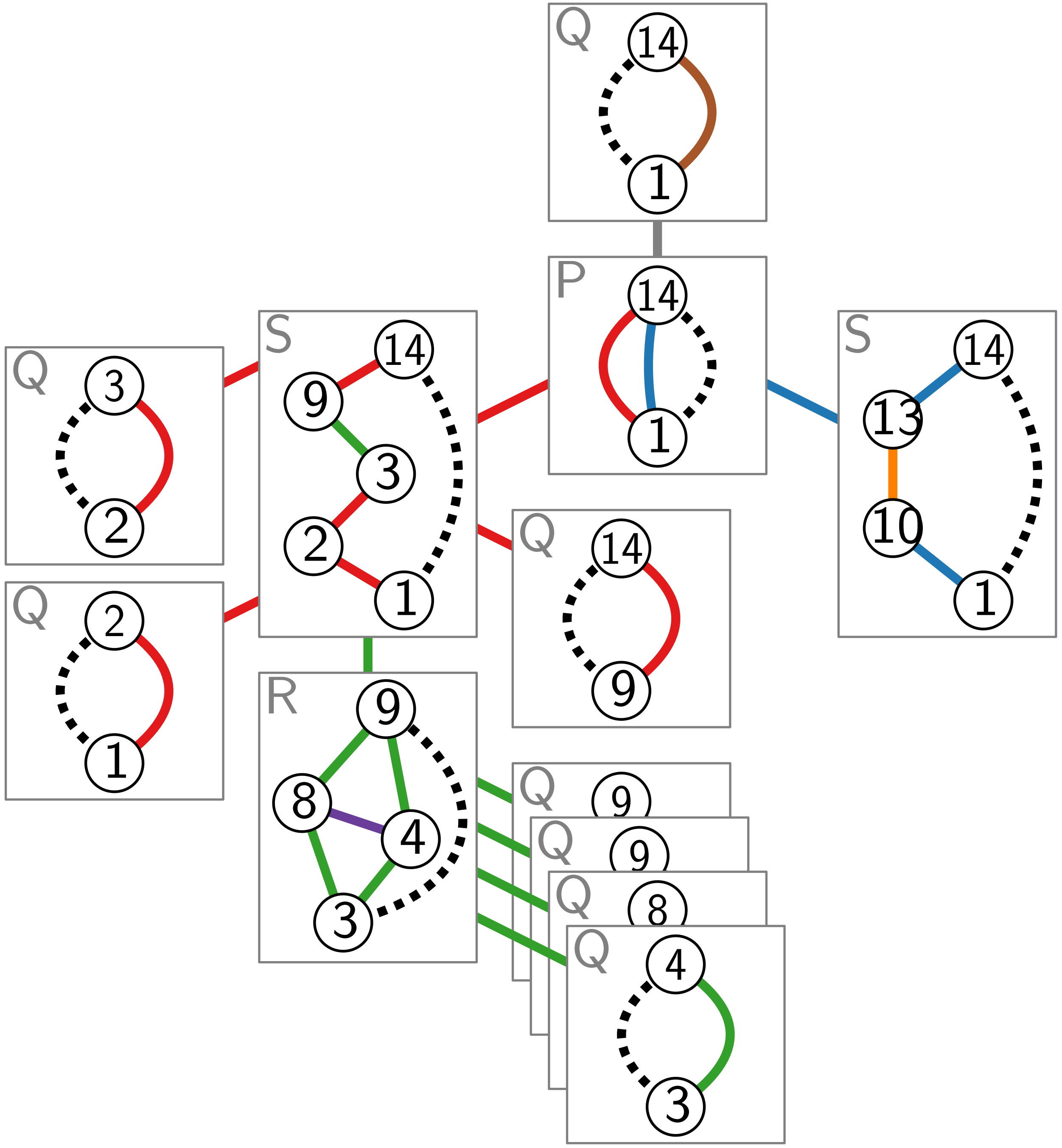

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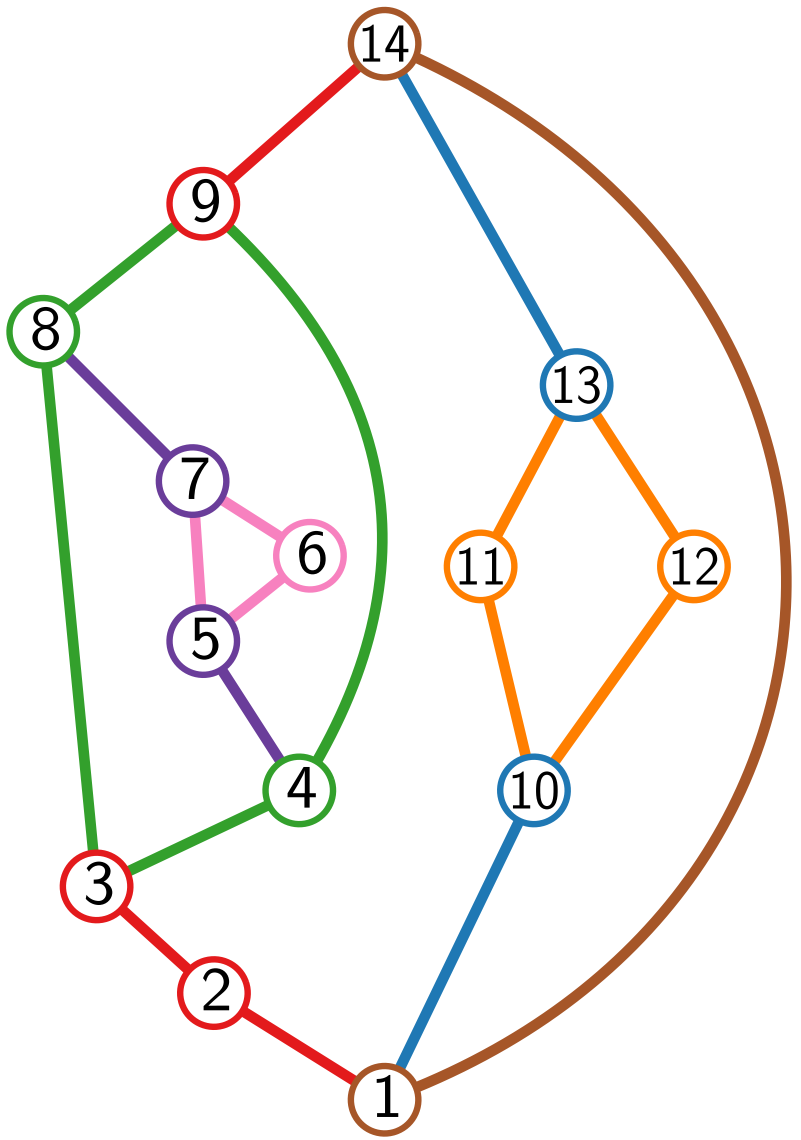
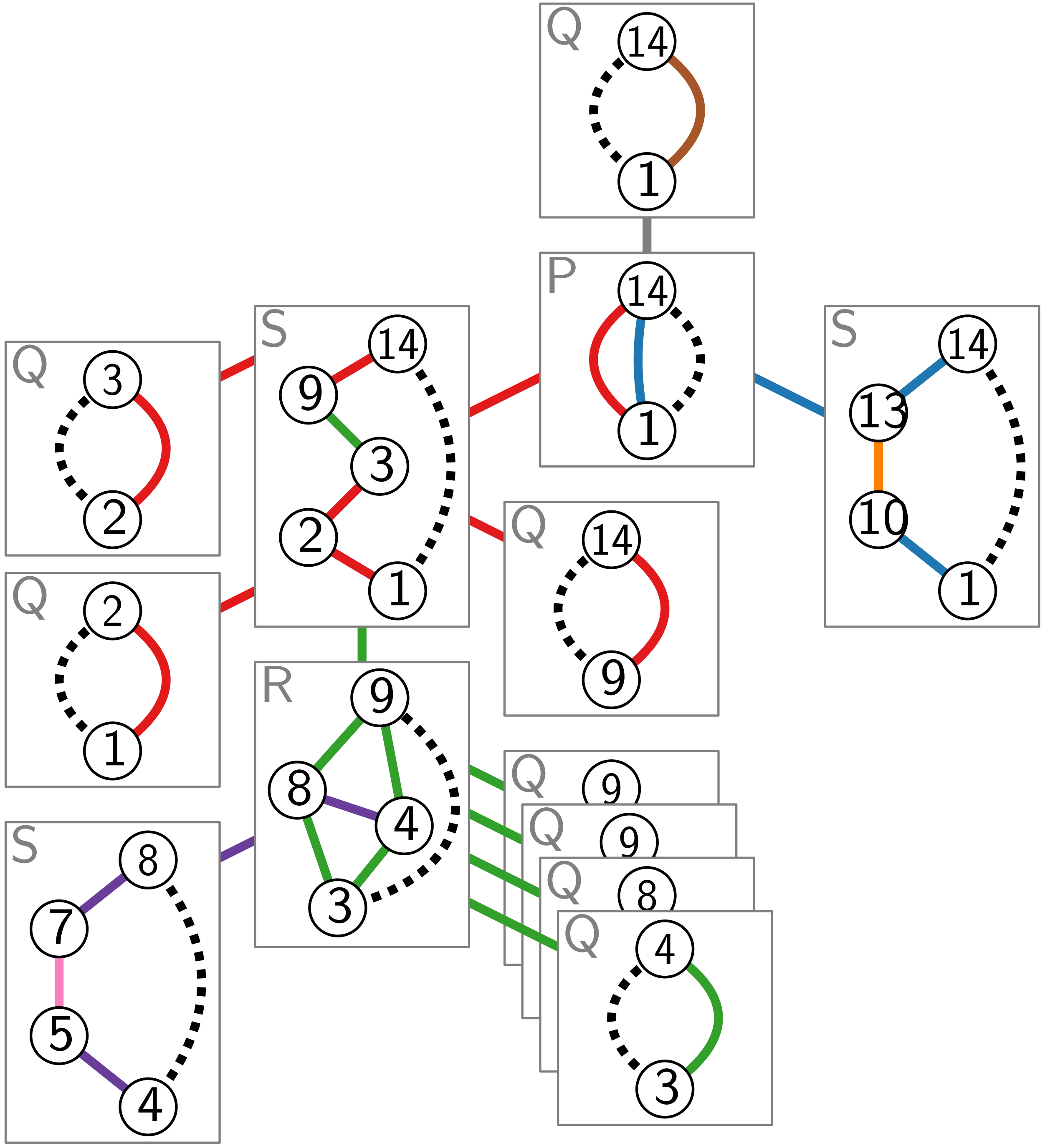
G





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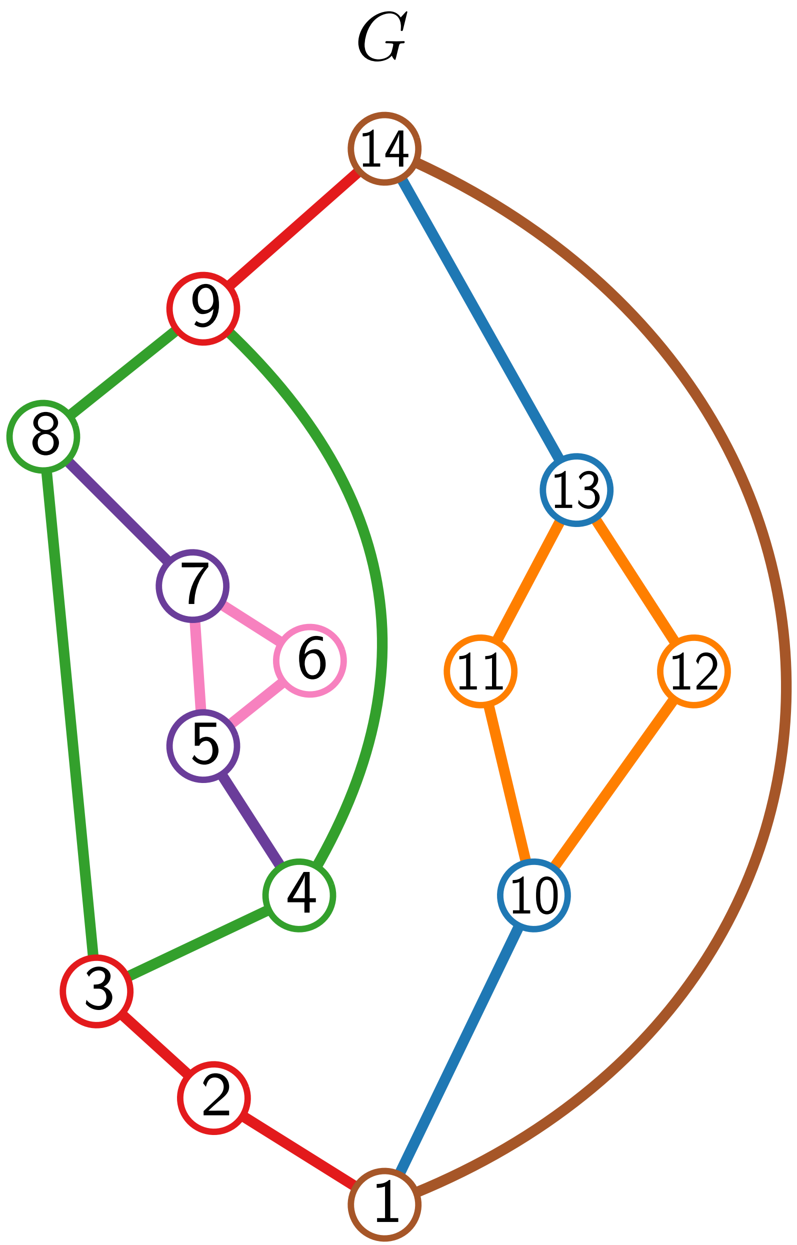

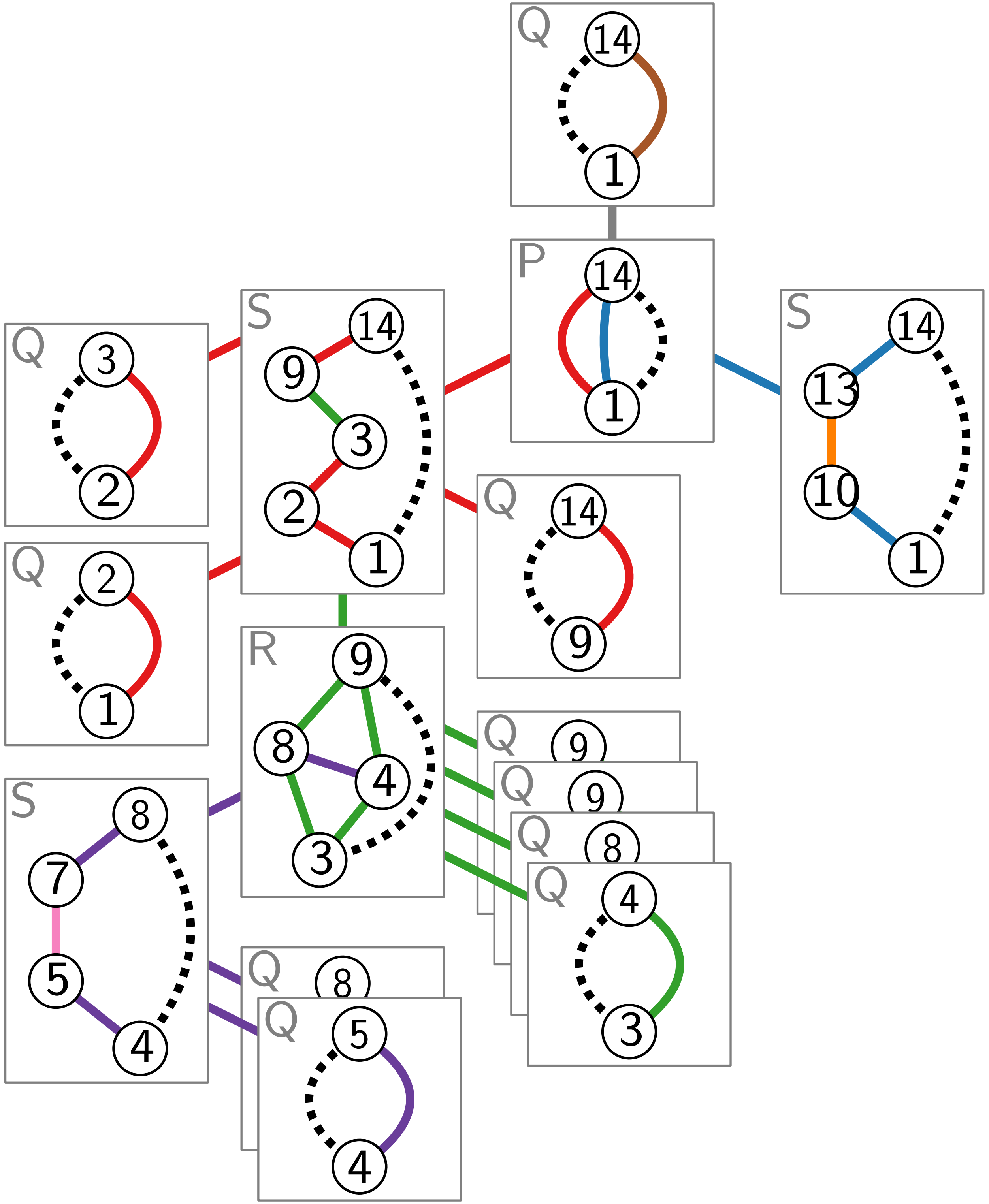

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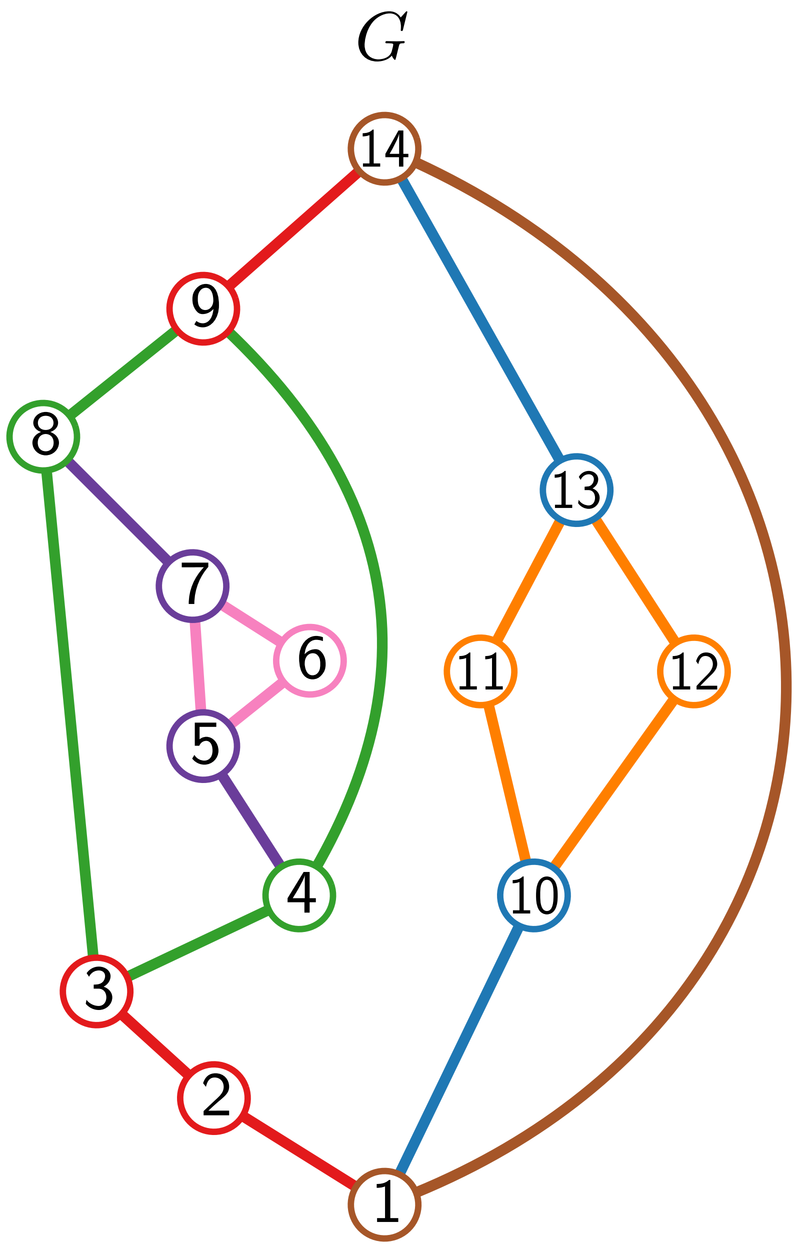

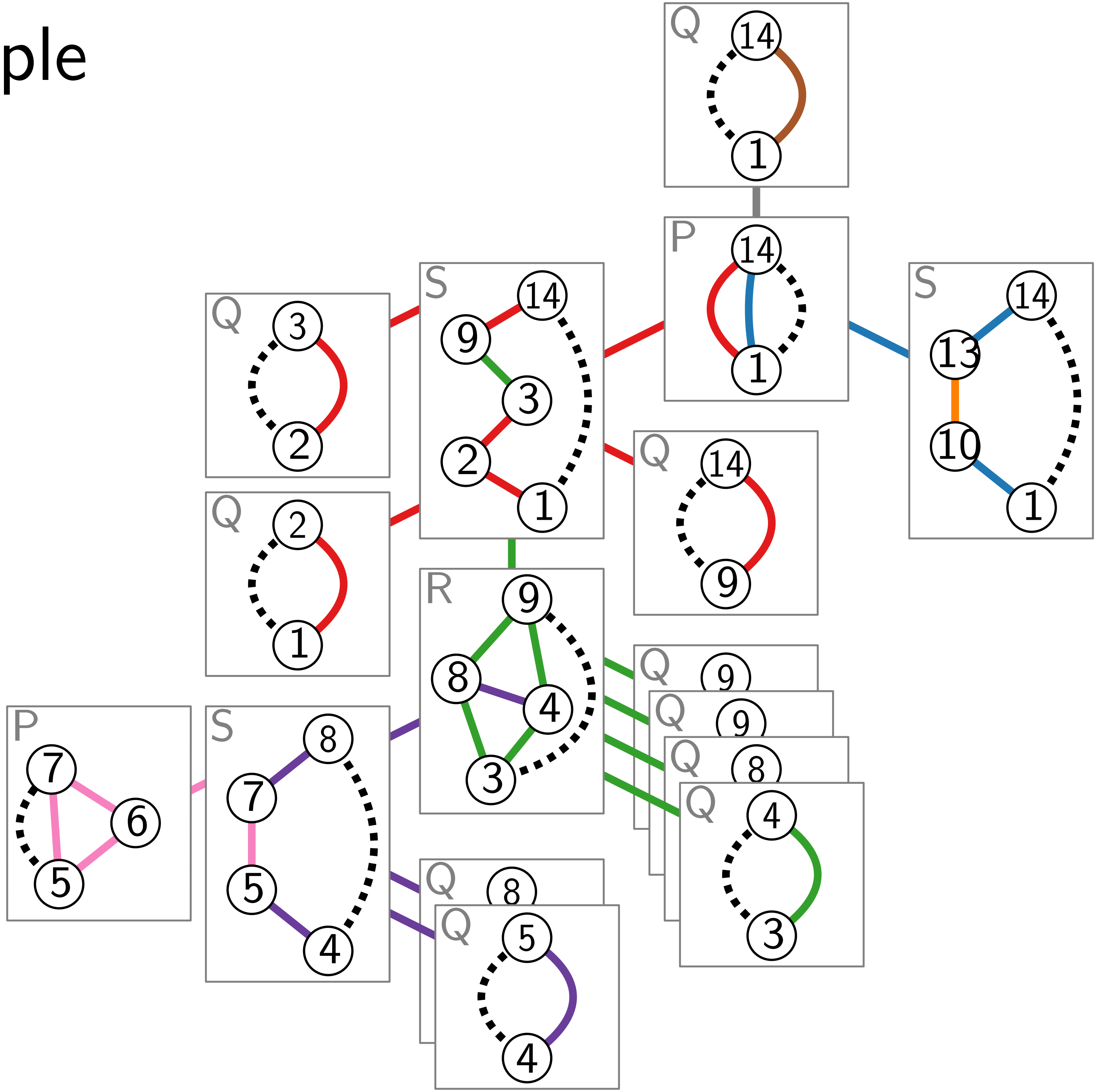

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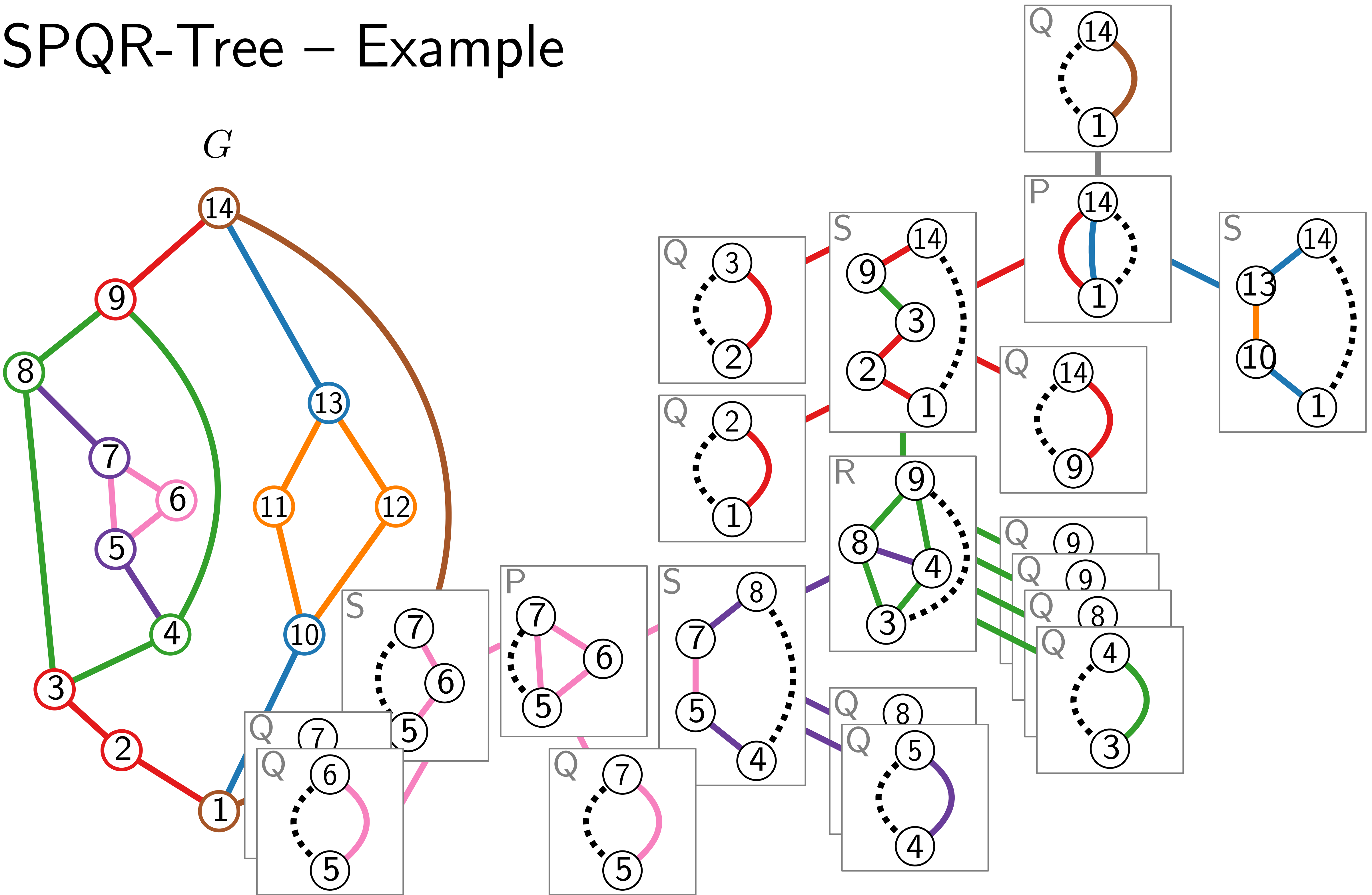

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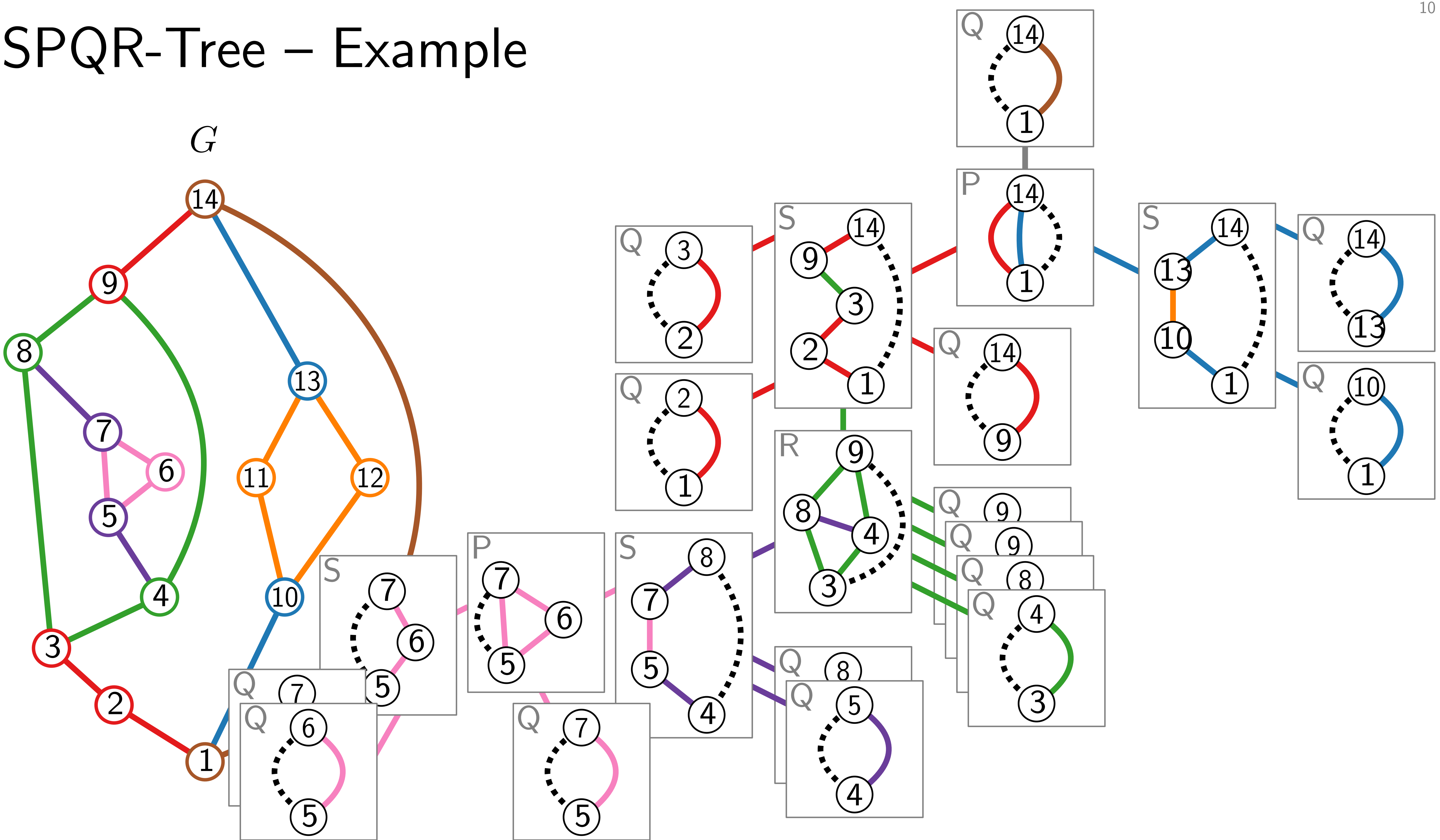

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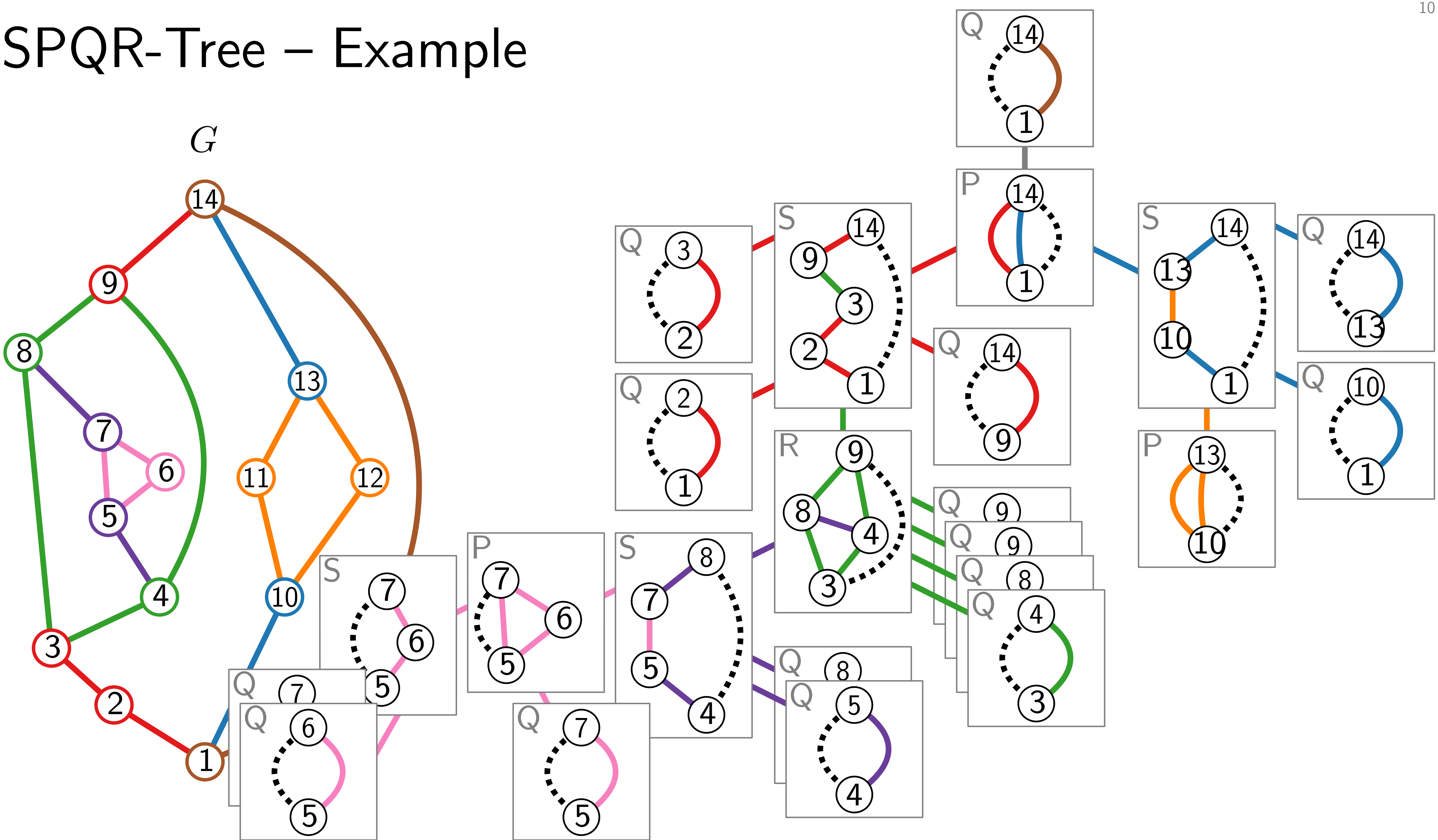

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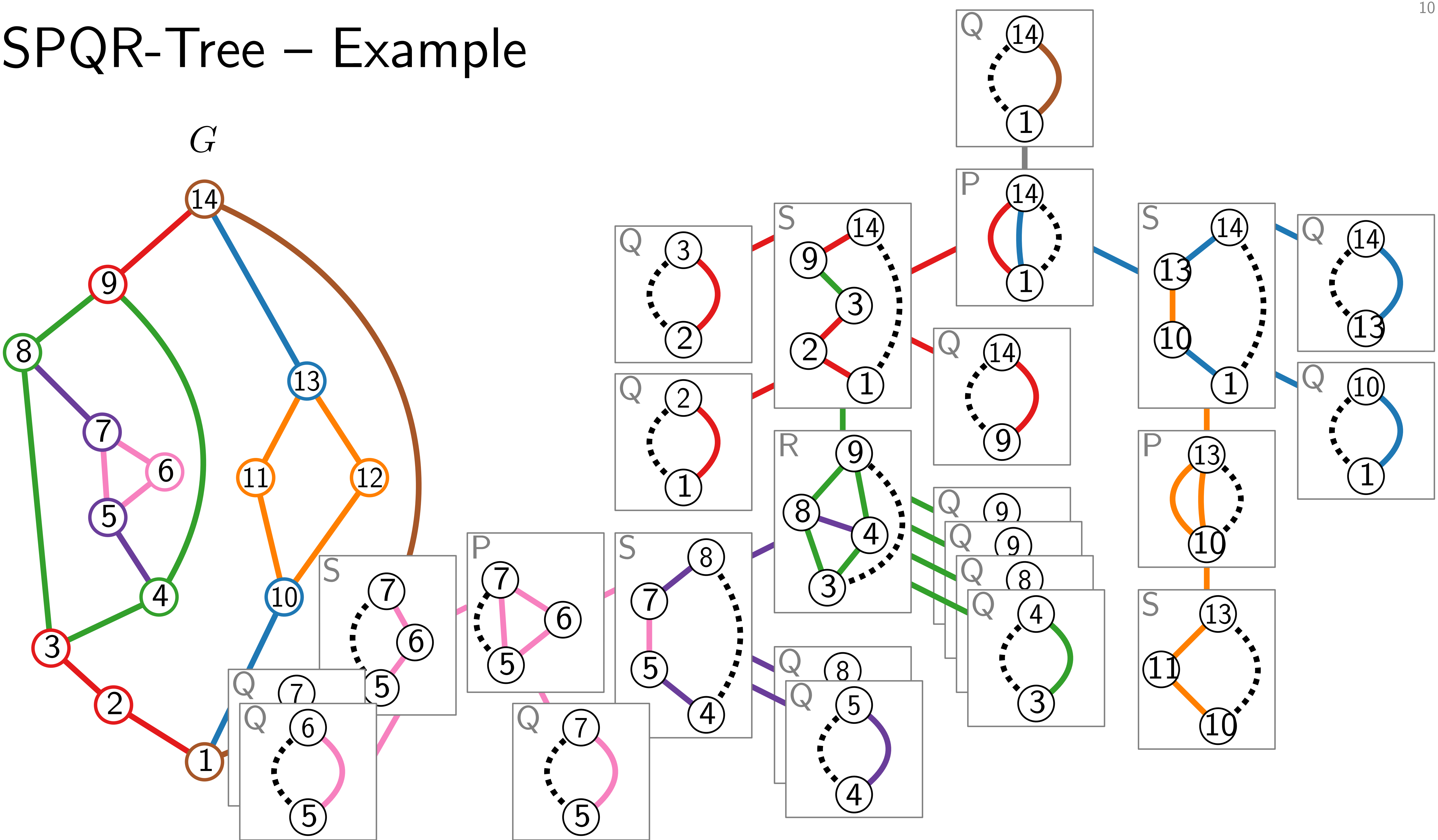

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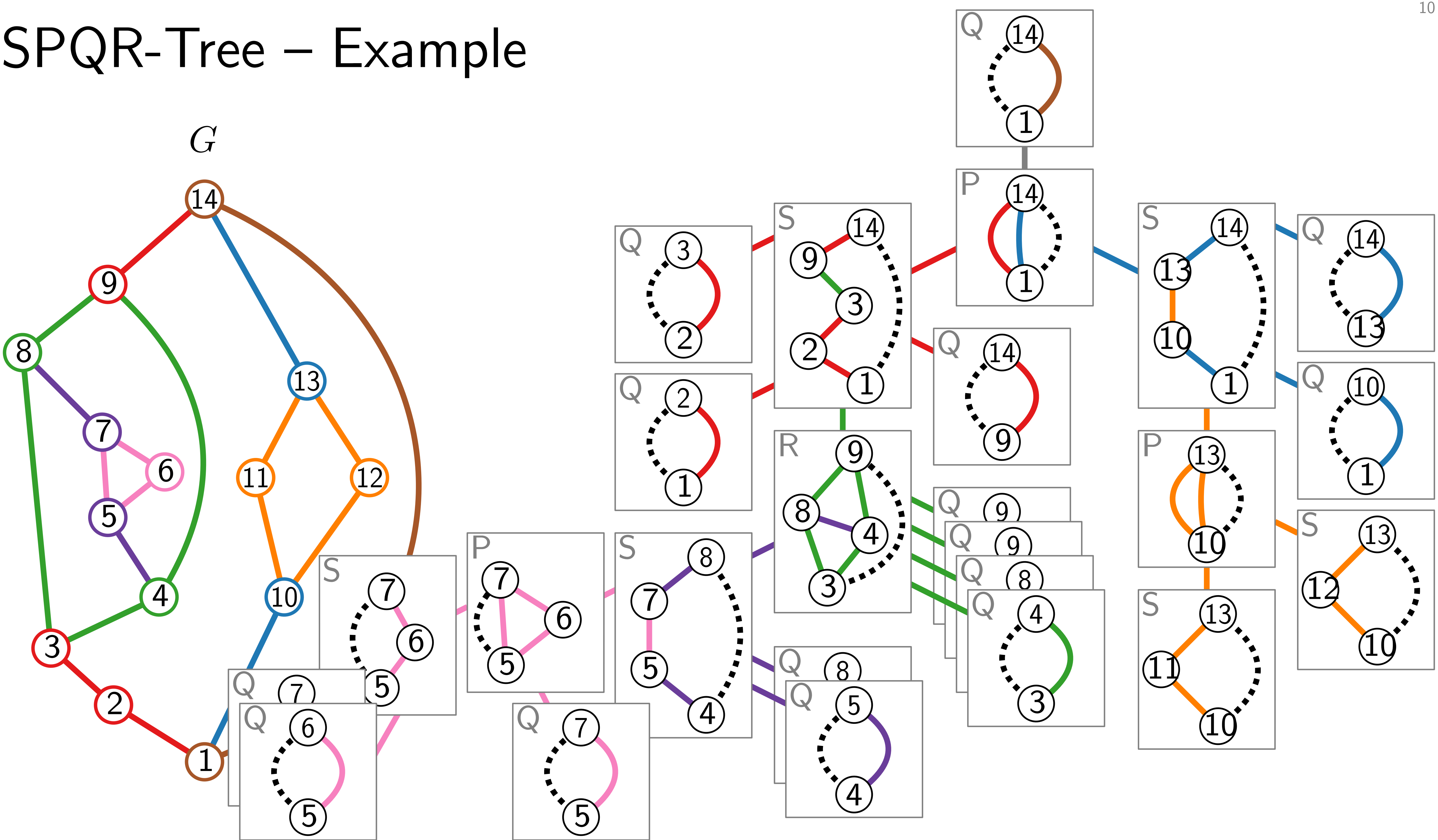

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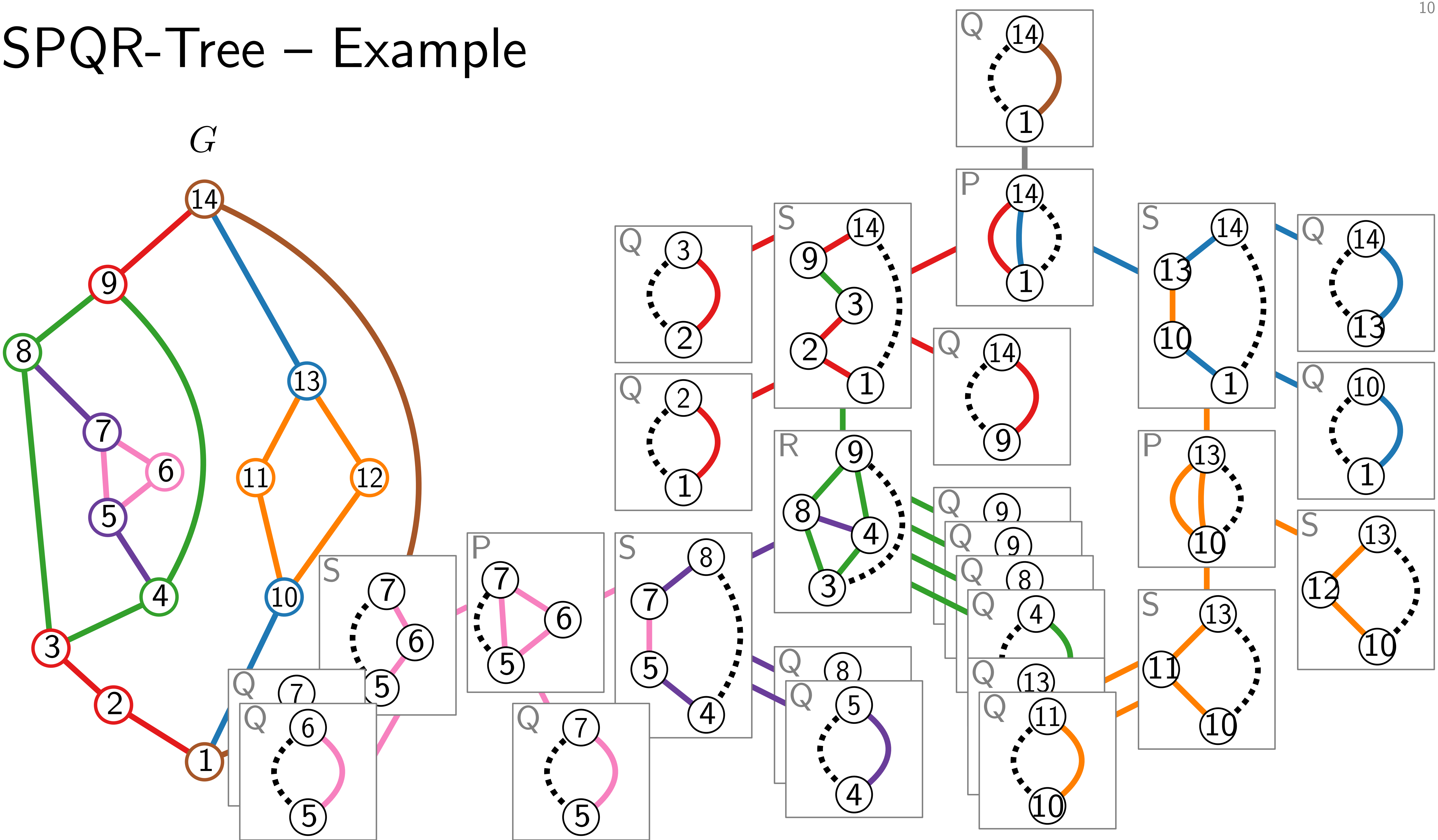

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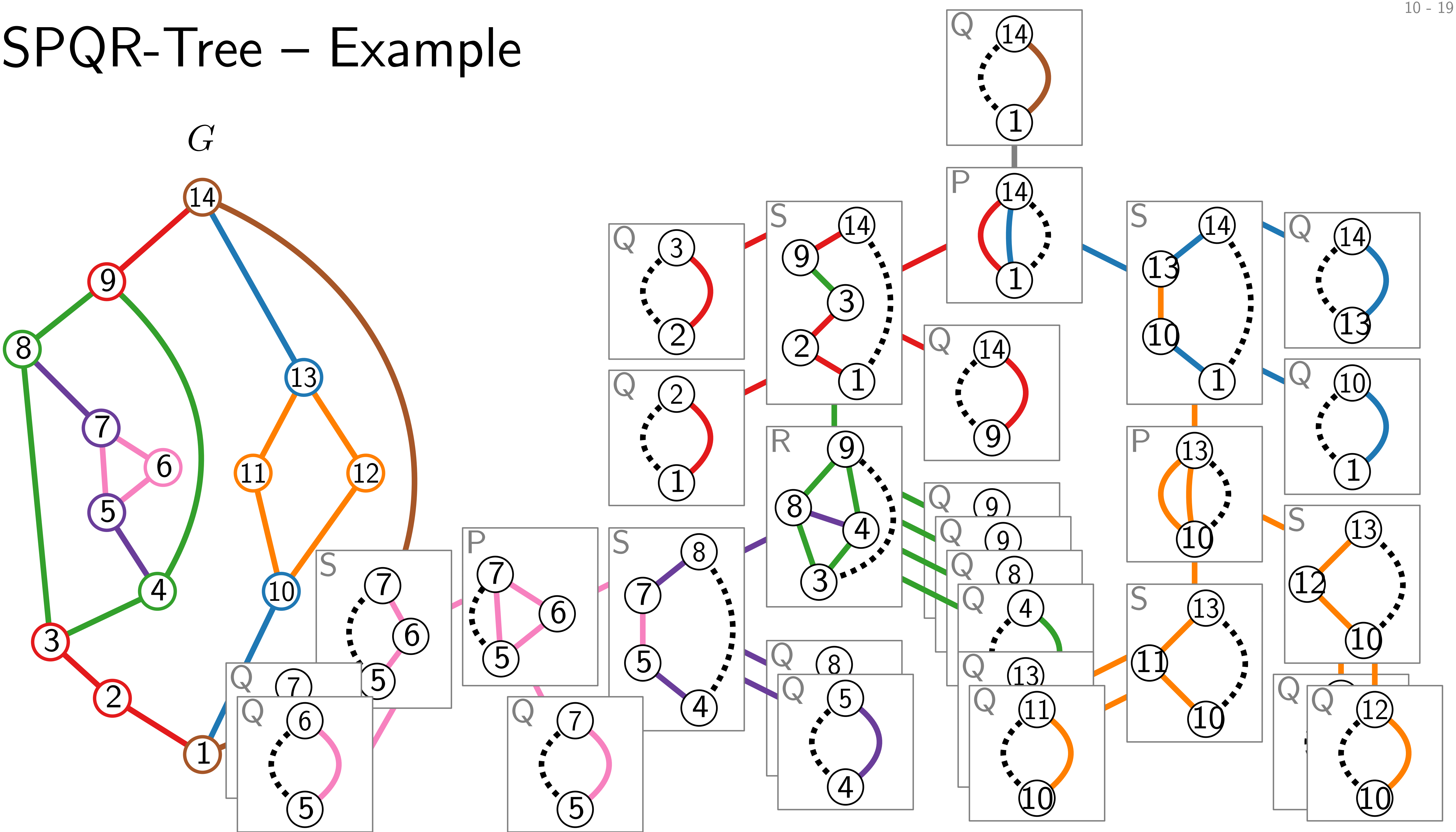

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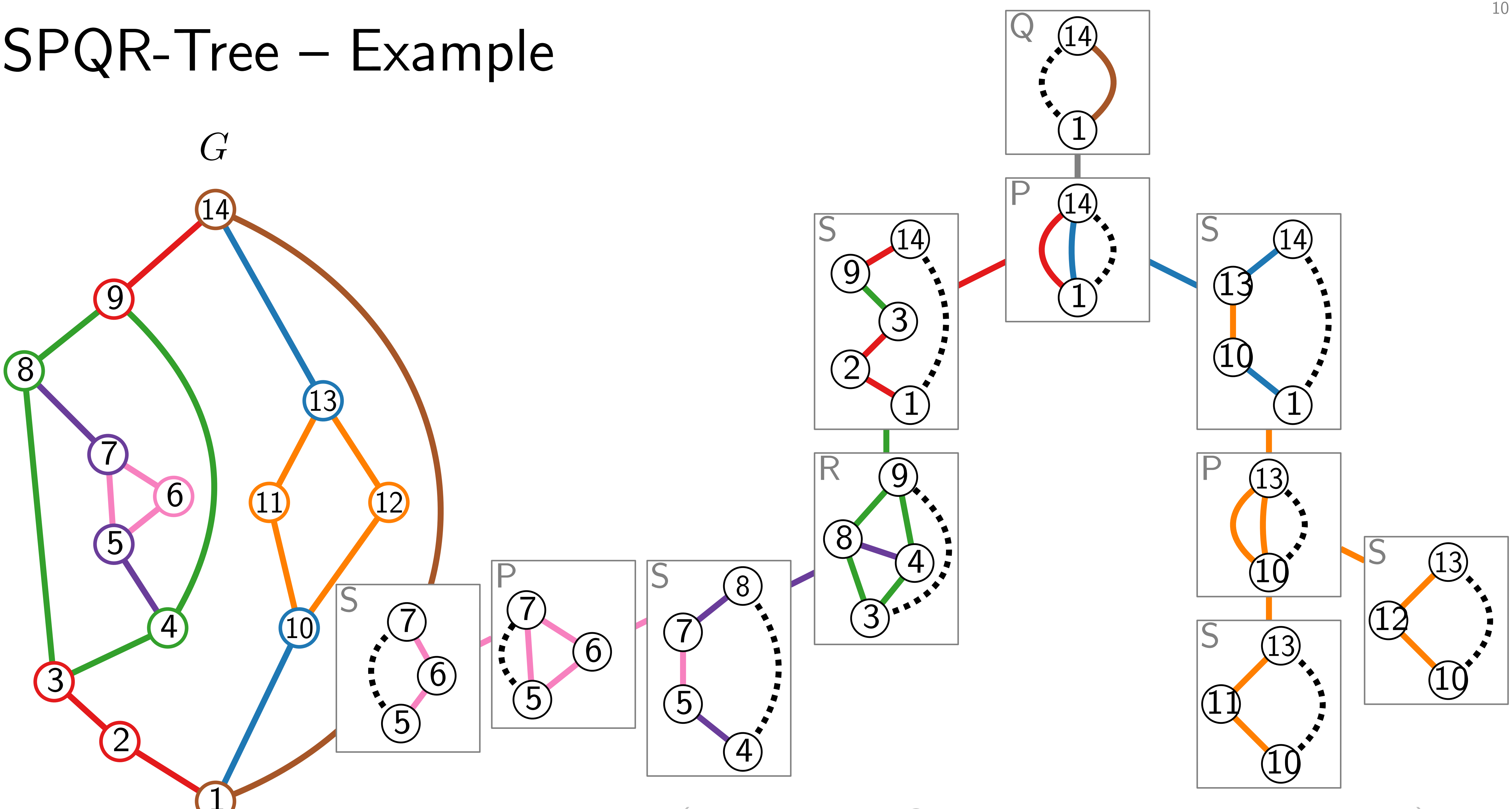
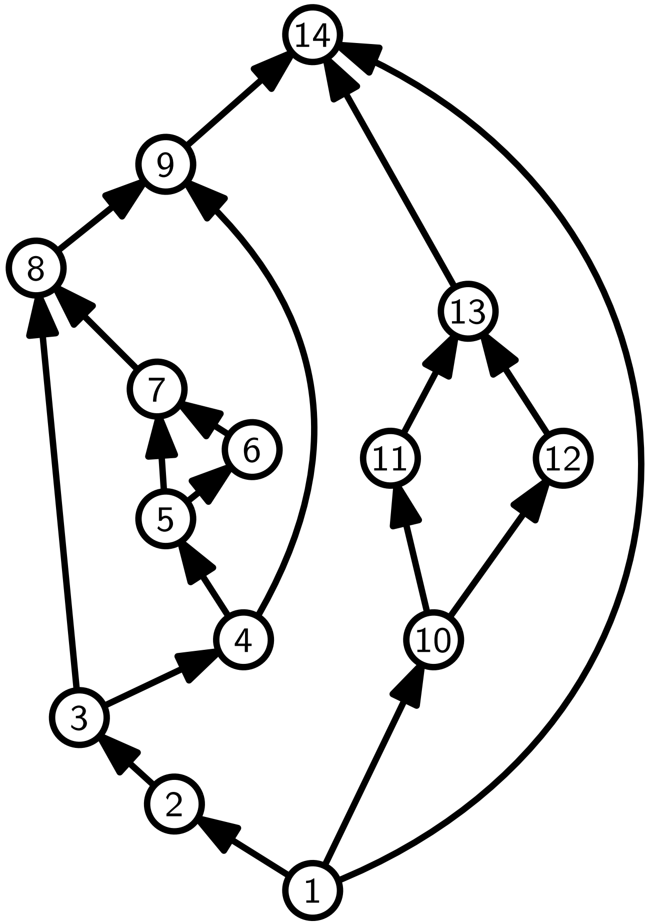


(tree without Q-nodes, but including the root)

Representation Extension for st-Graphs

Theorem 1'.
Rectangular ε-bar visibility representation extension can be solved in $\mathcal{O}(n^2)$ time for st-graphs.



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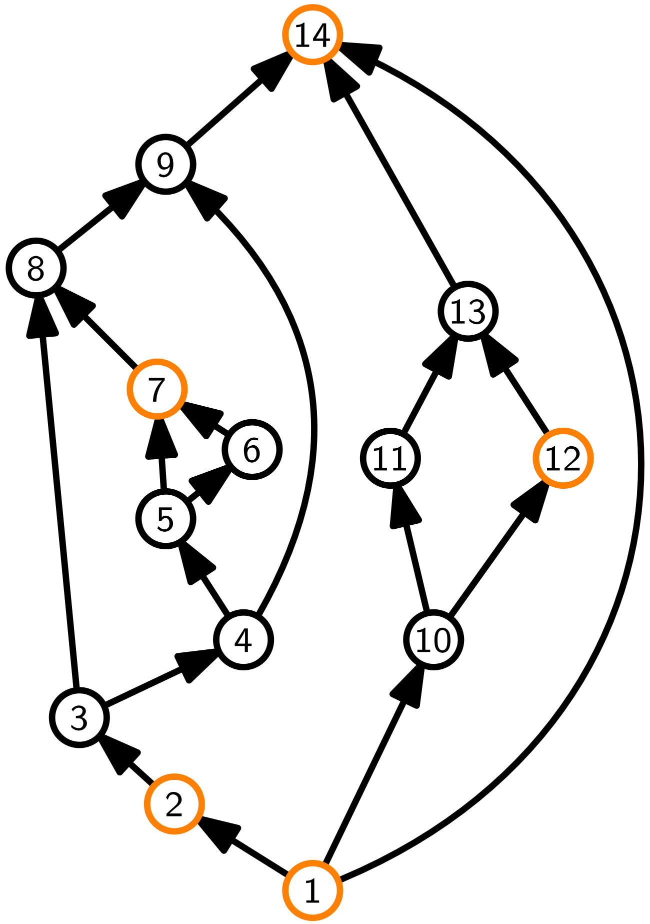

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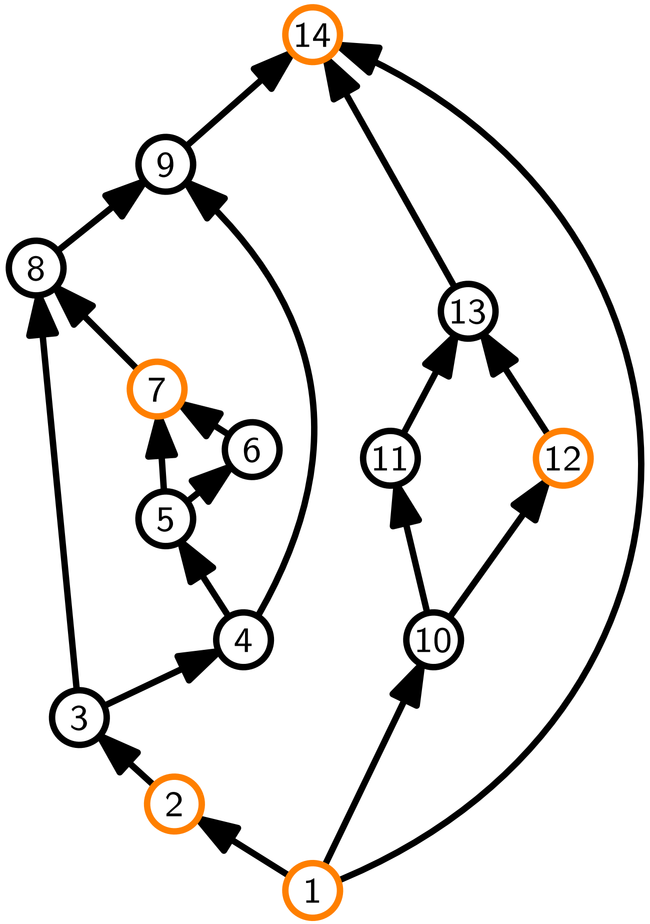

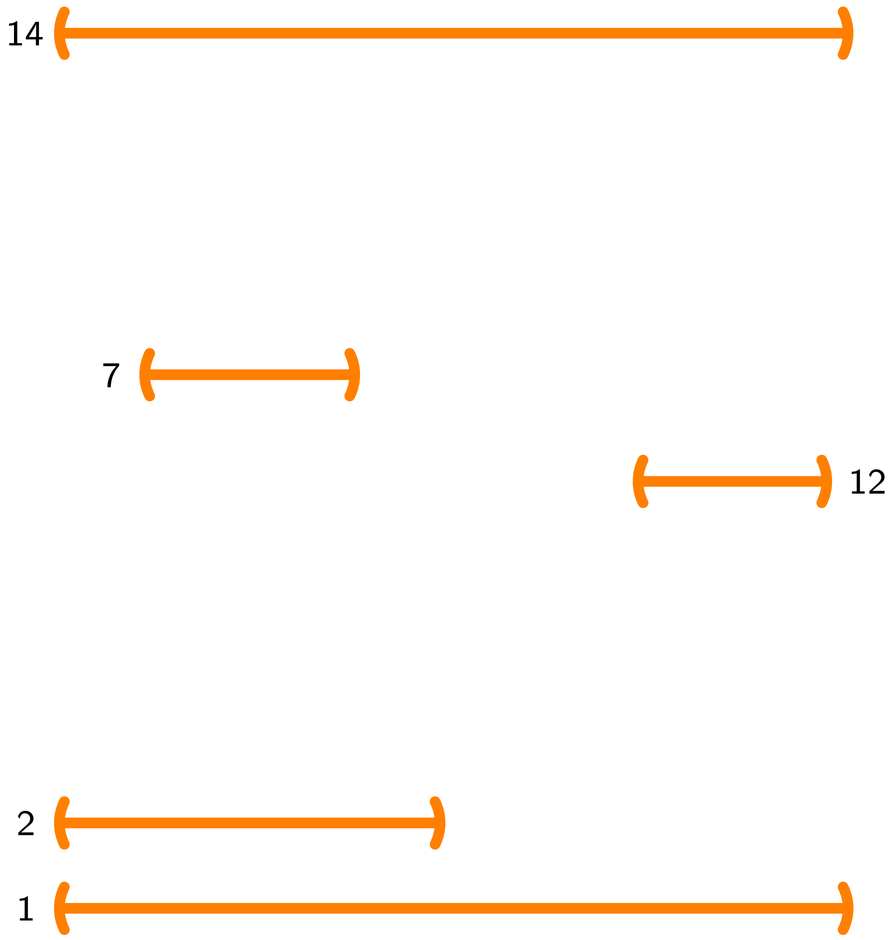

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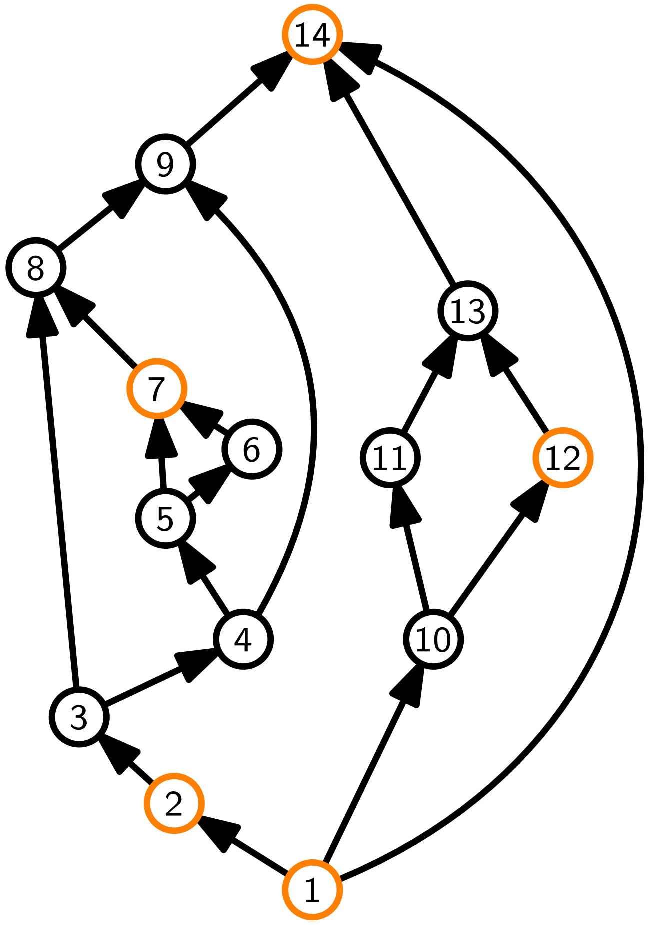

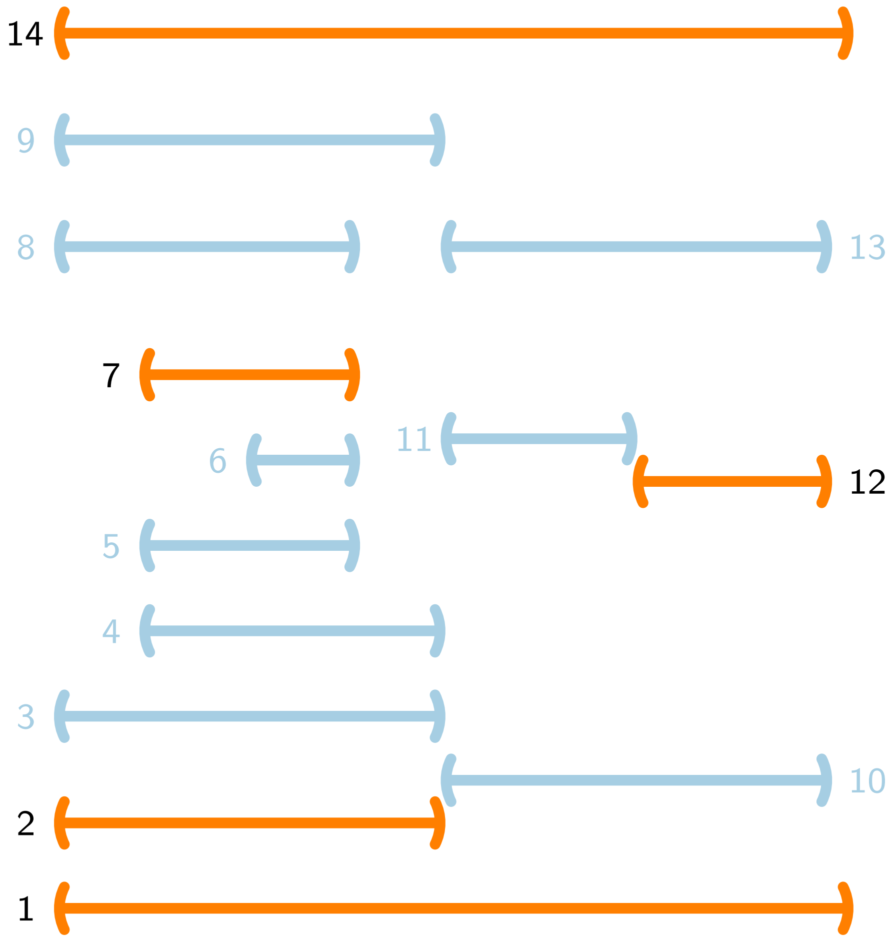

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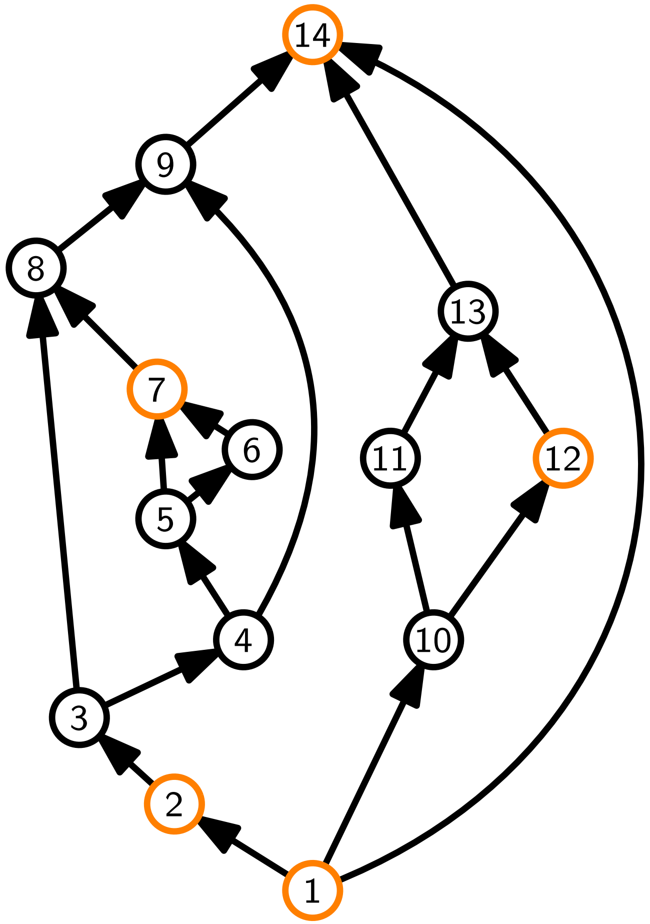

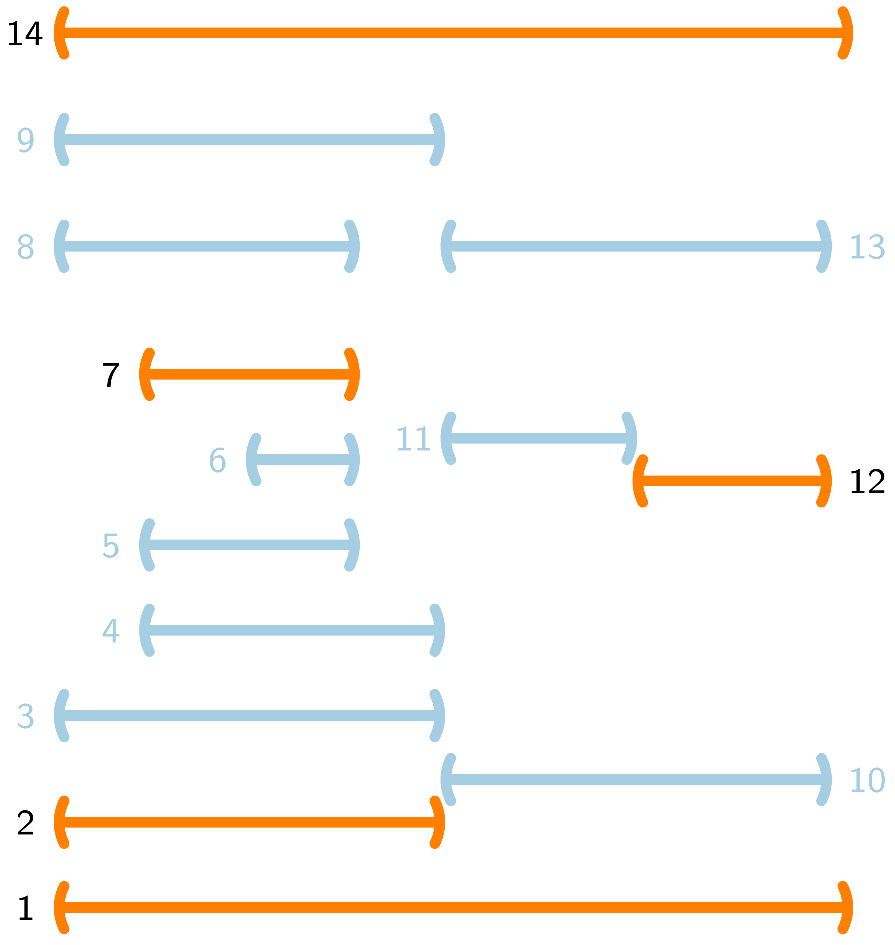

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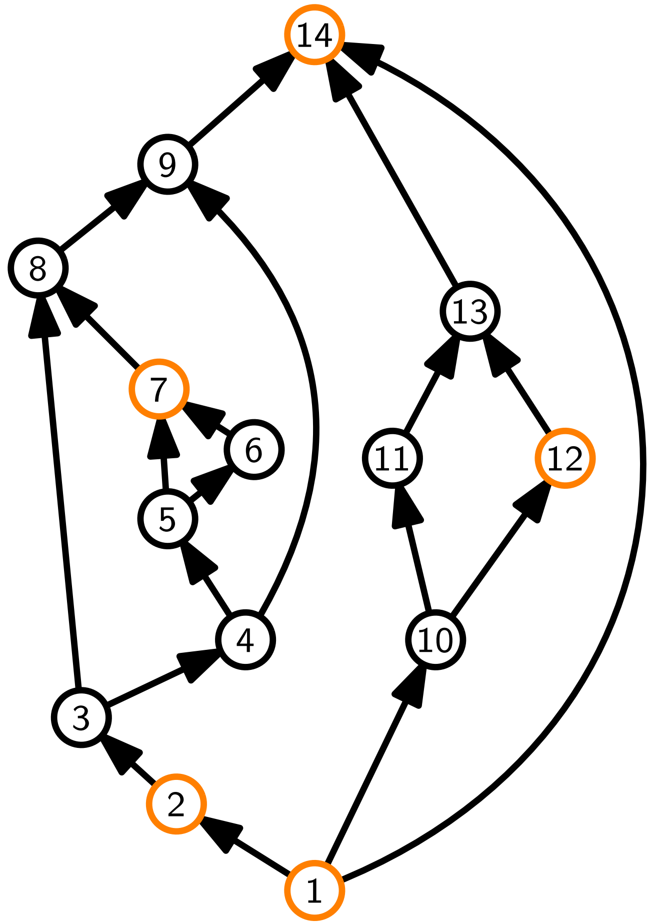

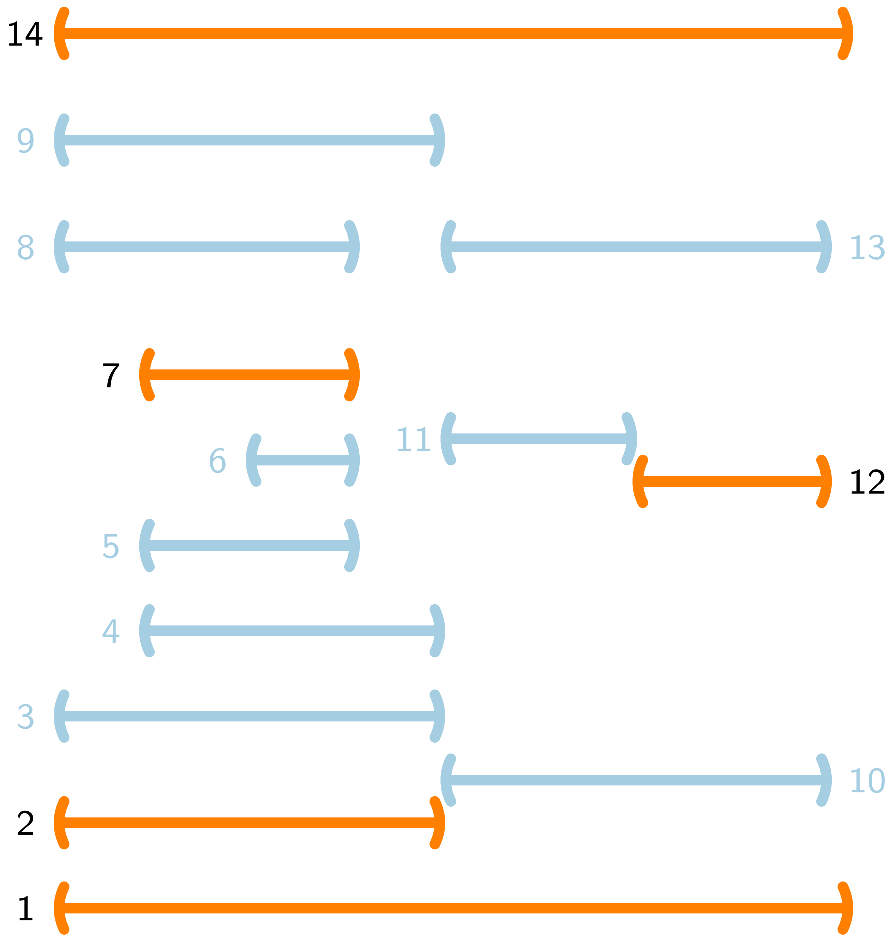

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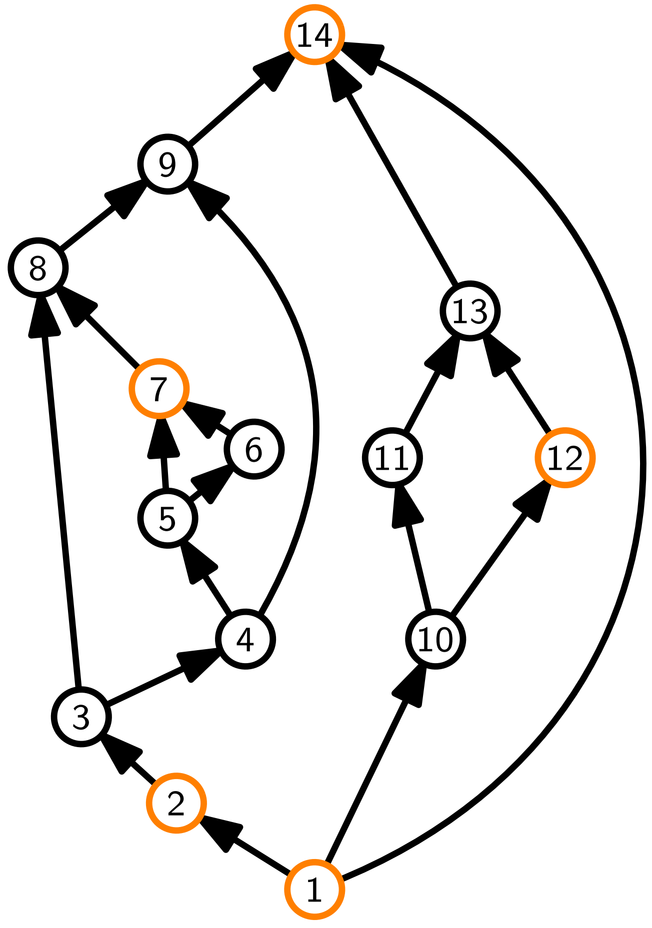

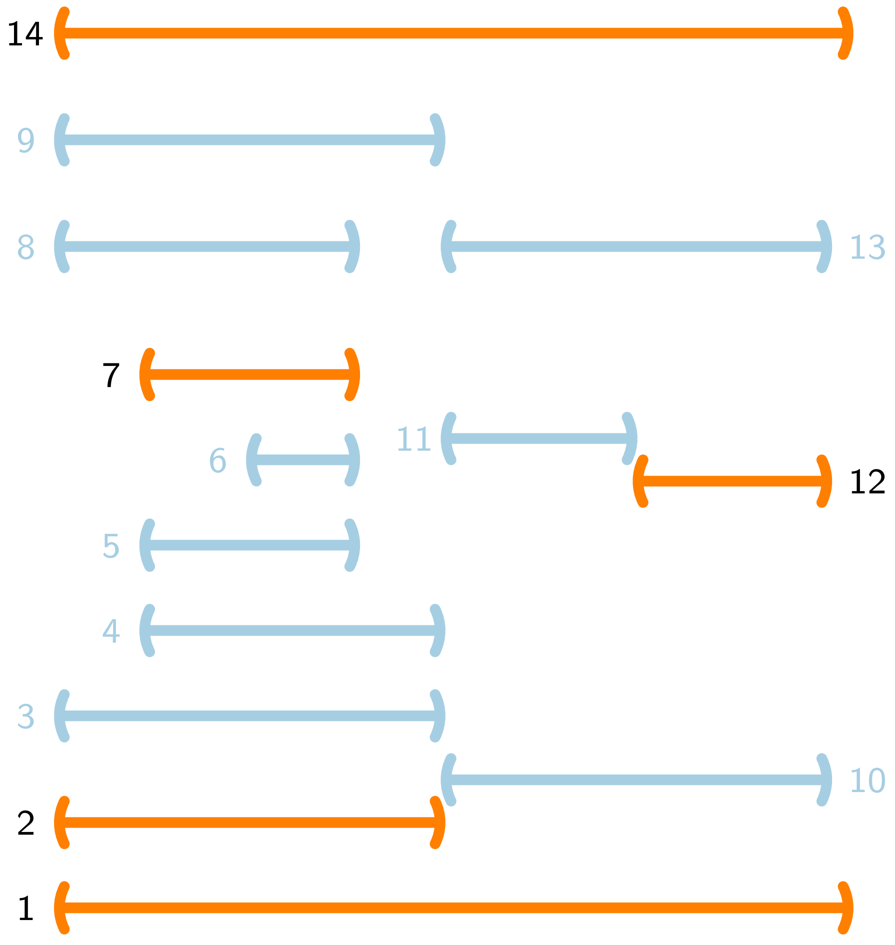

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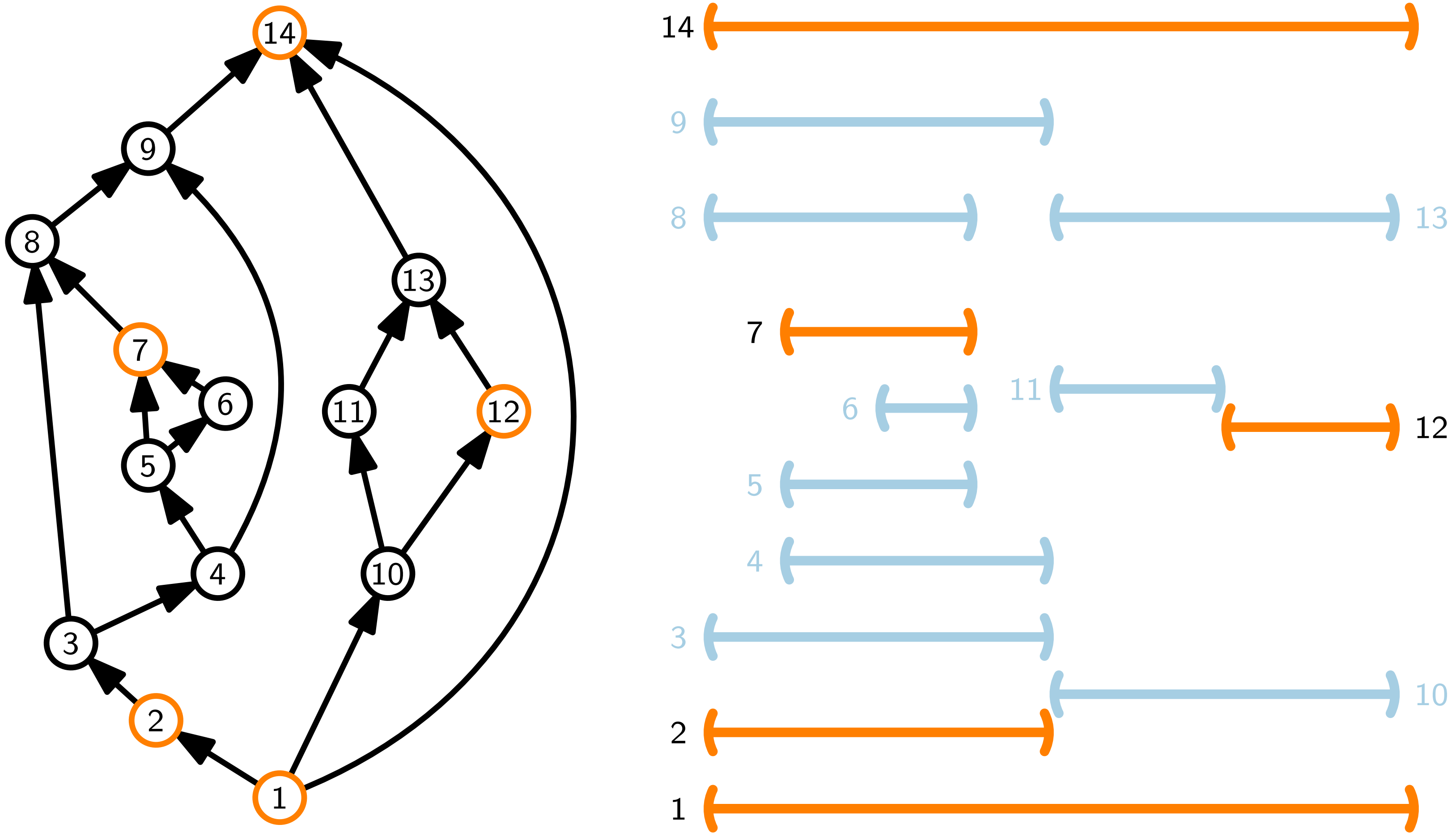

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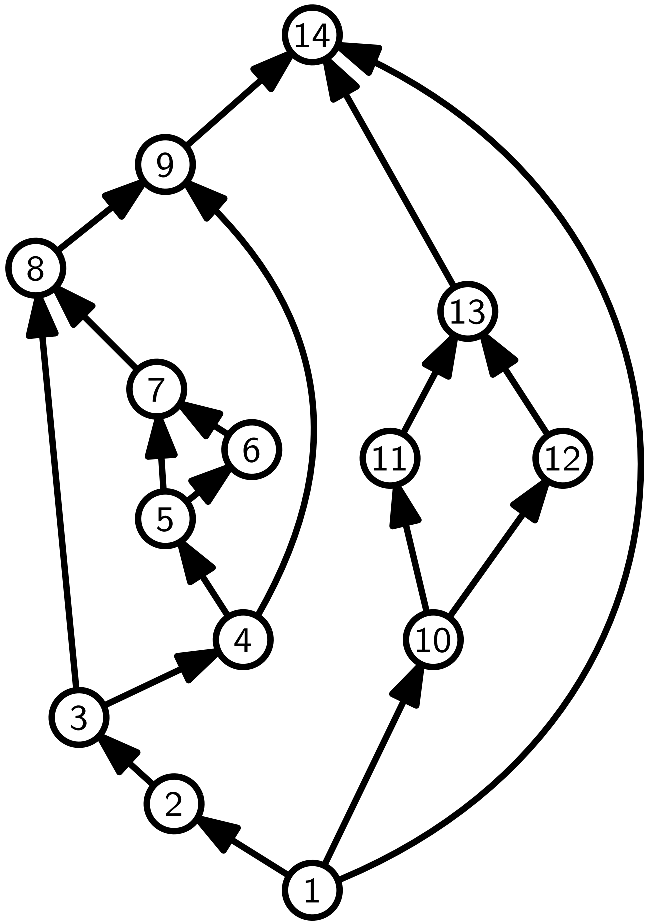
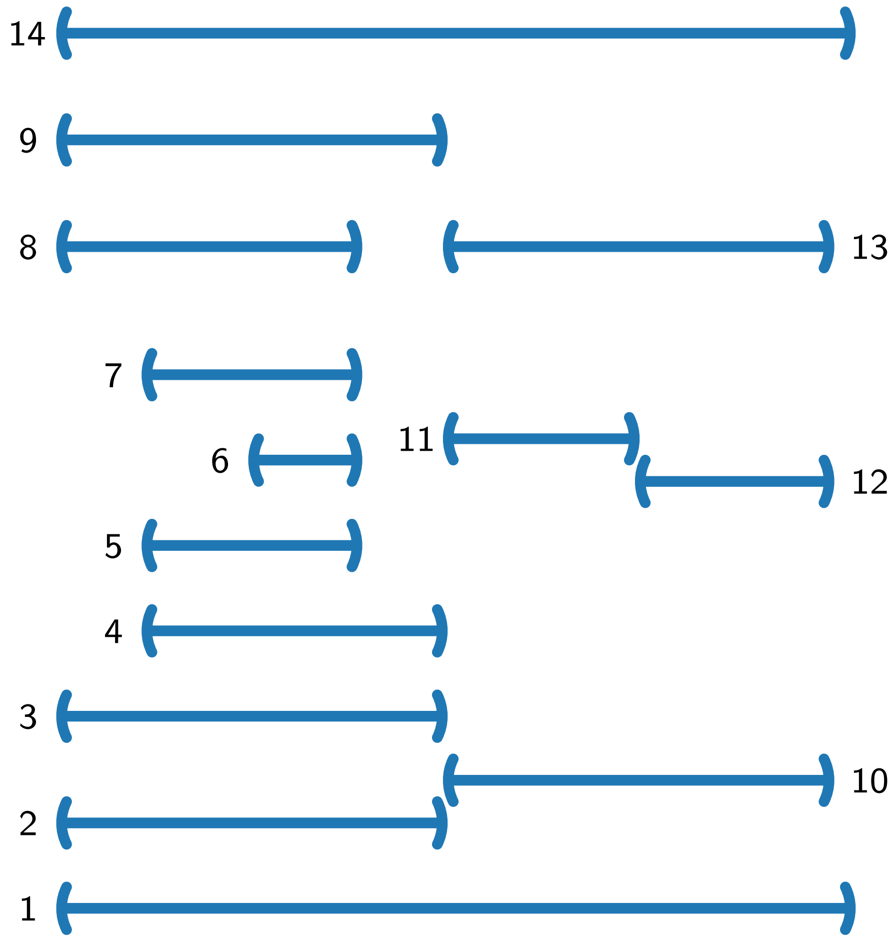
But Why Do SPQR-Trees Help?





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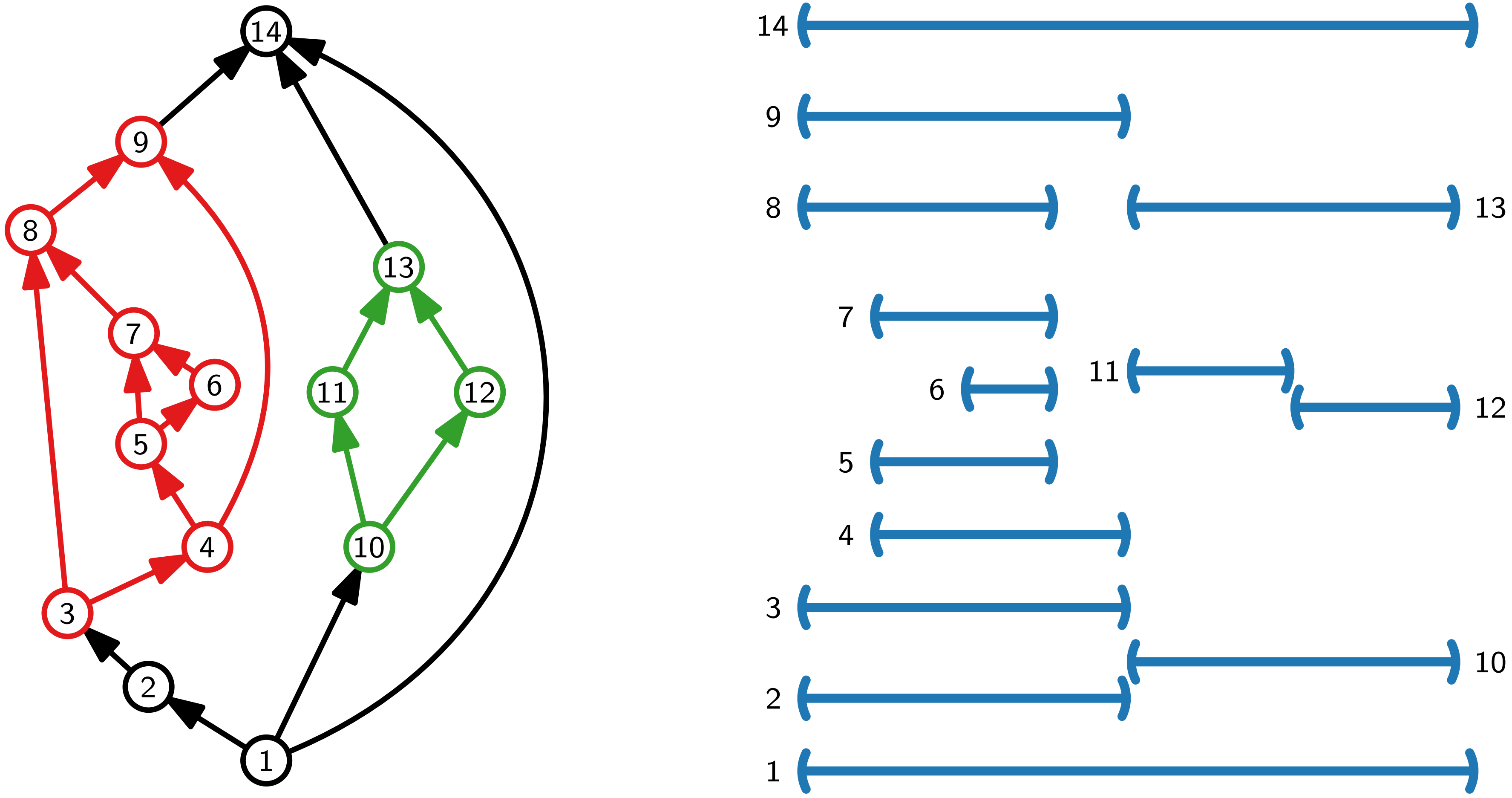

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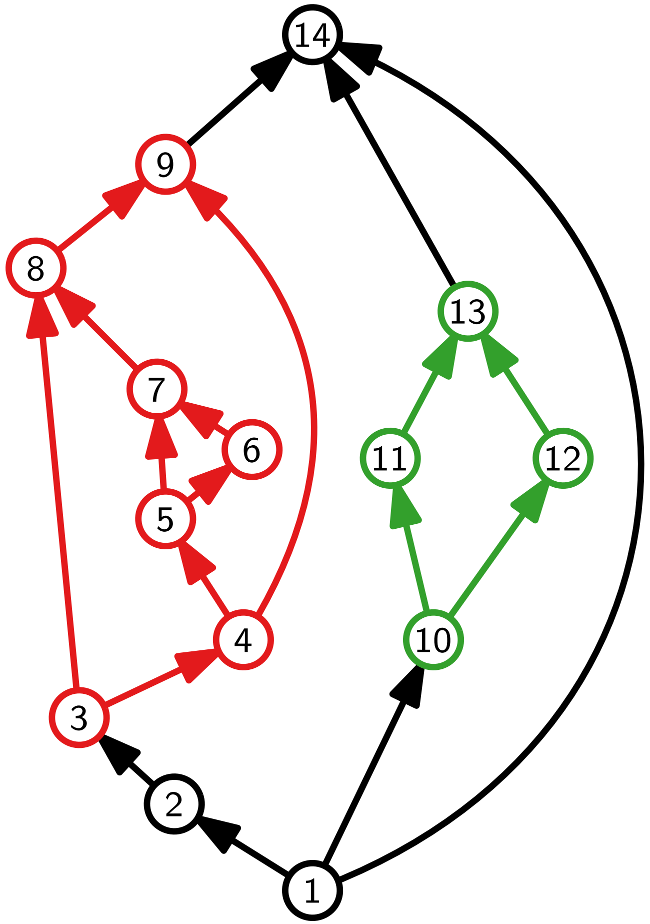

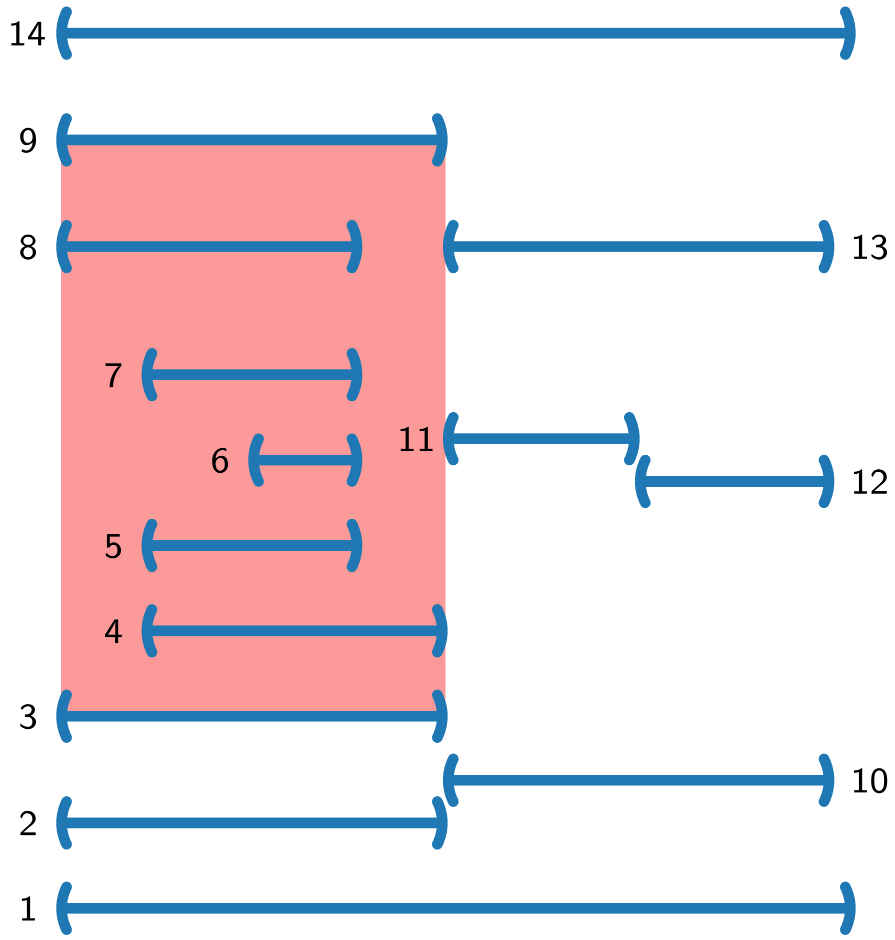

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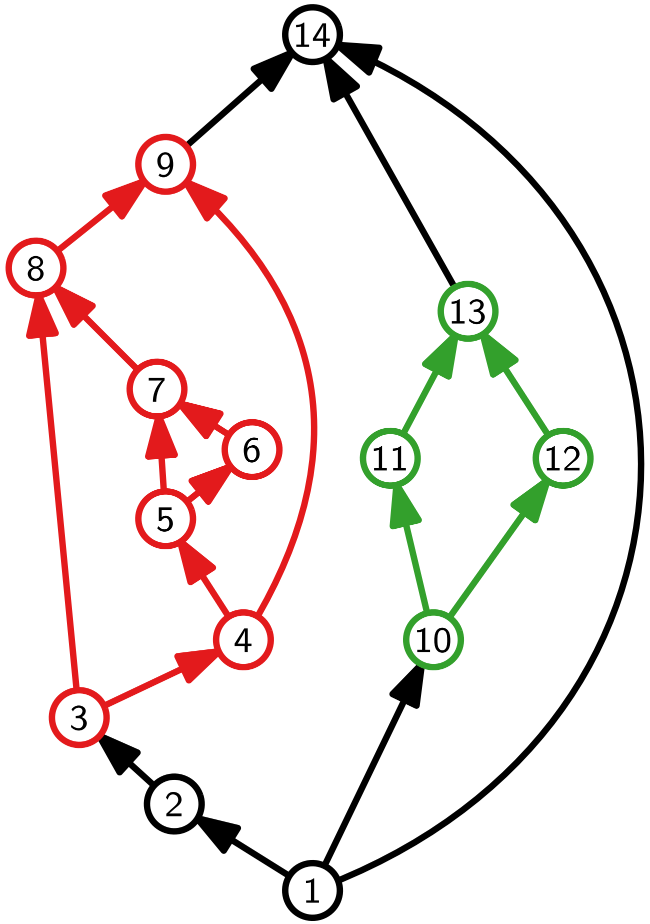

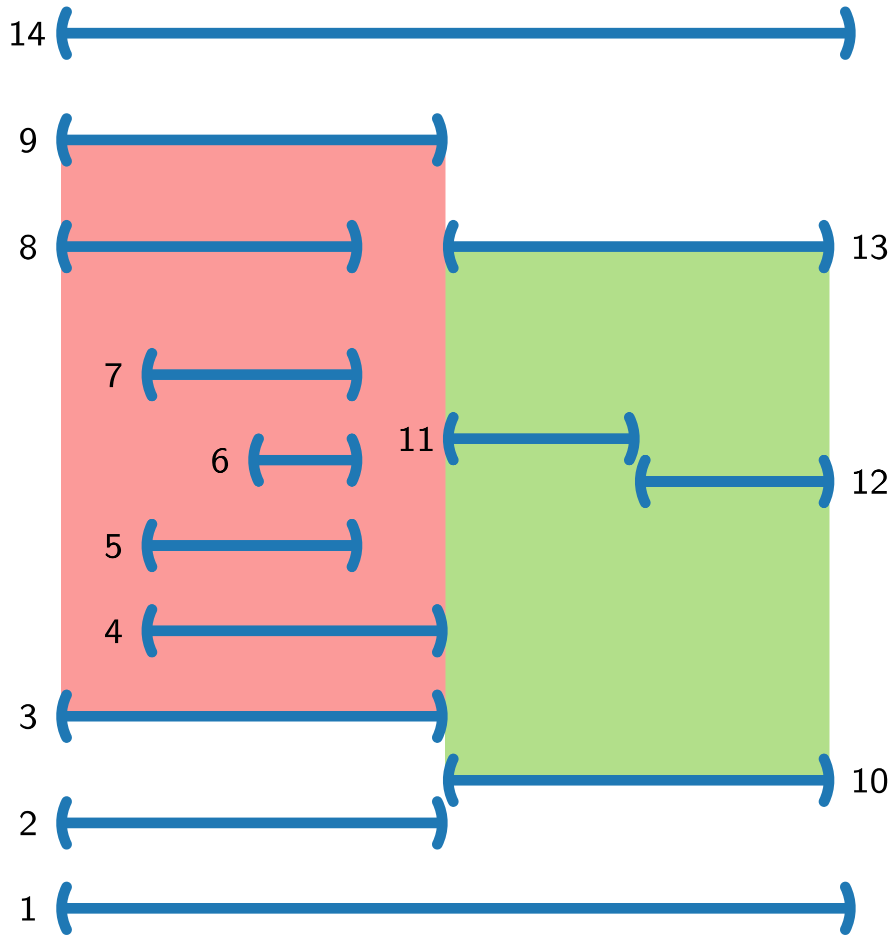

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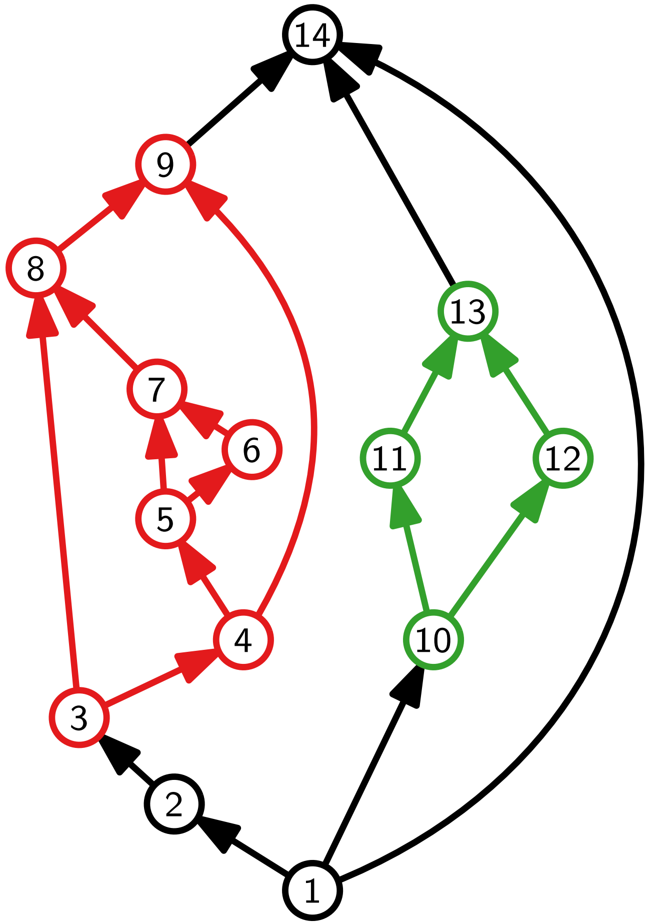
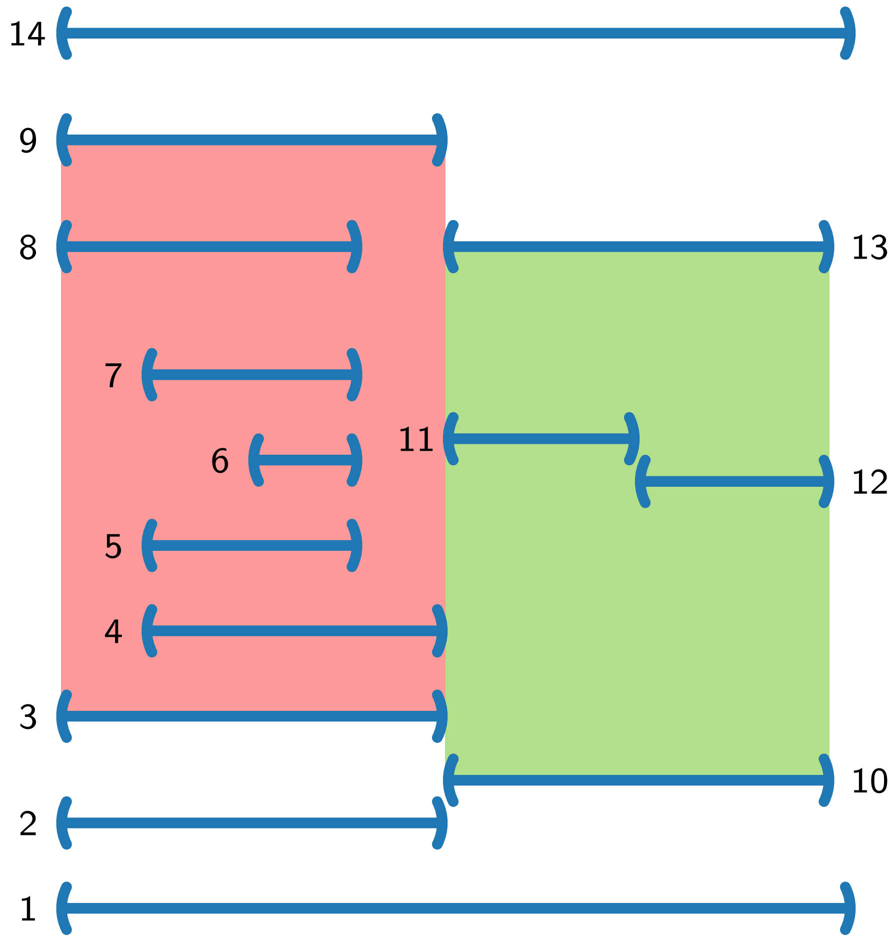
The SPQR-tree of an st-graph G induces a recursive **tiling** of any ε-bar visibility representation of G.





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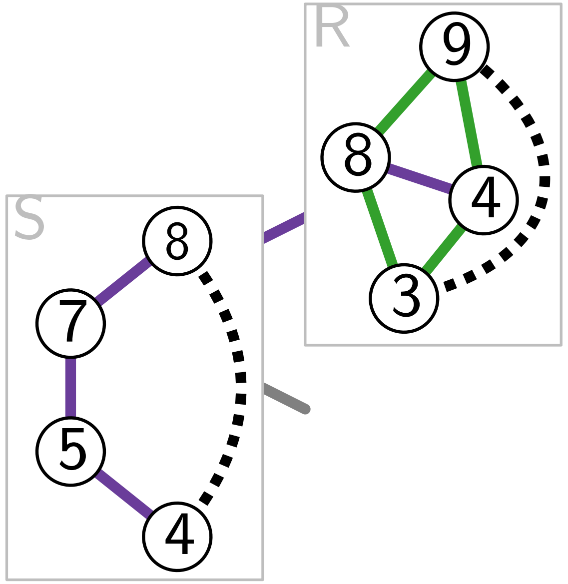
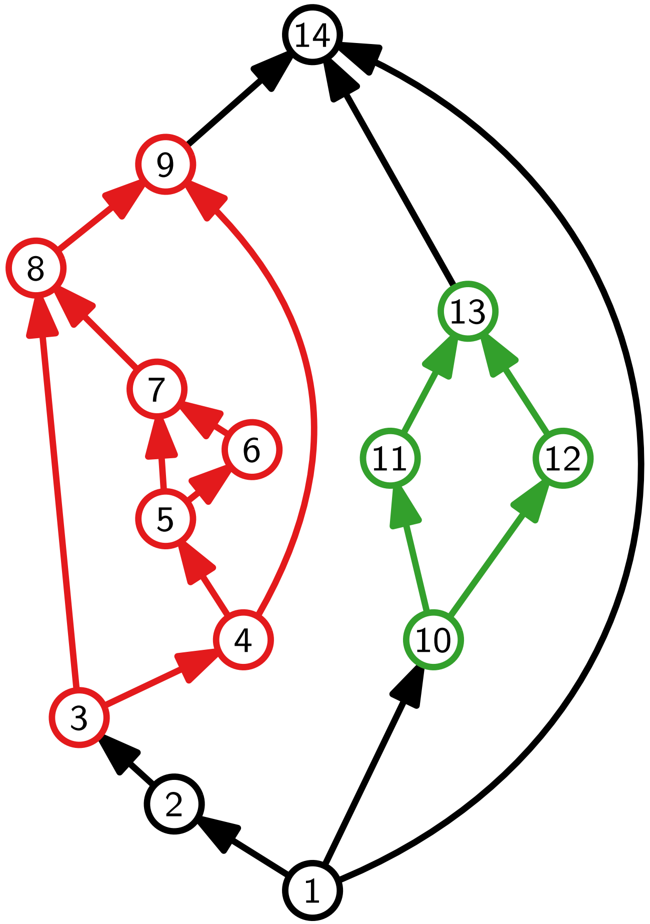
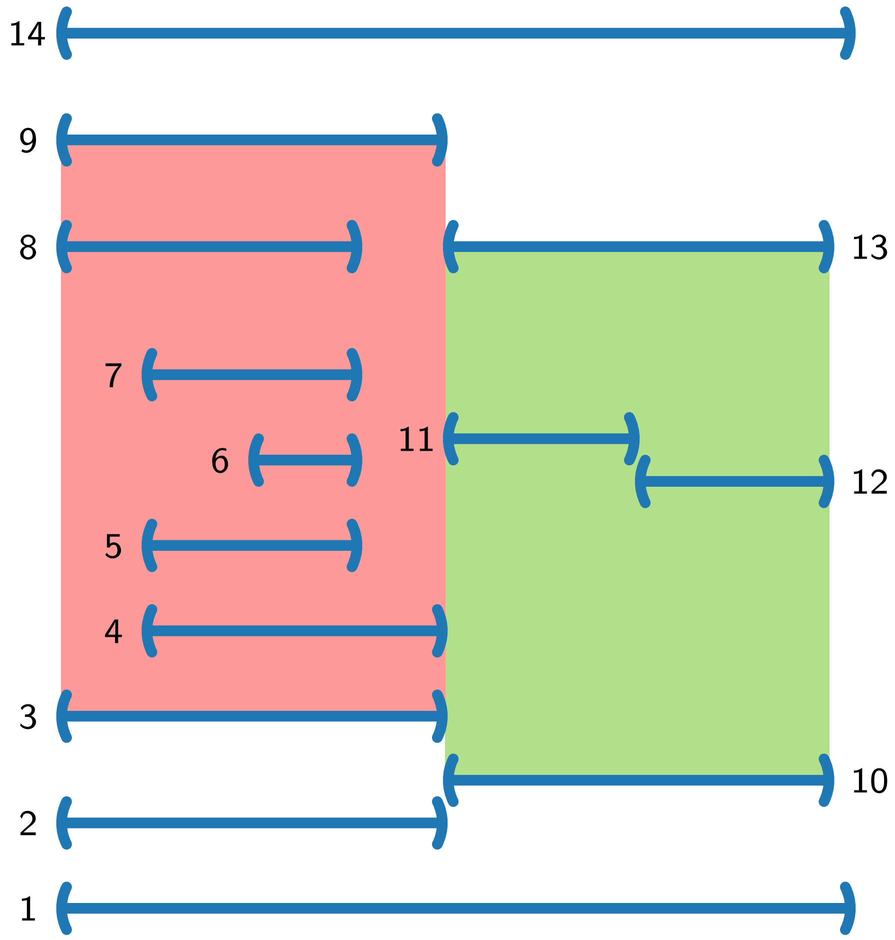


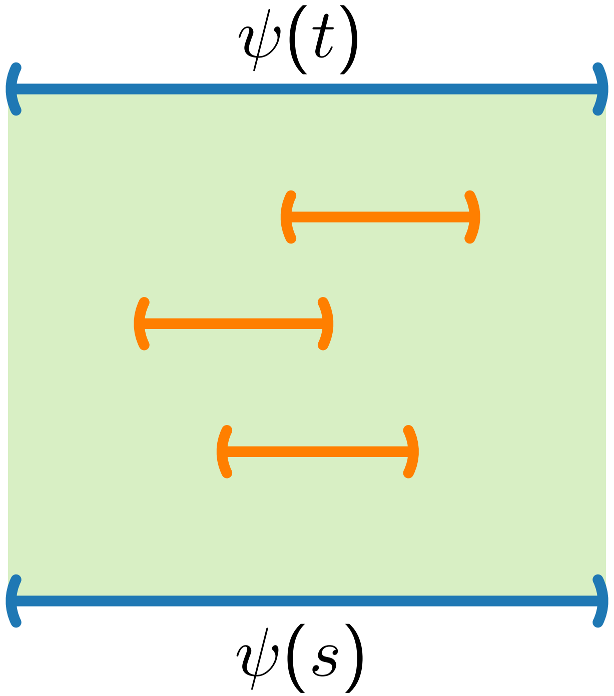
Solve tiles bottom-up.





Tiles

Convention. **Orange** bars are from the given partial representation.



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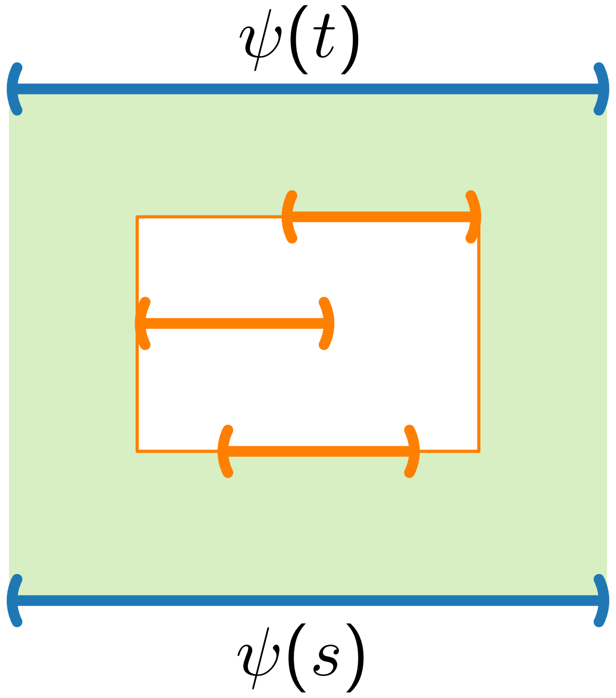

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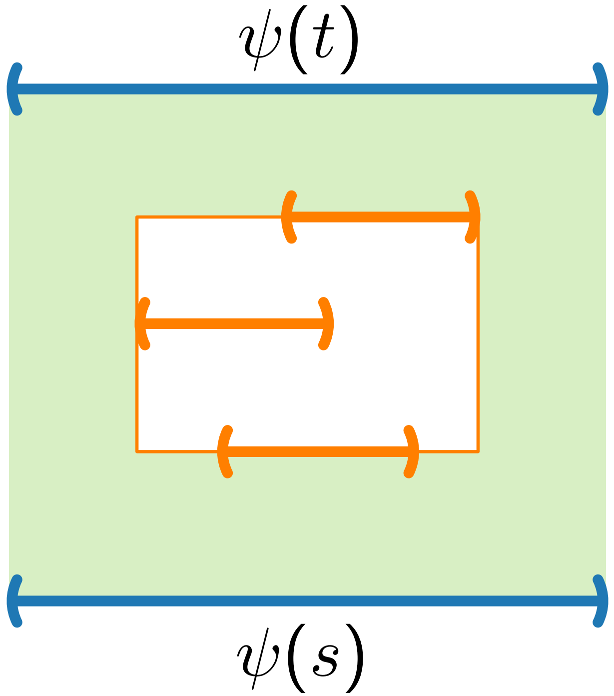

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The bounding box (tile) of any solution ψ **contains** the bounding box of the partial representation.

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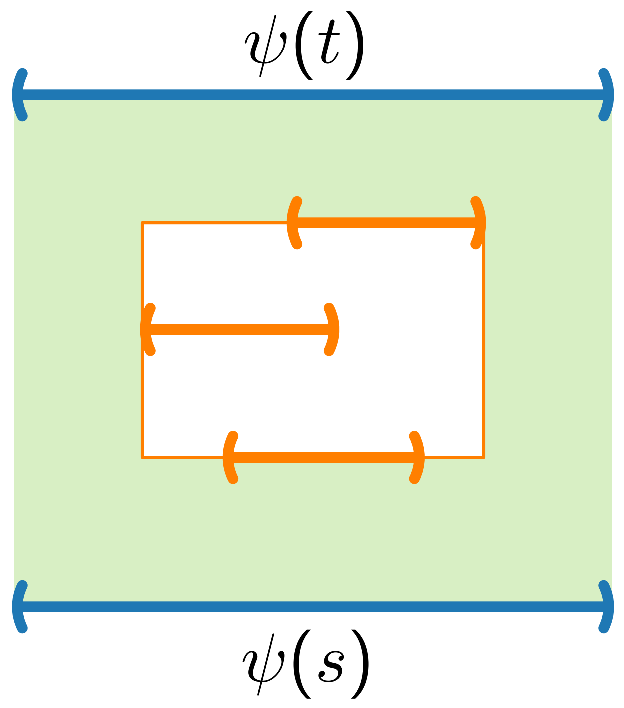

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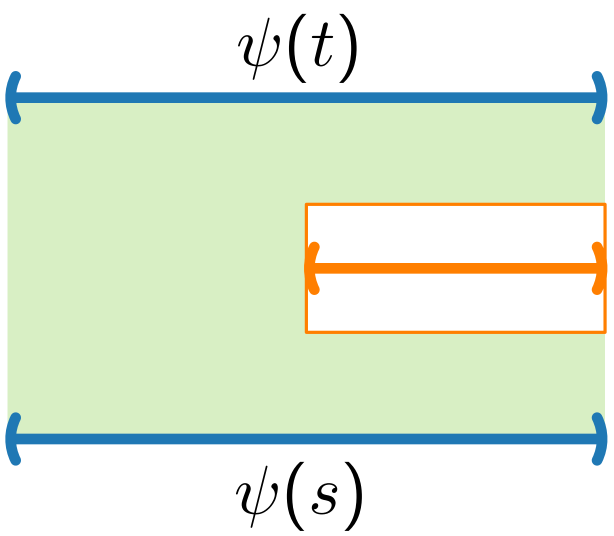
How many different **types** of tiles are there?

Types of Tiles



- Left **L**oose
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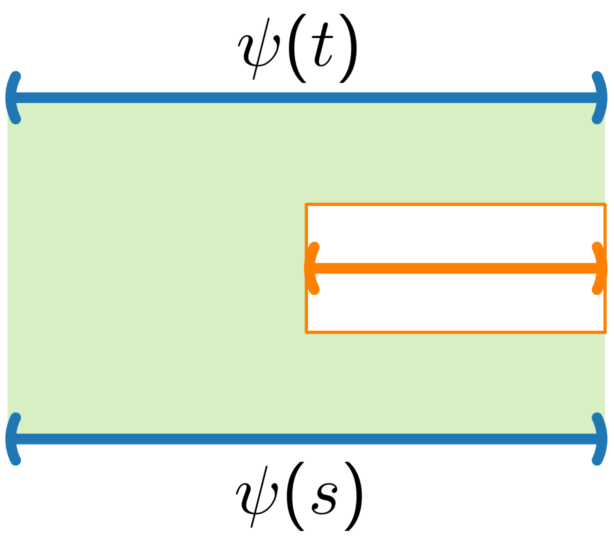

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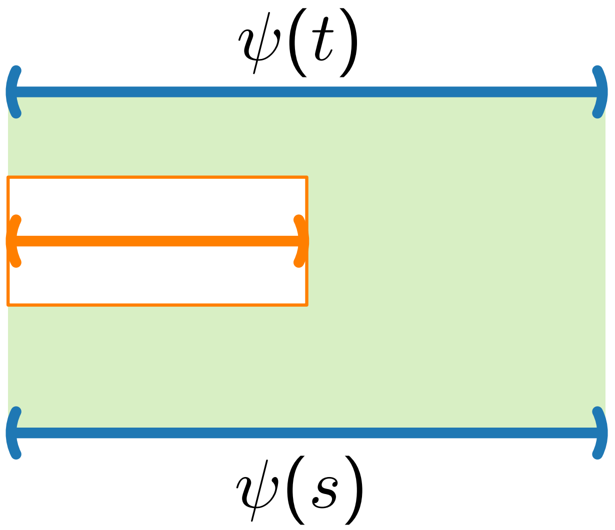

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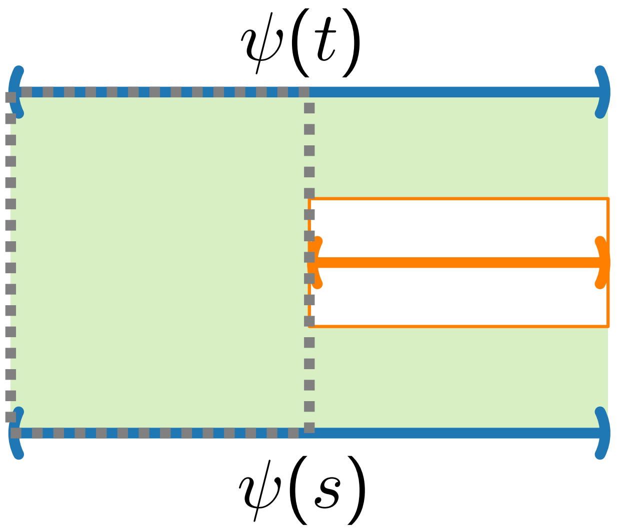

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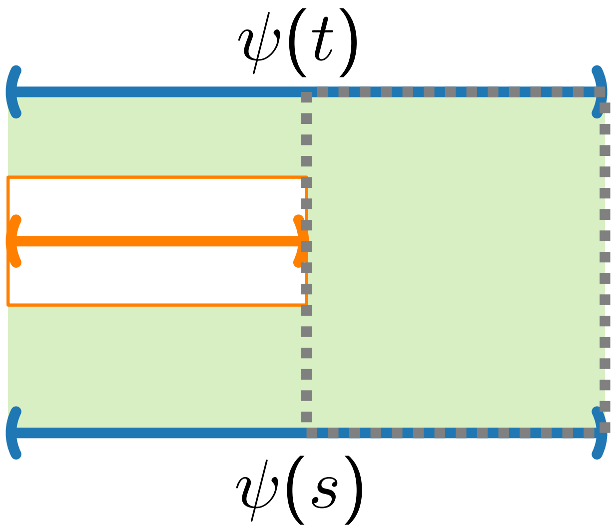

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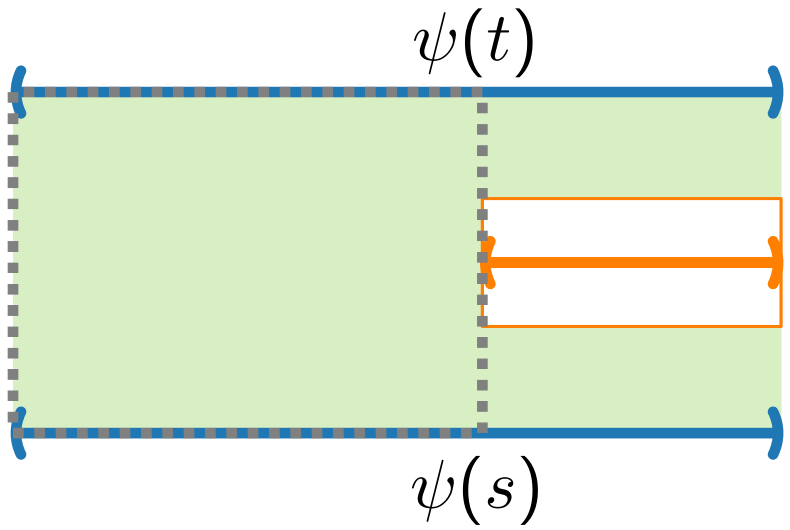

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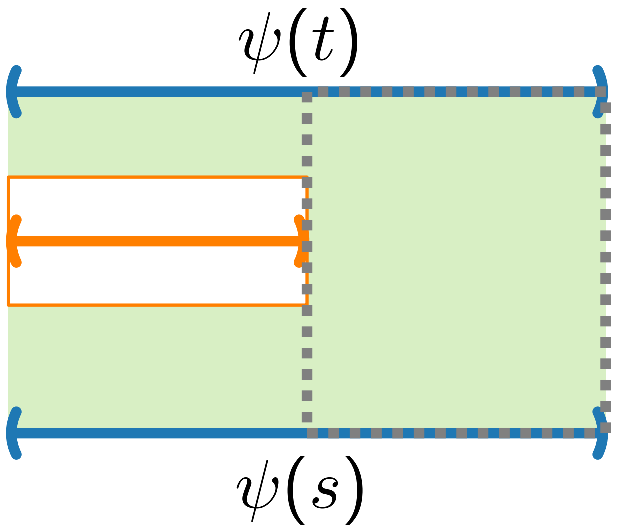

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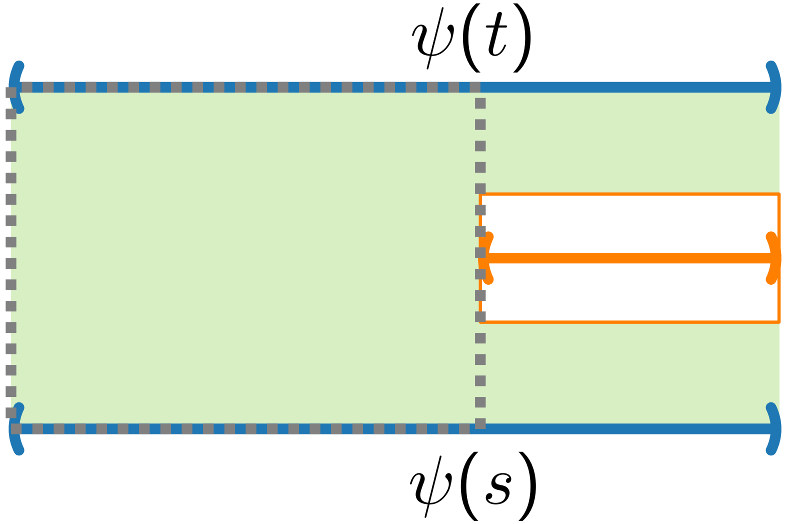

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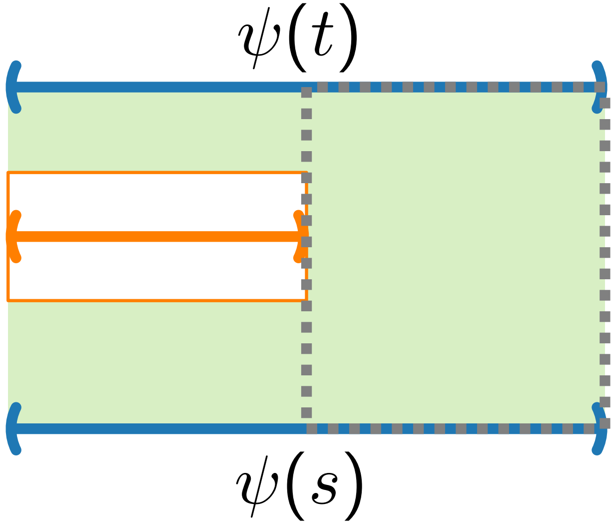
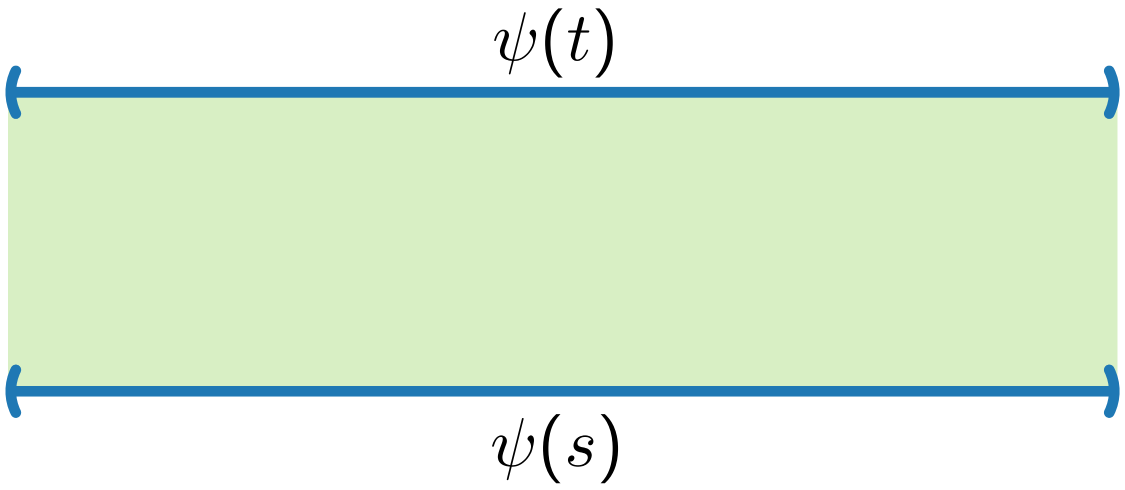
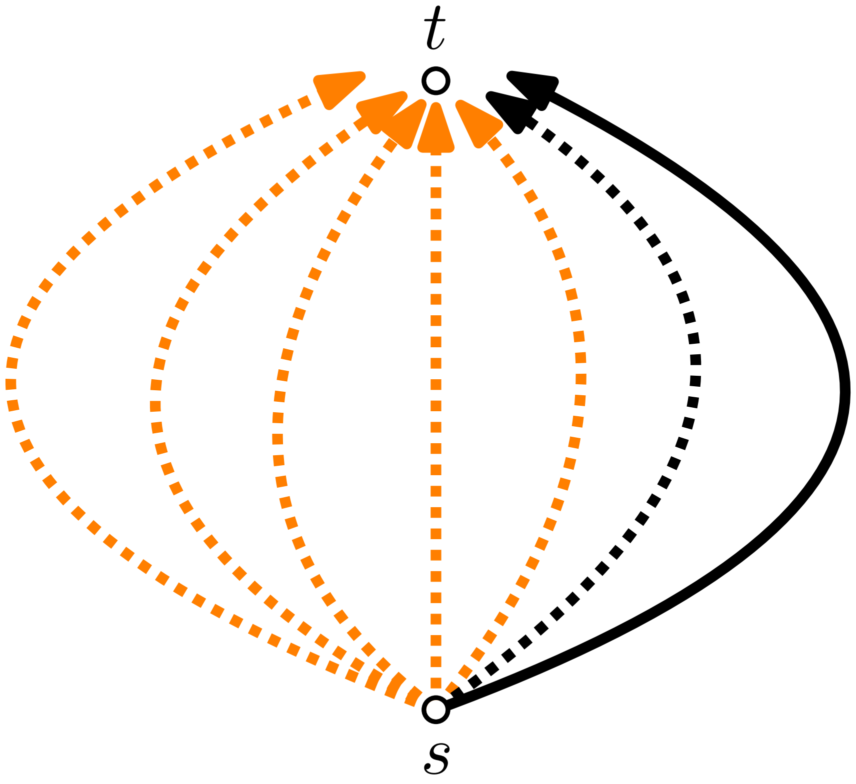


Four different types: **FF, FL, LF, LL**

P-Nodes





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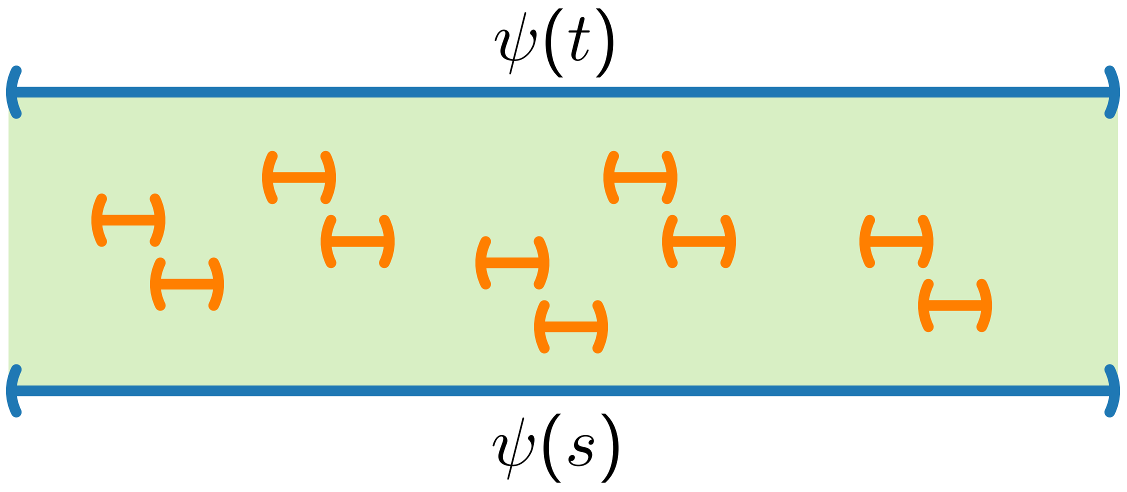

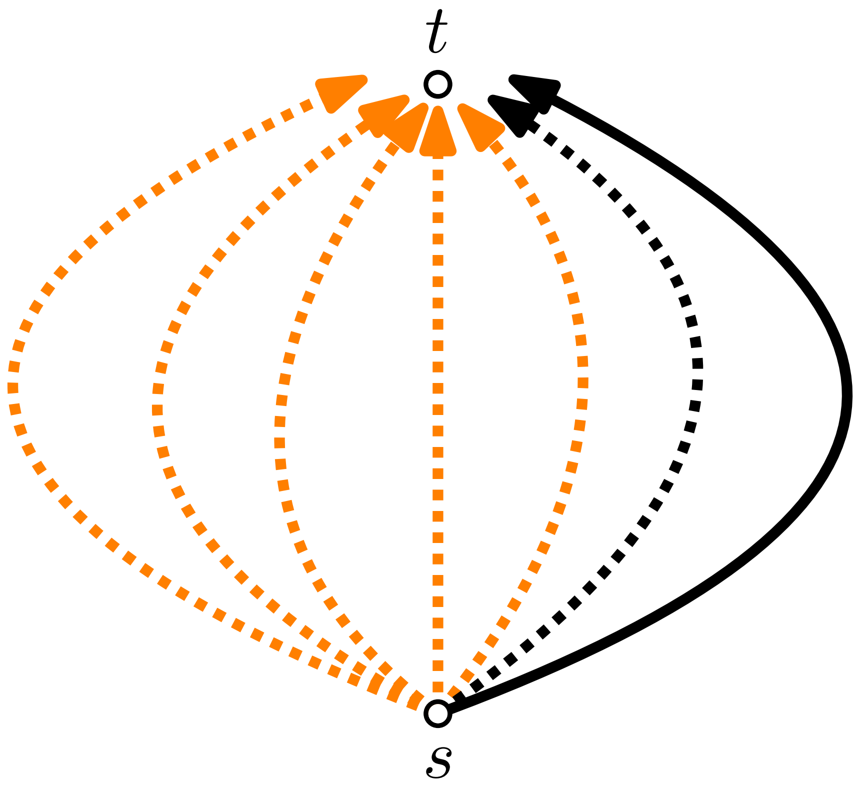

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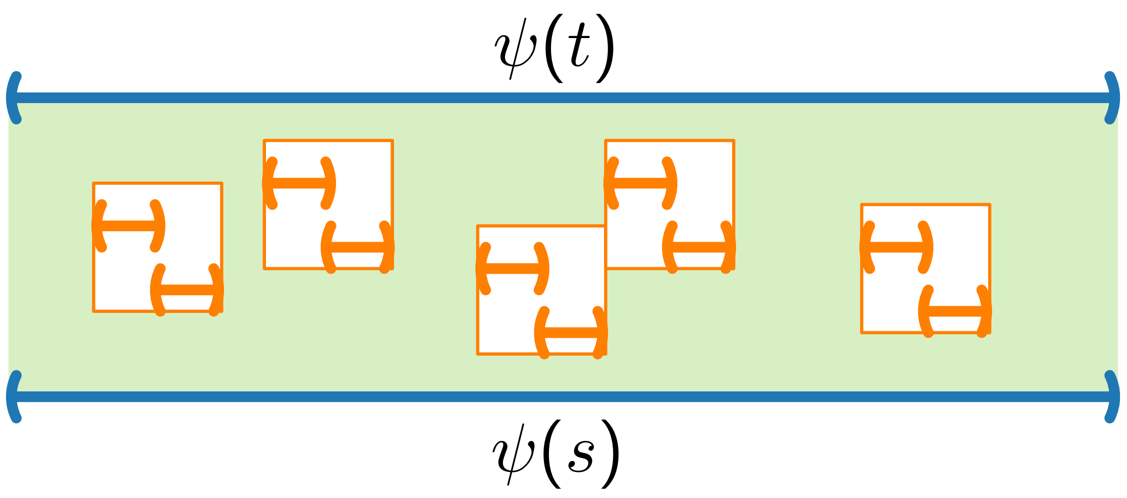

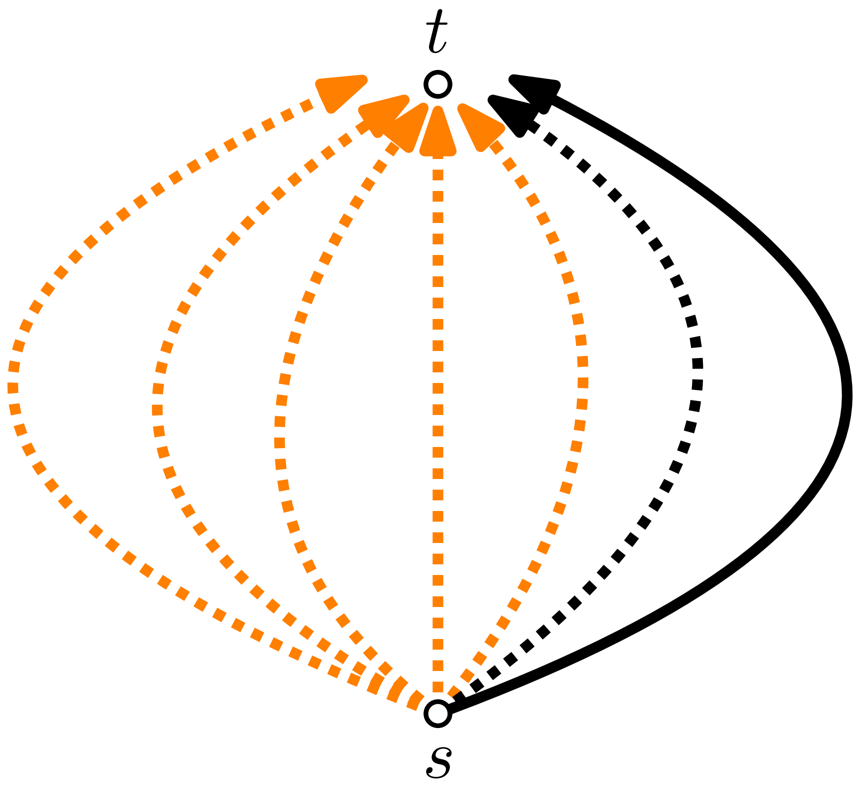

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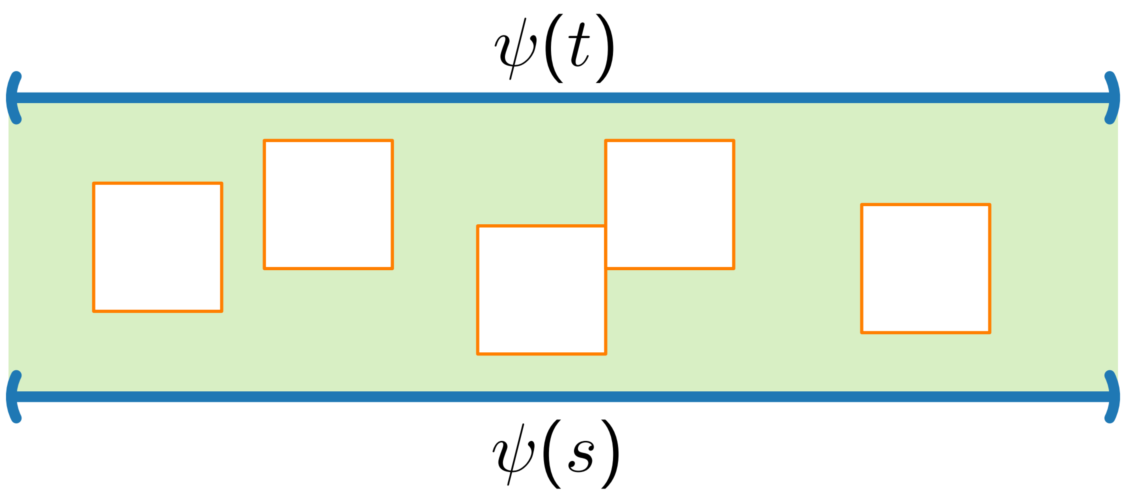

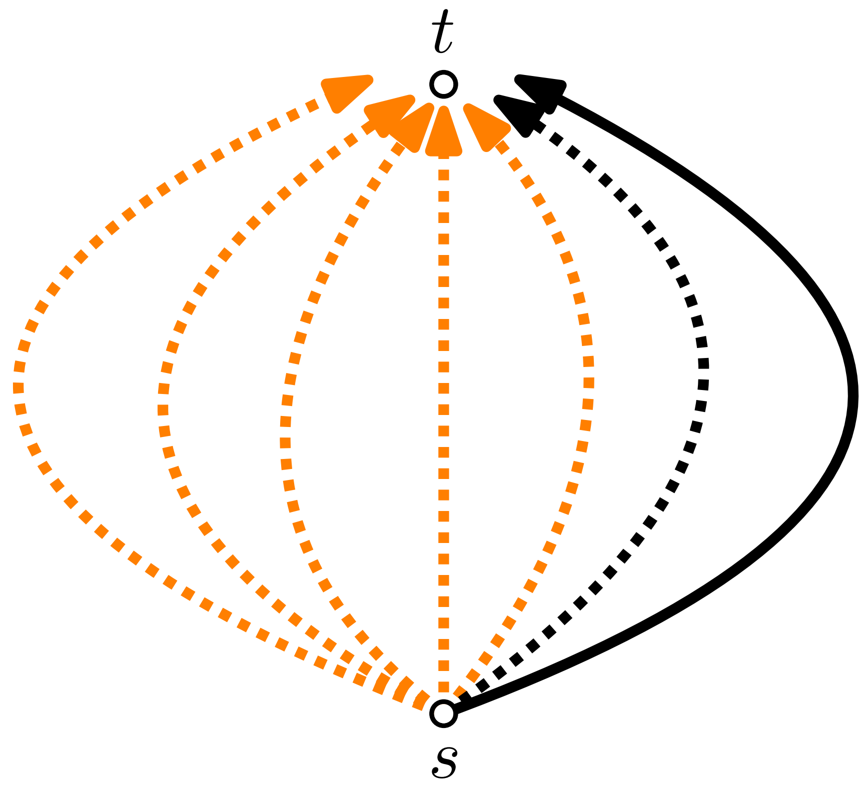

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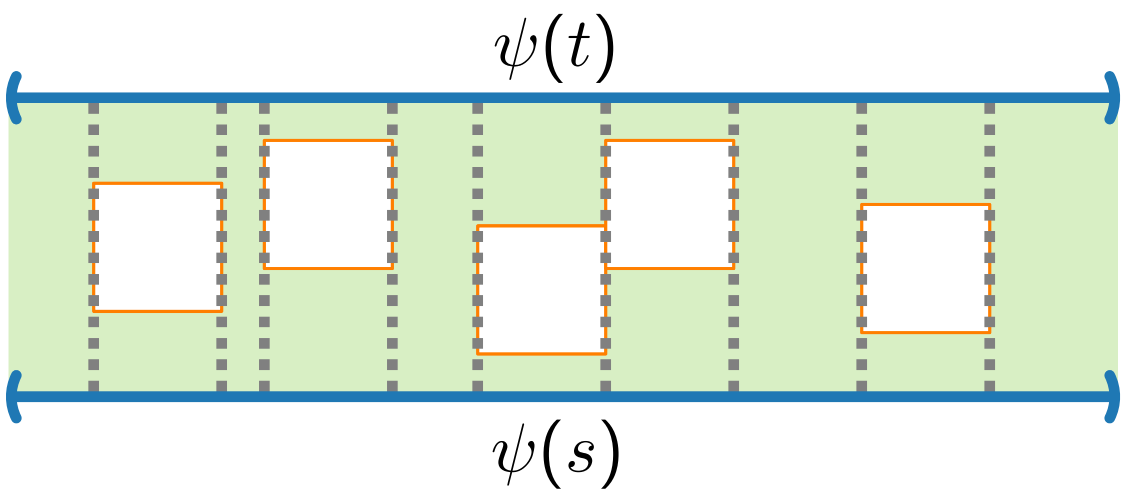

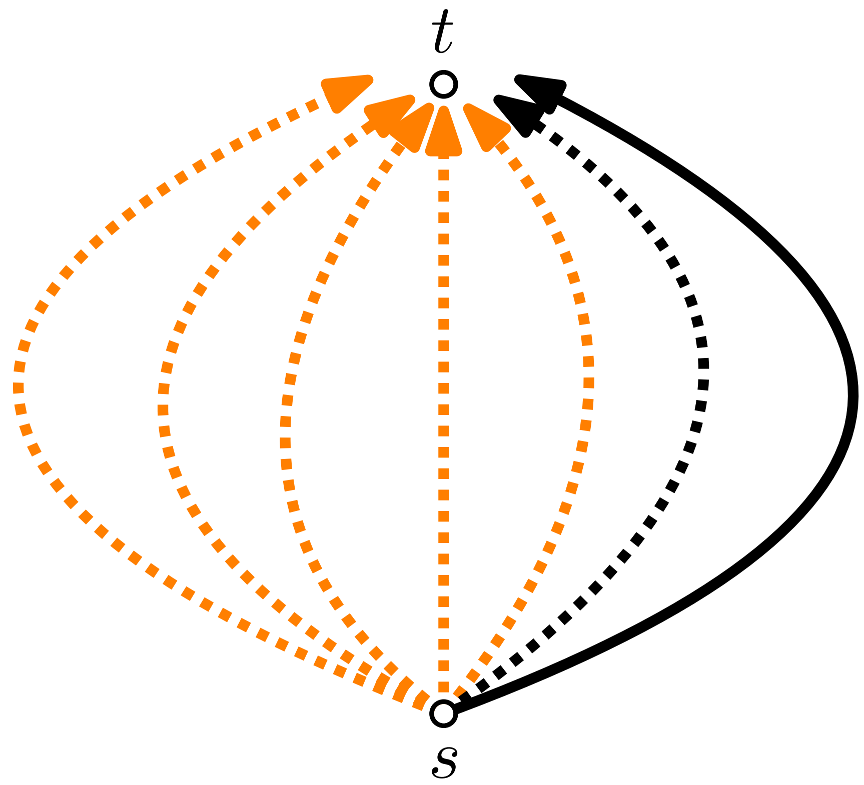

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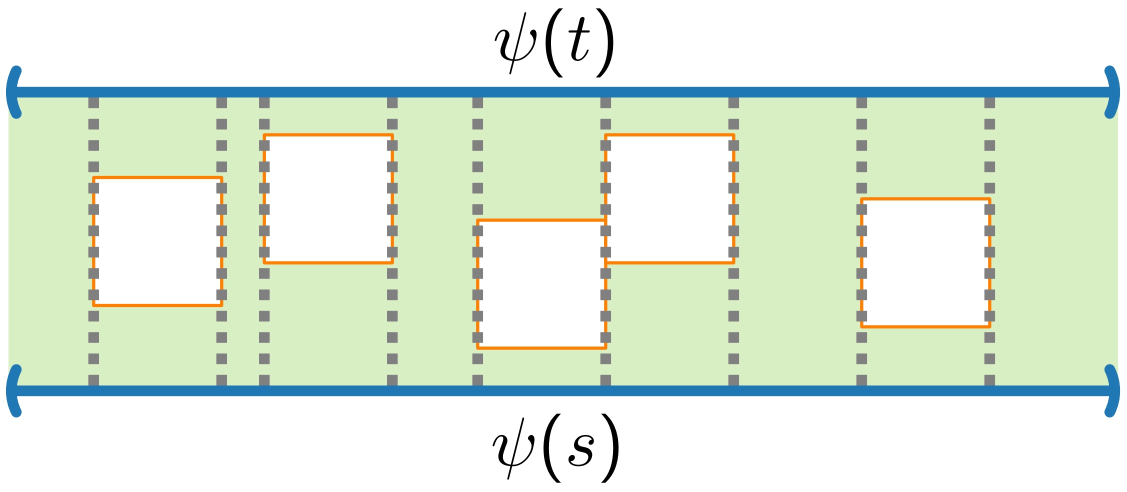
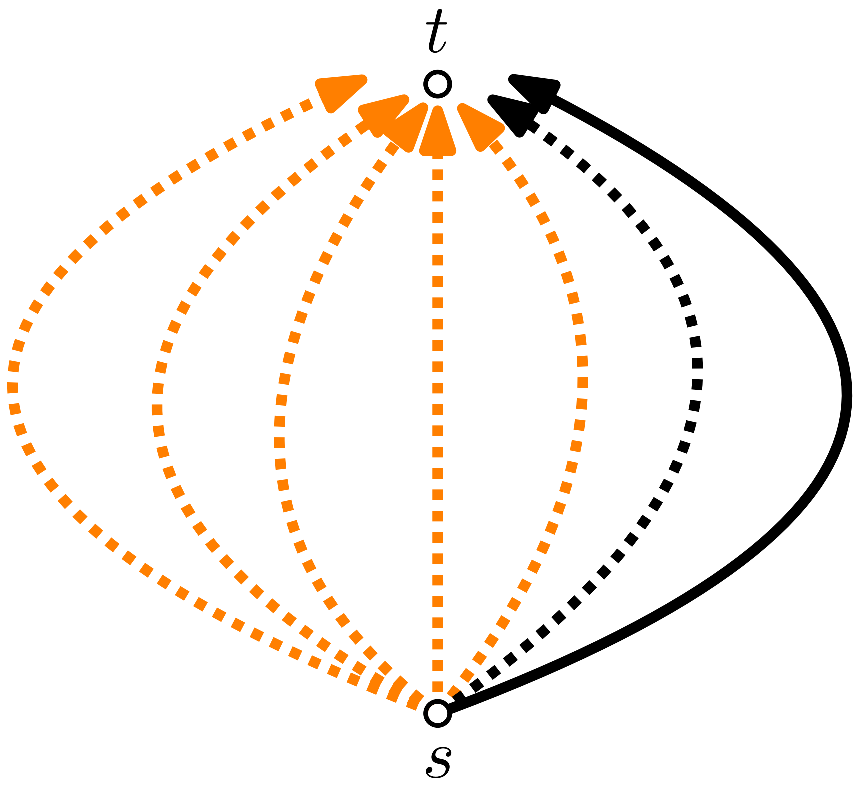


- Children of **P**-node with prescribed bars occur in given left-to-right order



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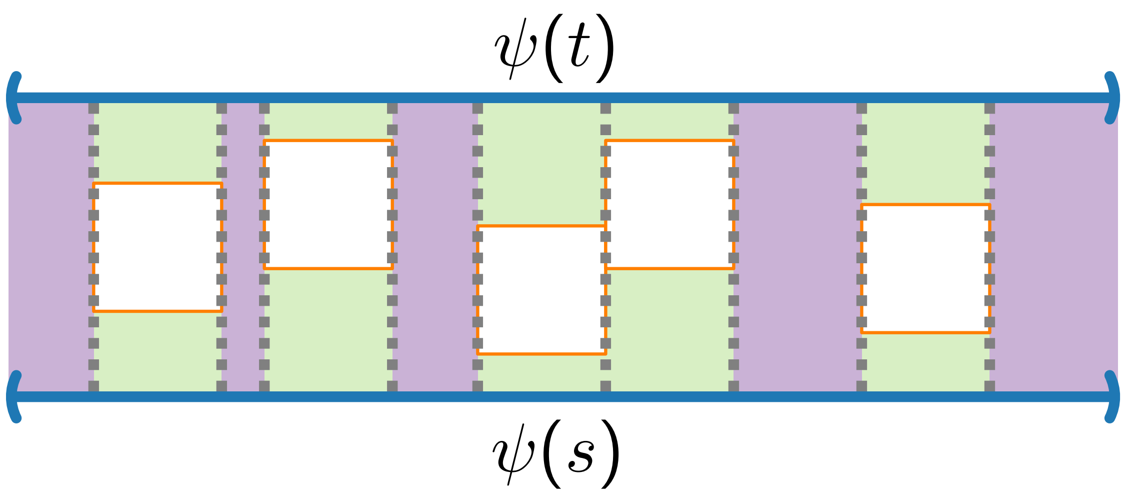
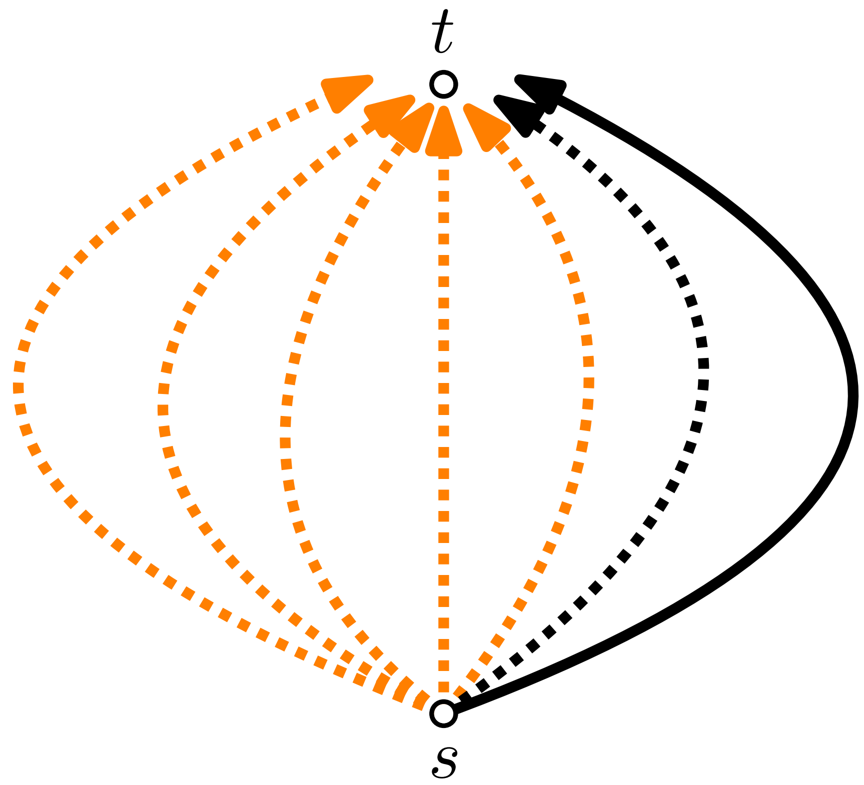


- Children of **P**-node with prescribed bars occur in given left-to-right order
- But there might be some gaps. . .



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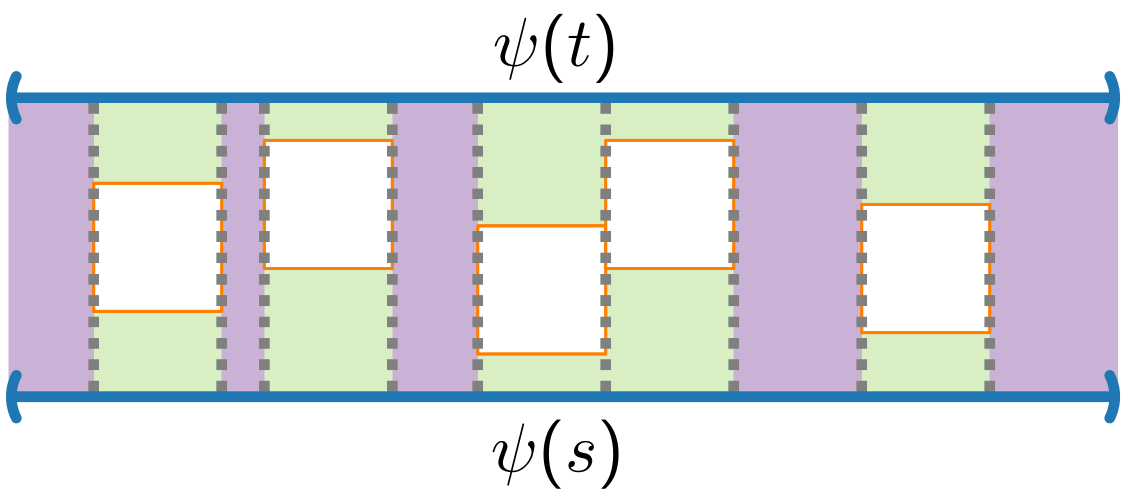

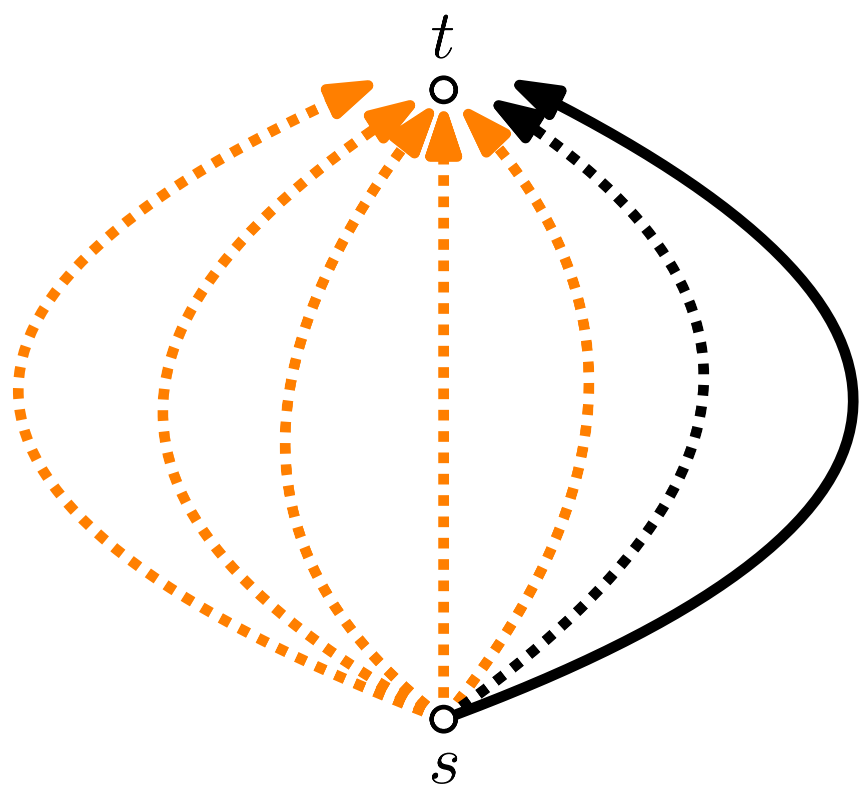

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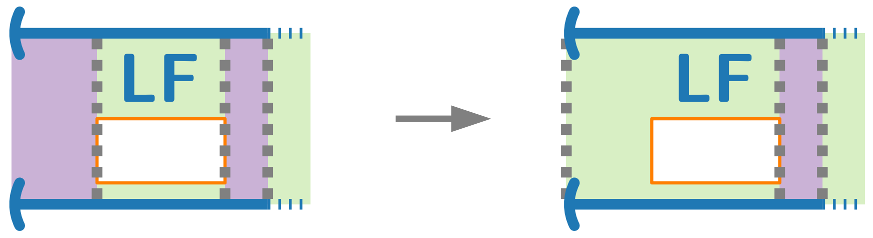
Idea.
Greedily *fill* the gaps by preferring to "stretch" the children with prescribed bars.



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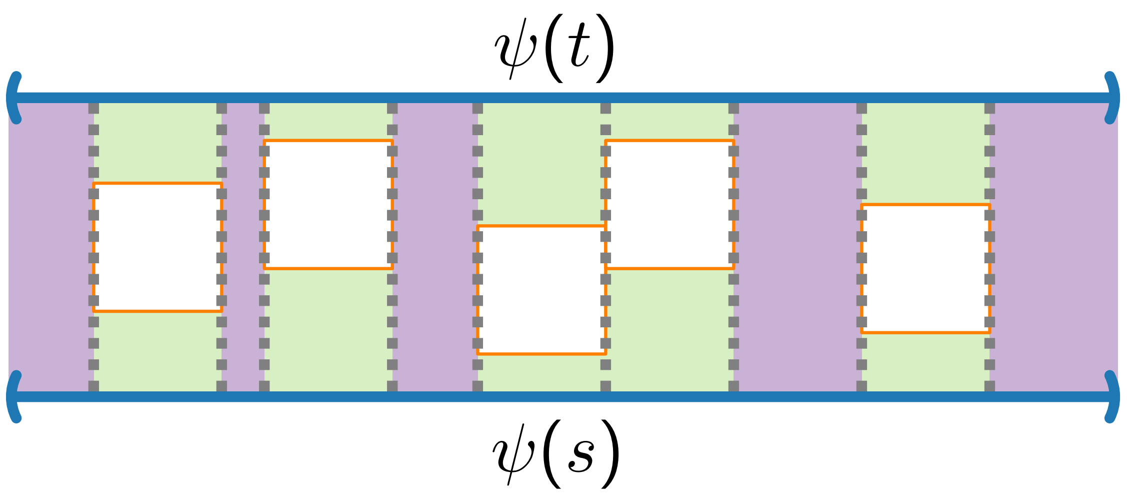

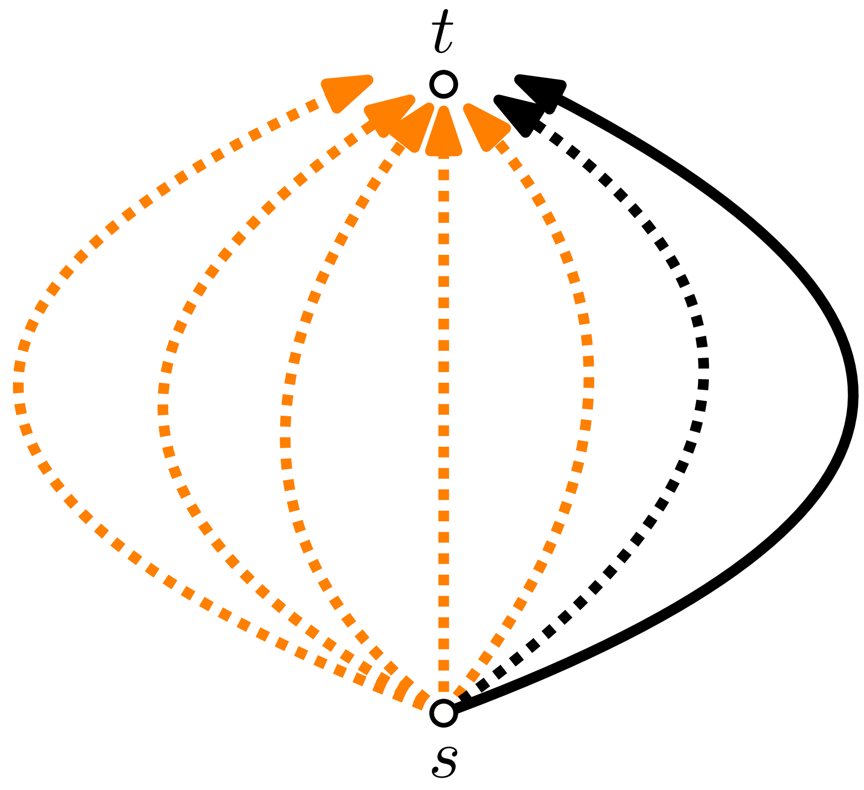

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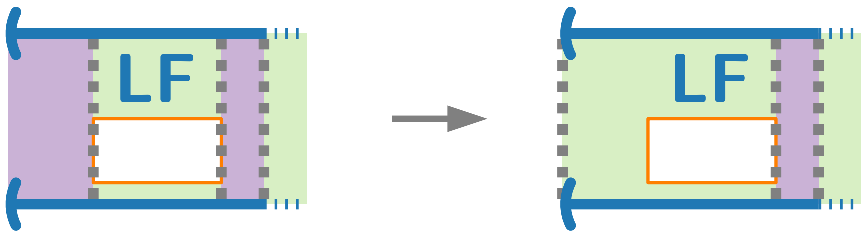
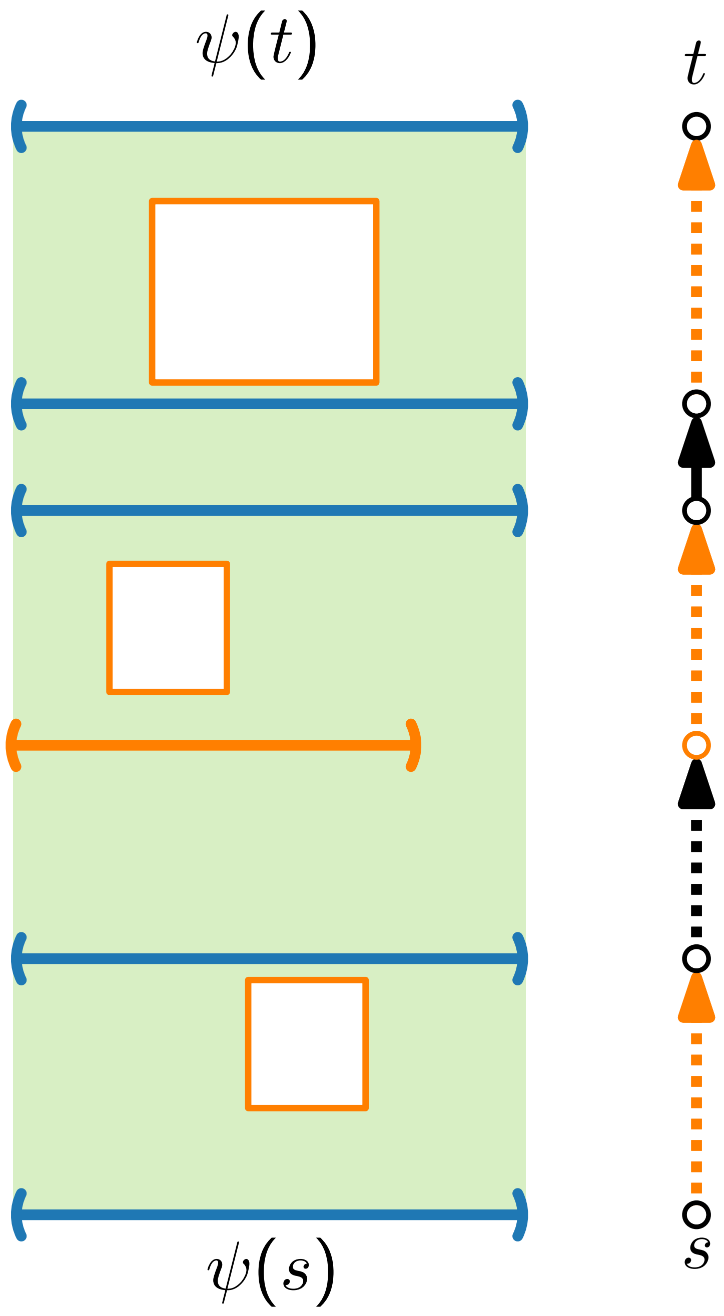


Outcome.
After processing, we must know the valid types for the corresponding subgraphs.

S-Nodes



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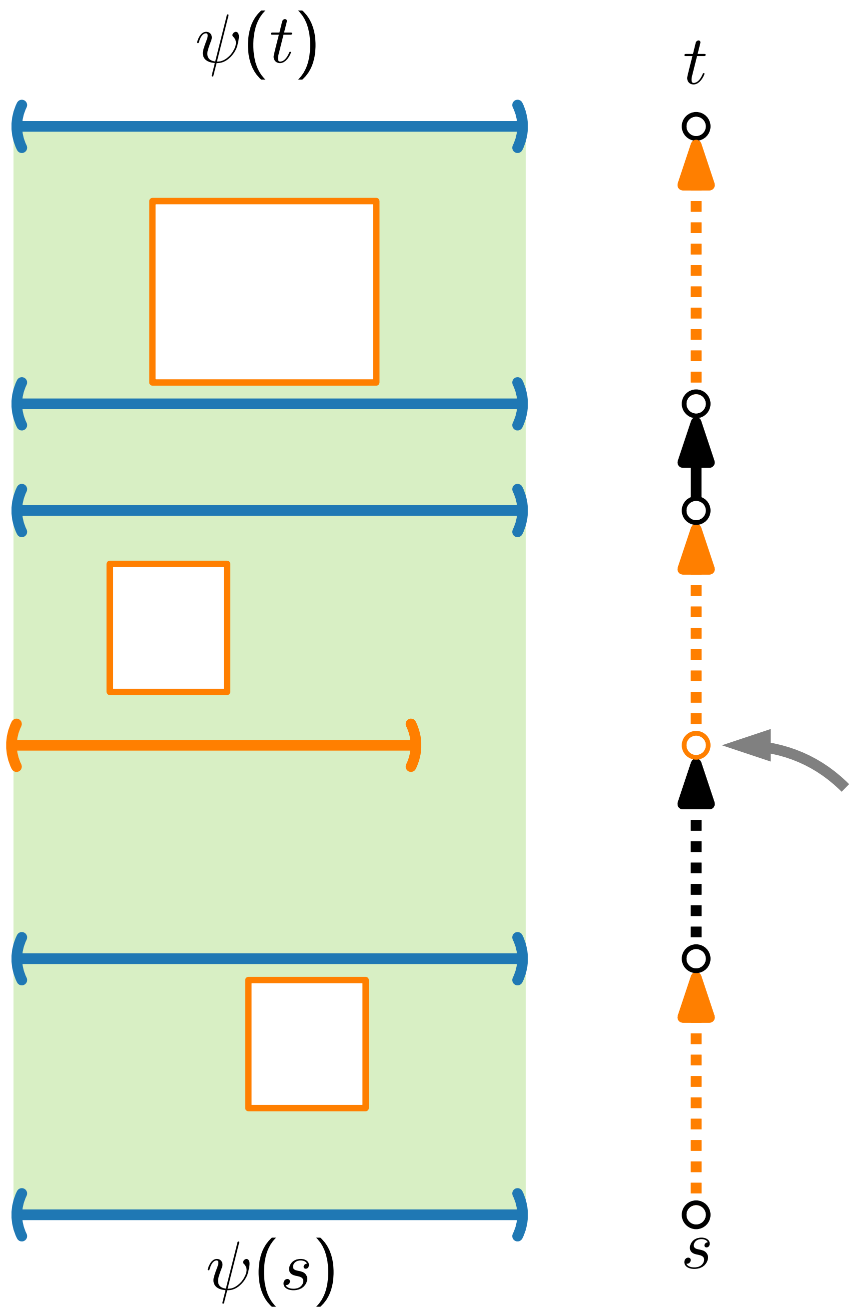
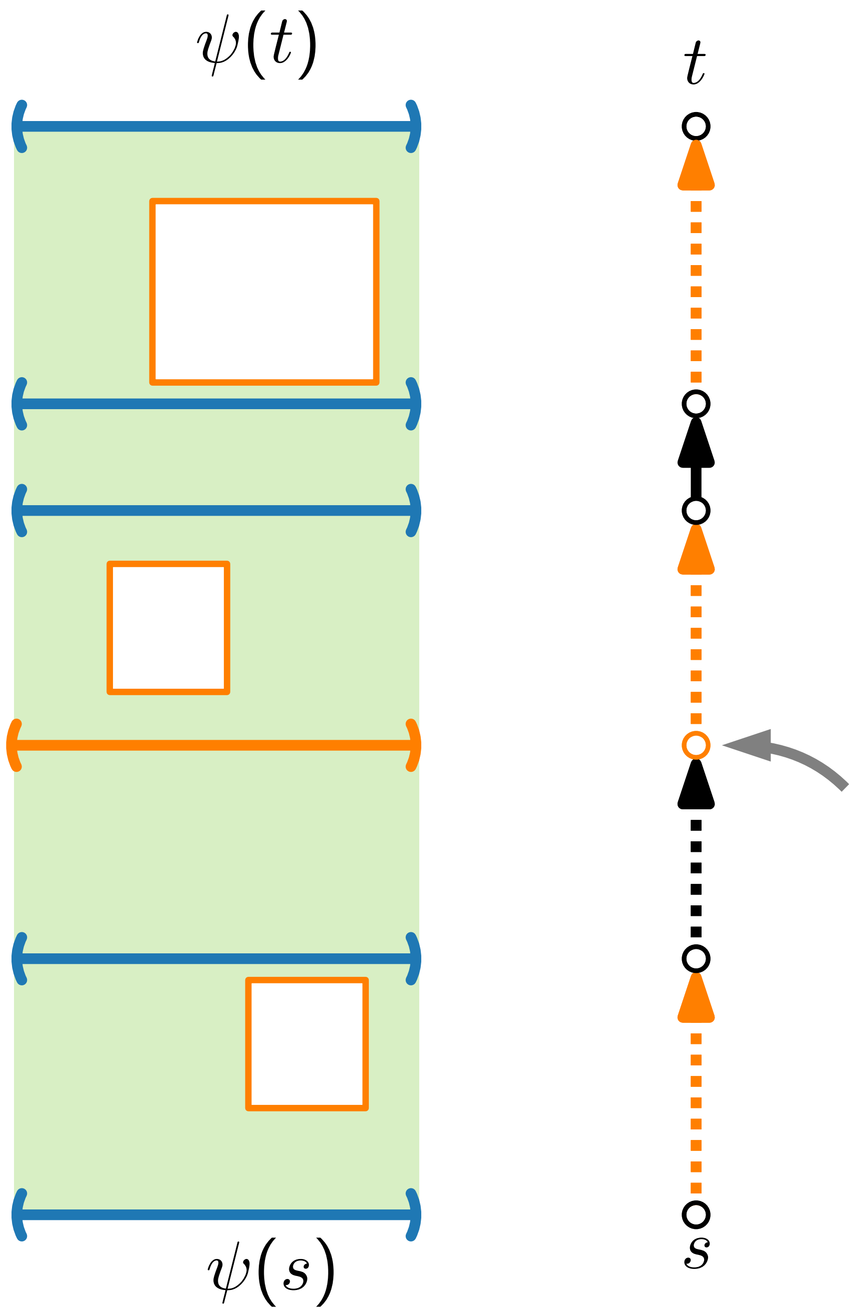


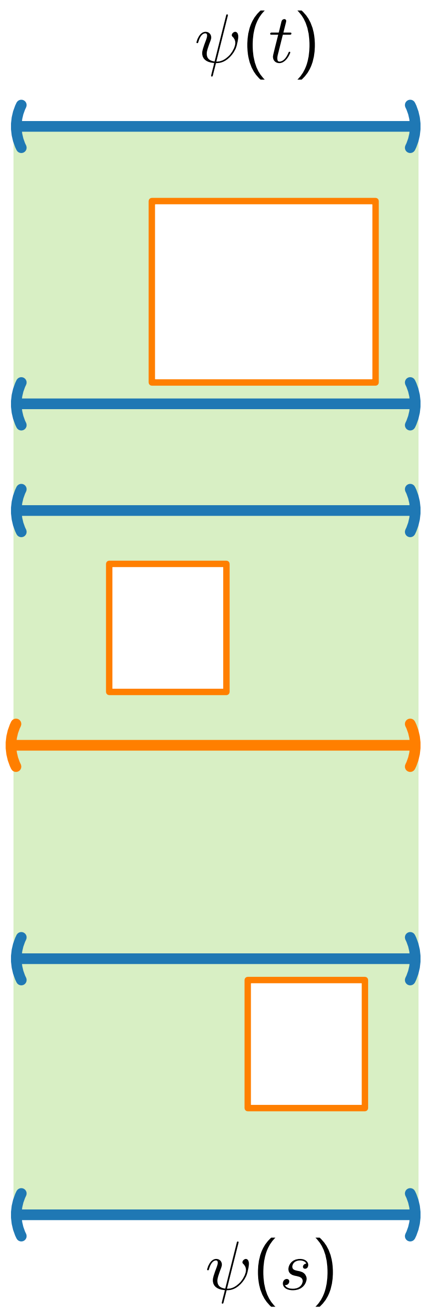
This fixed vertex means we can only make a Fixed-Fixed representation!

S-Nodes



This fixed vertex means we can only make a **F**ixed-**F**ixed representation!

S-Nodes



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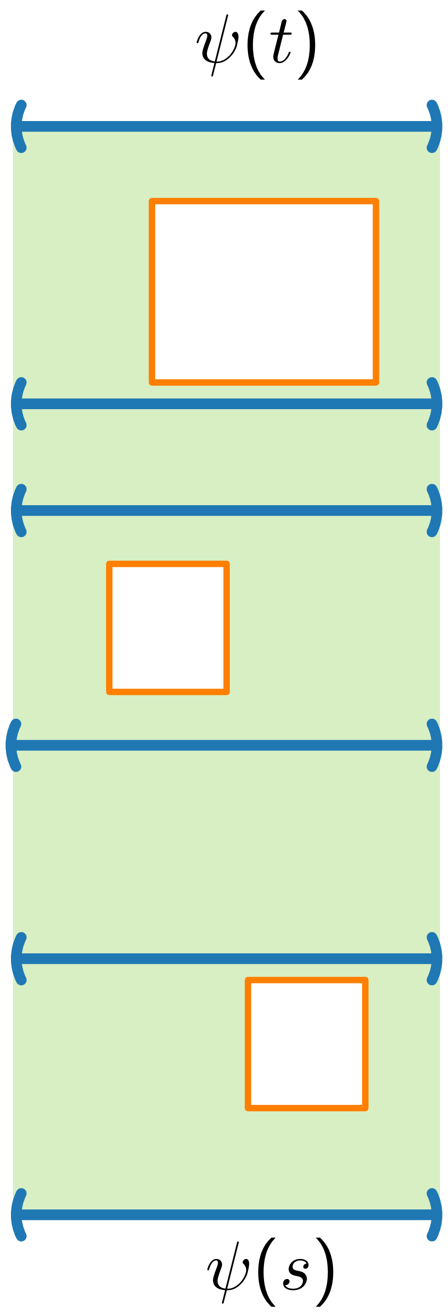

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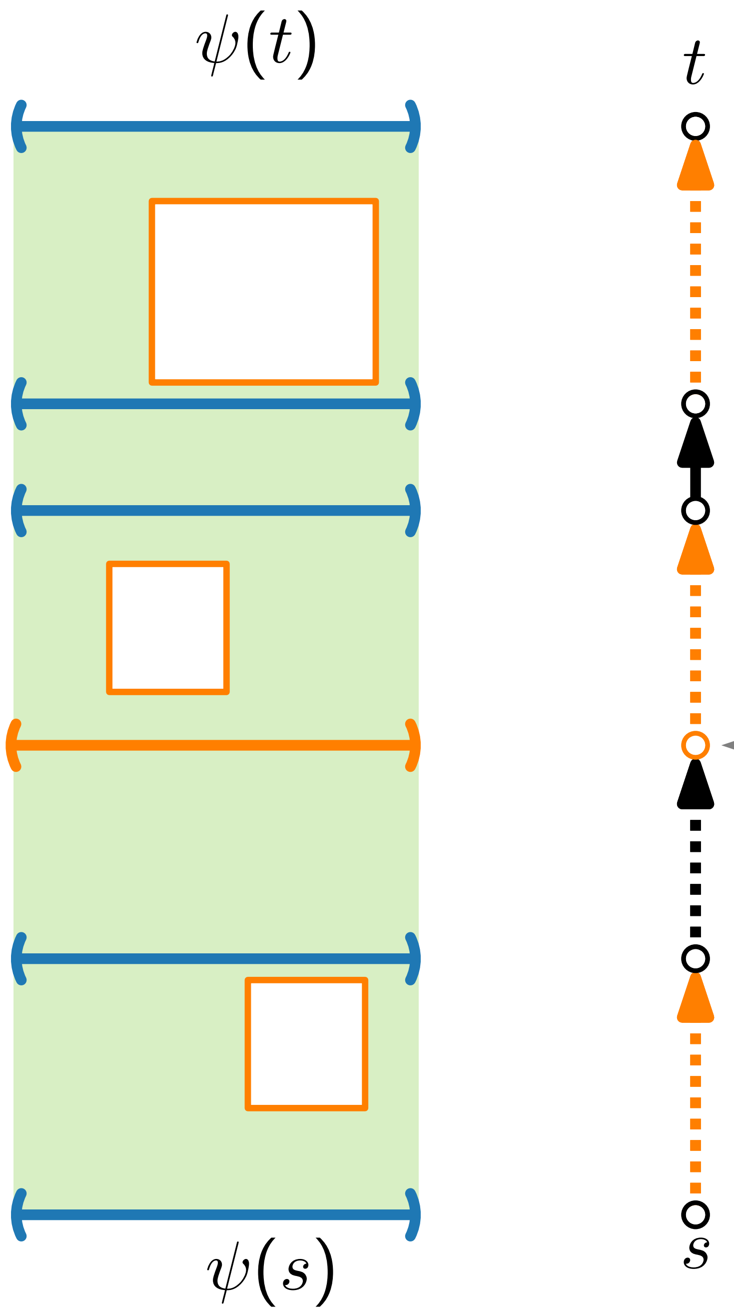
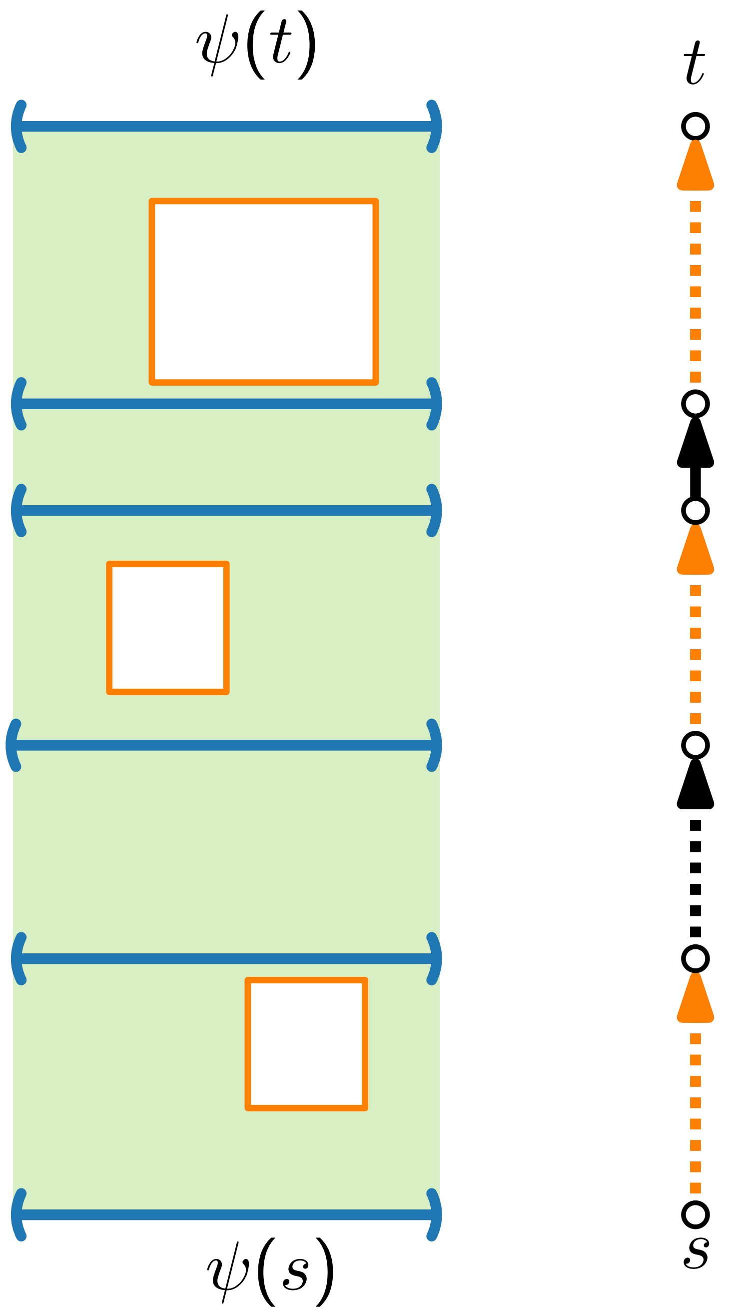
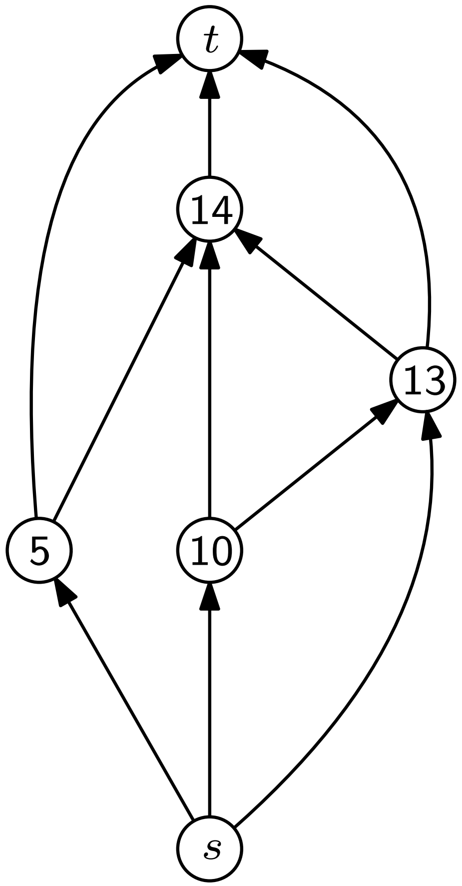


Here we have a chance to make all (**LL, FL, LF, FF**) types.

This fixed vertex means we can only make a **F**ixed-**F**ixed representation!



R-Nodes



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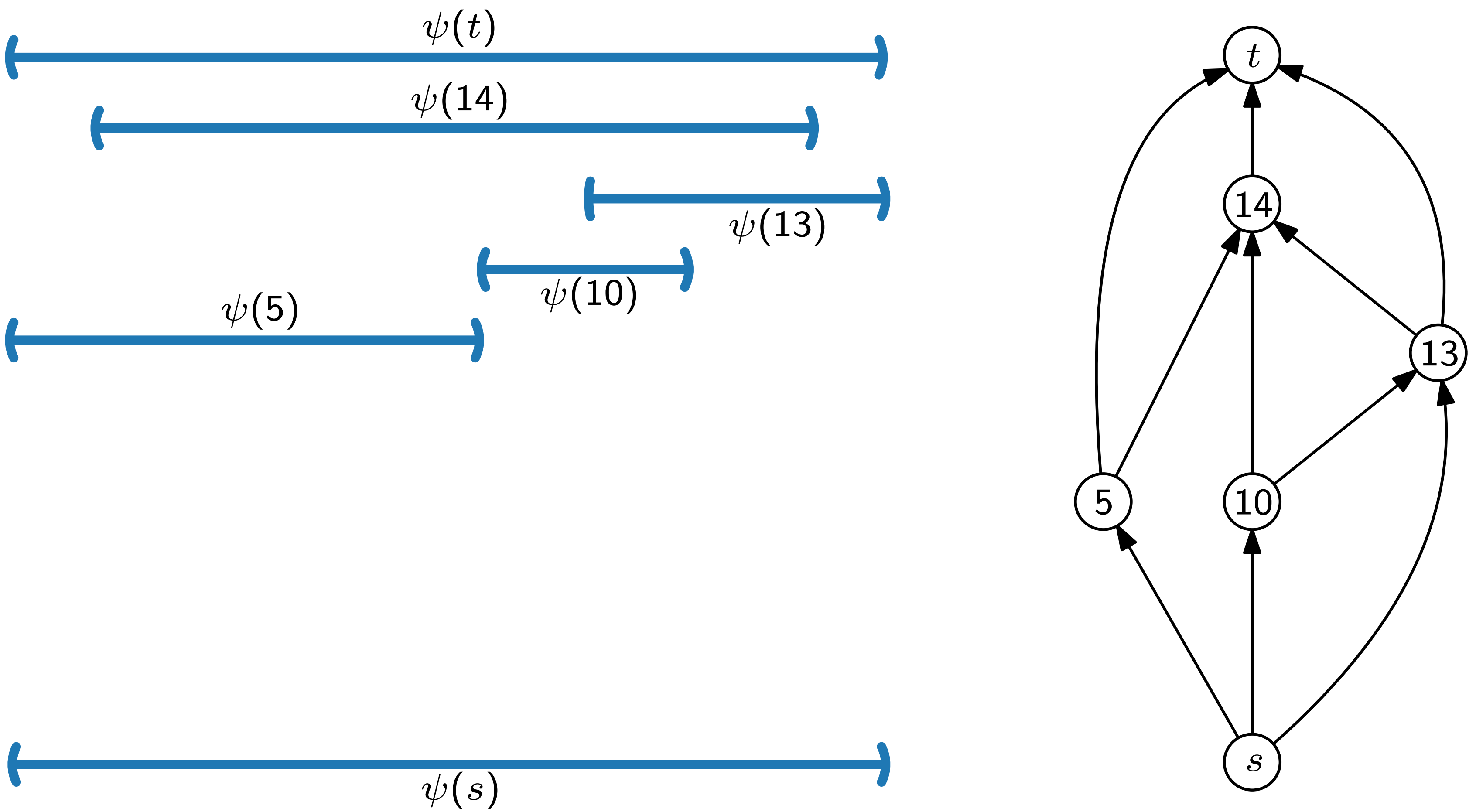

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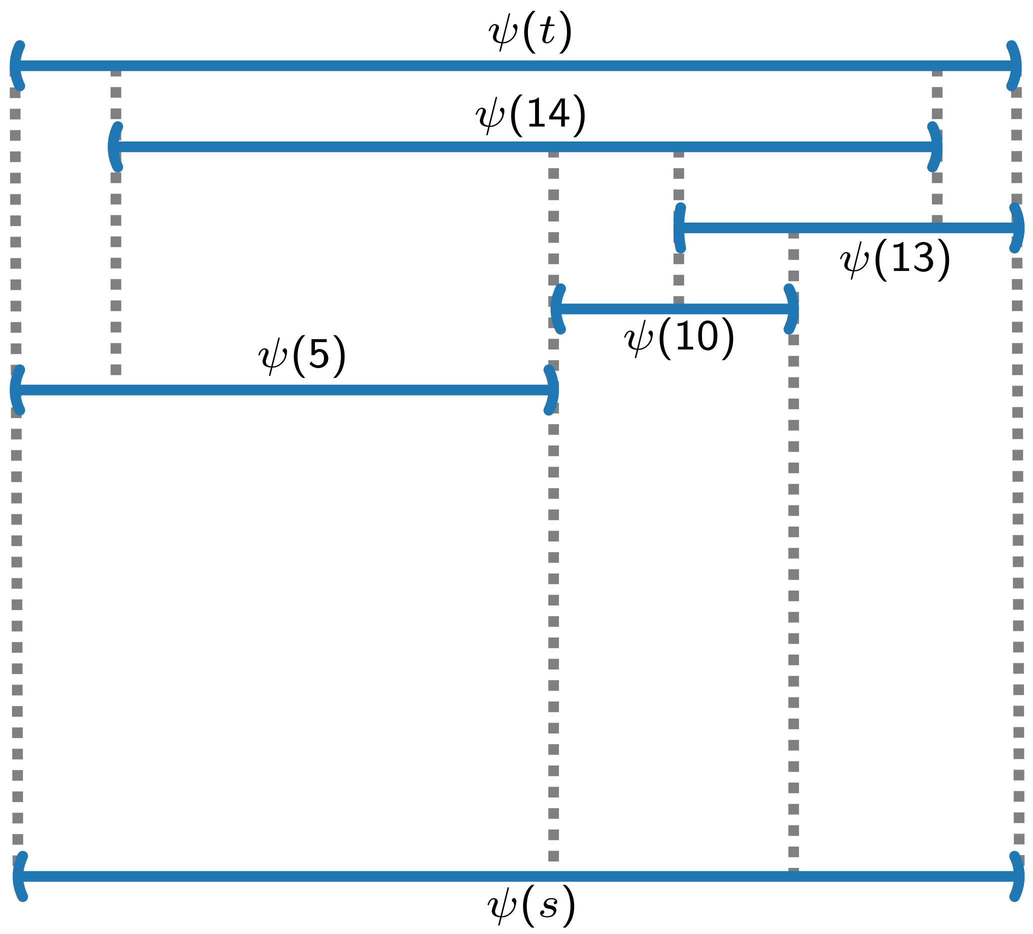

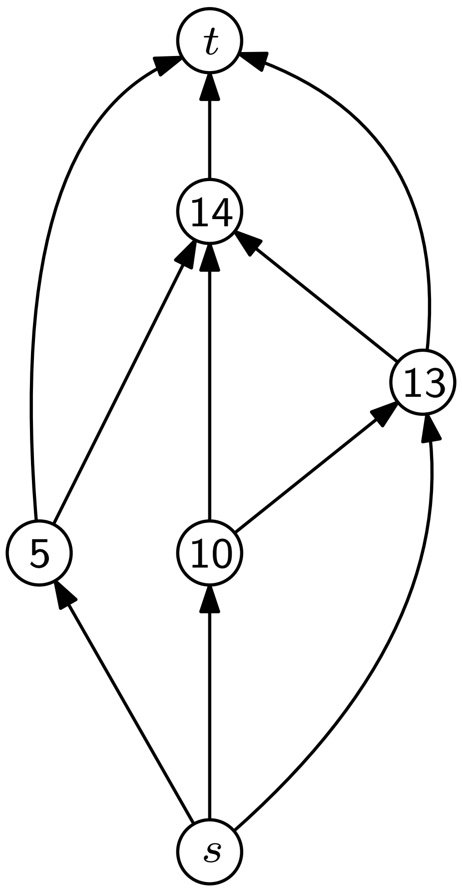

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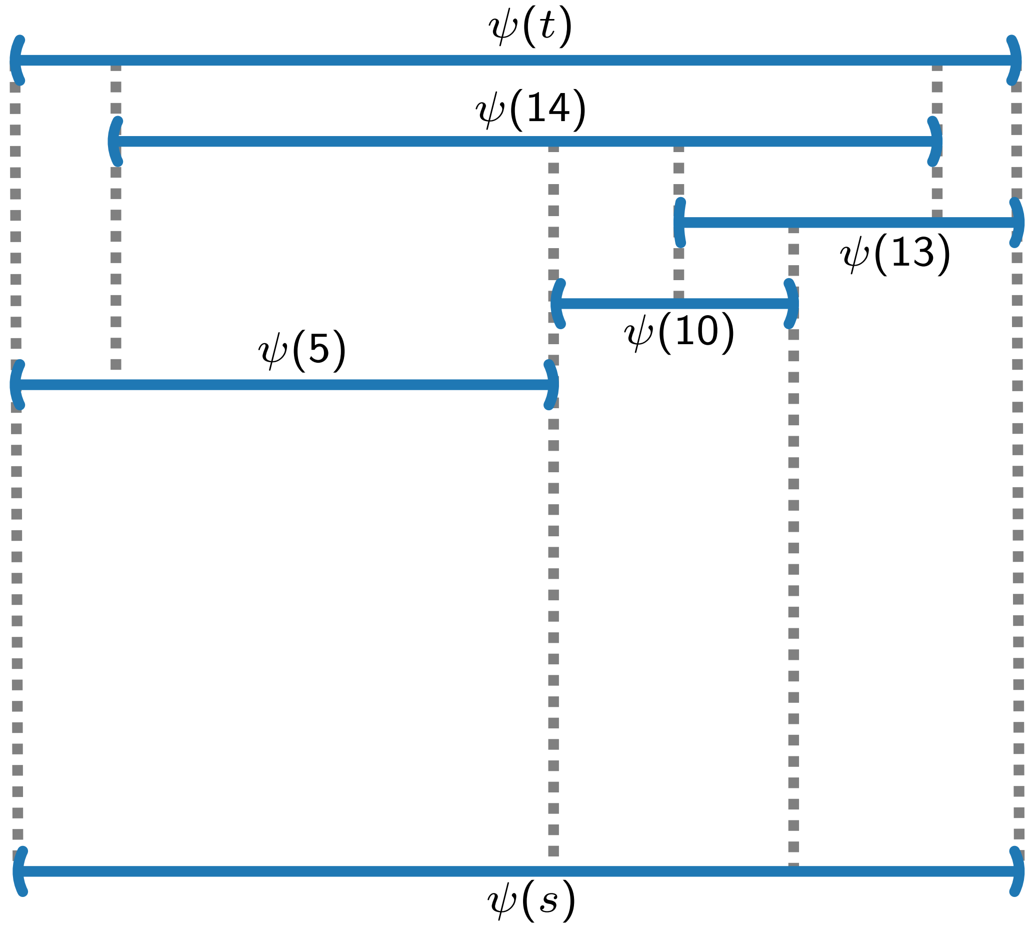

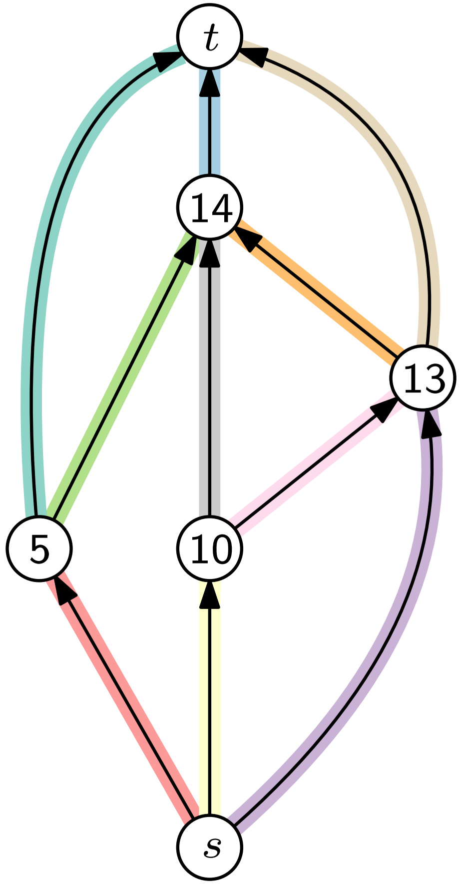

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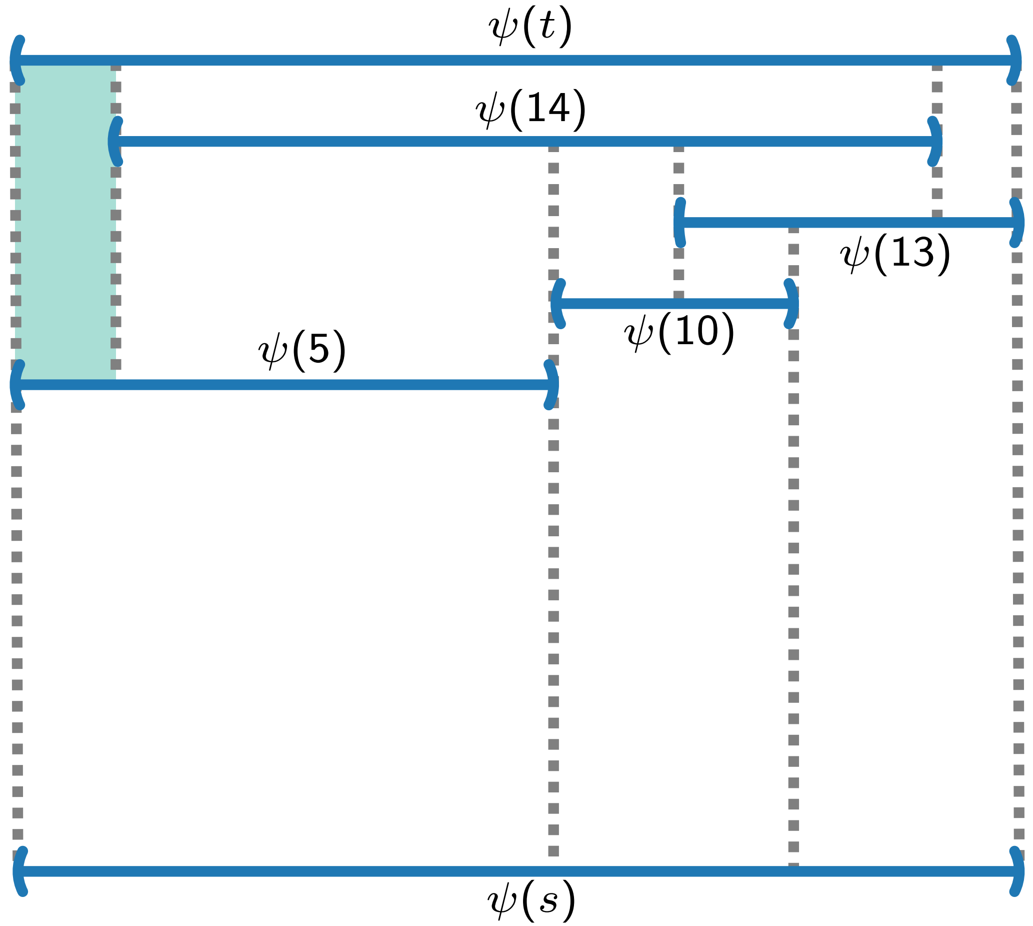

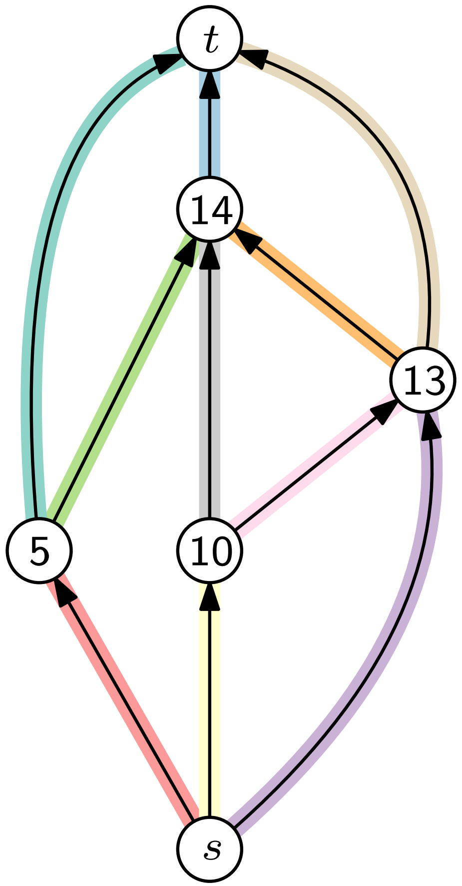

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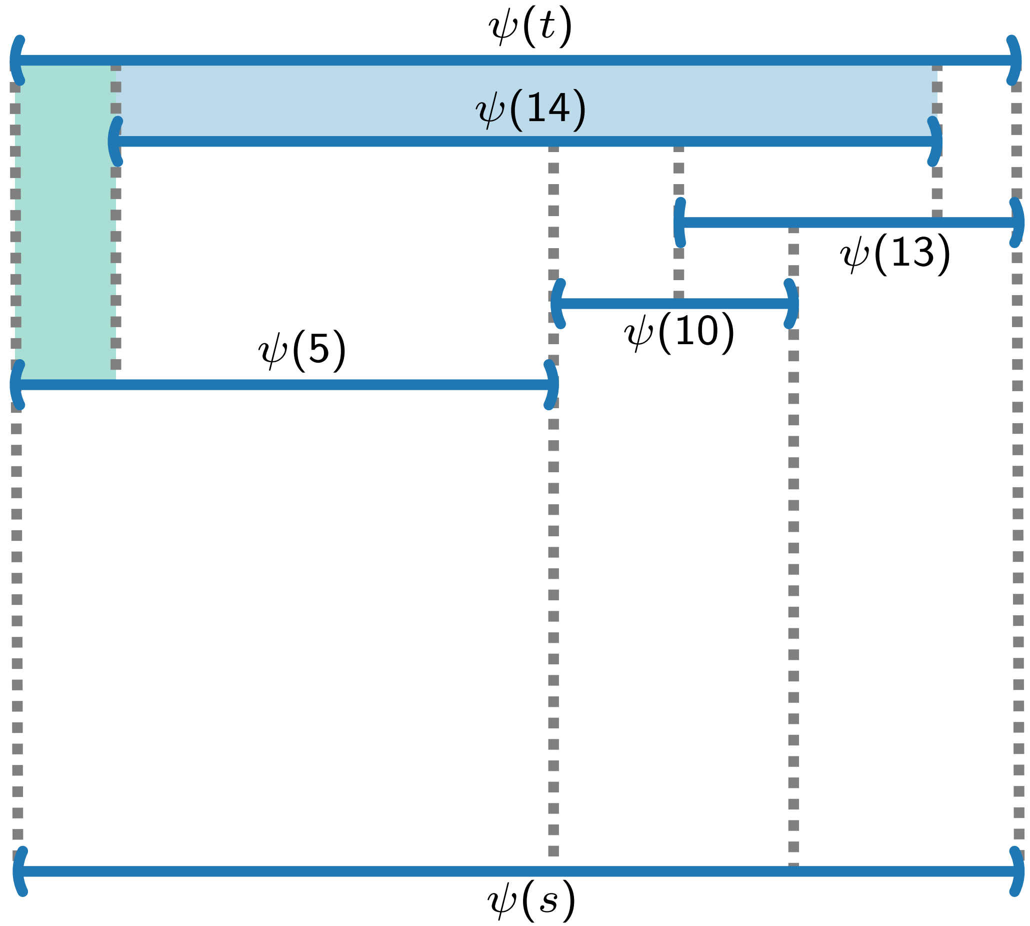
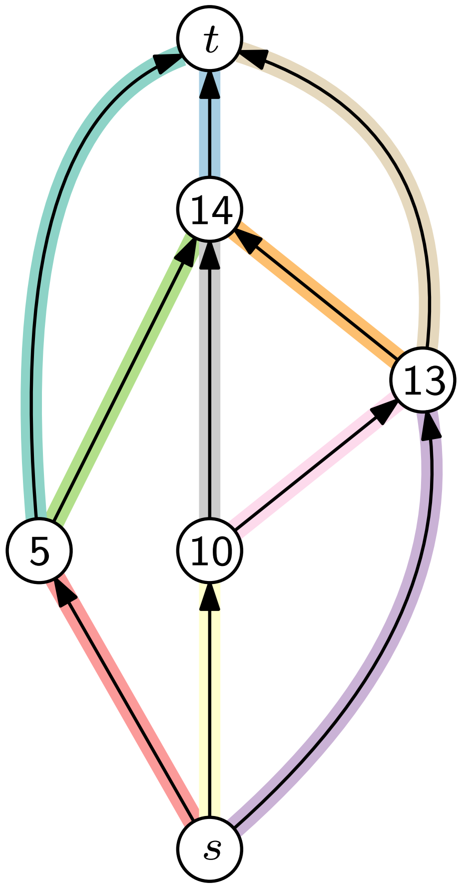
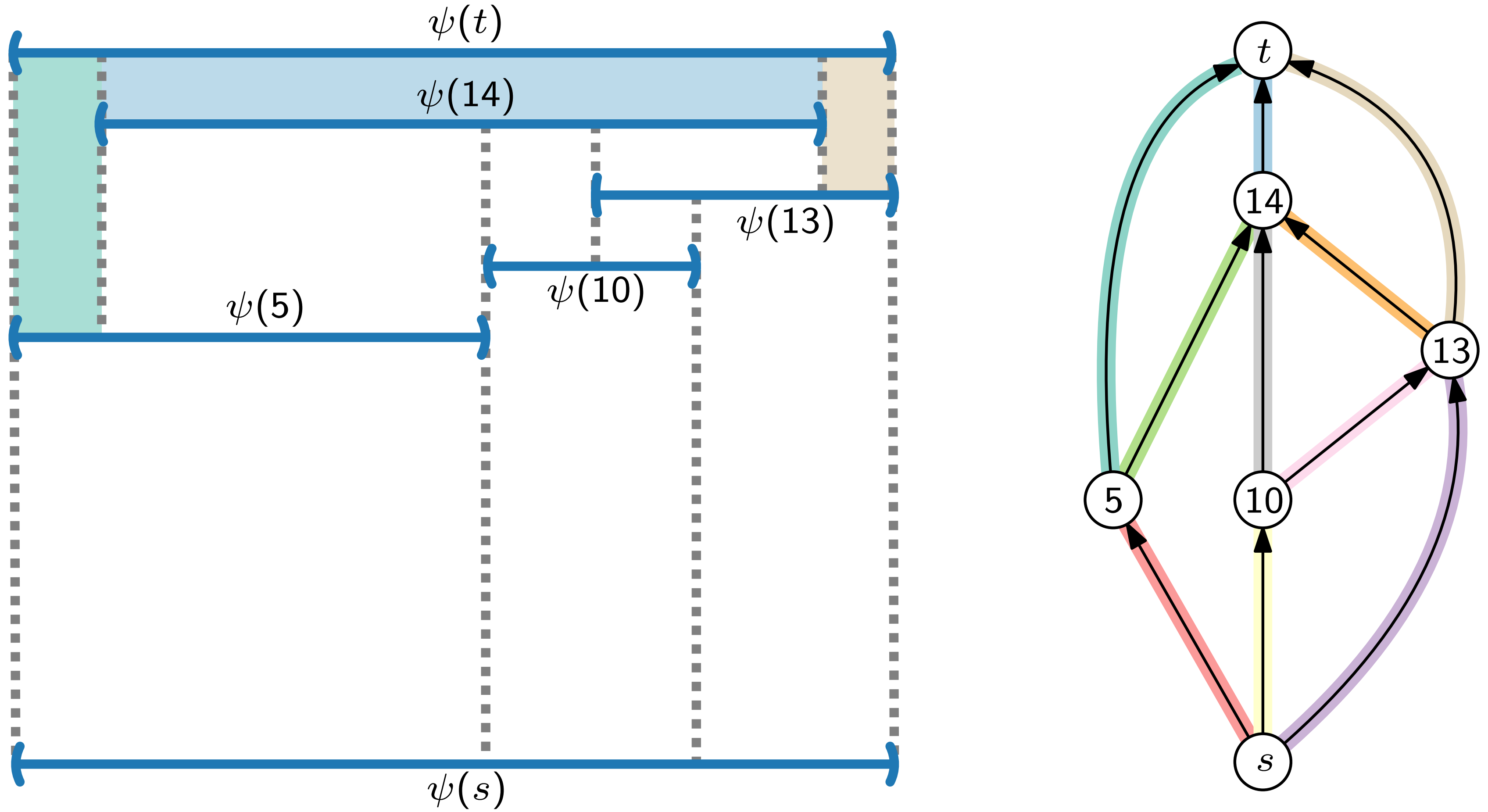
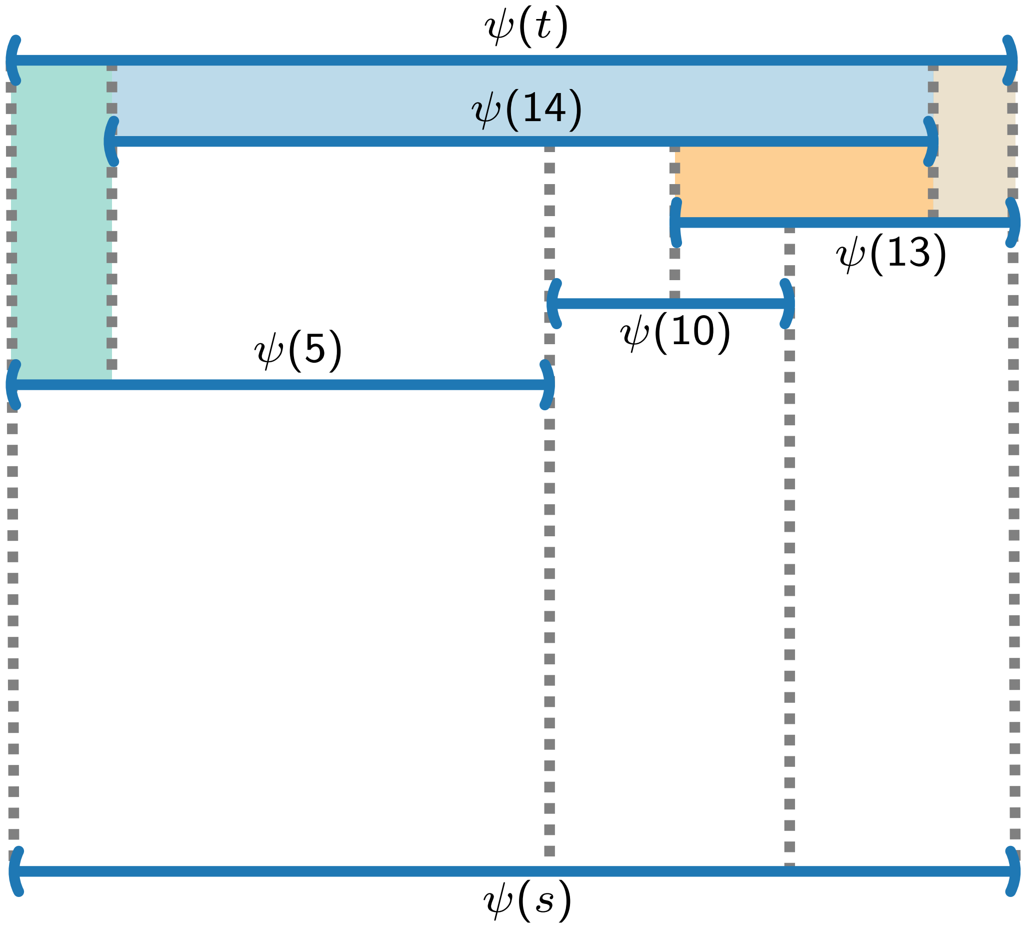


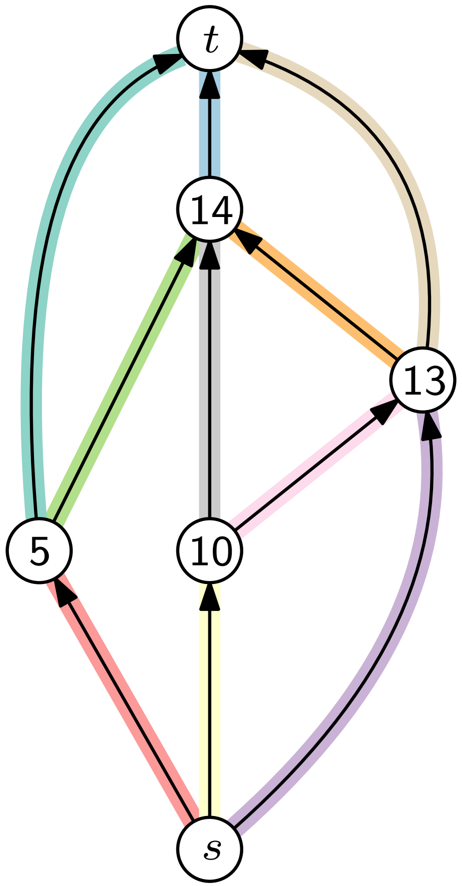


R-Nodes

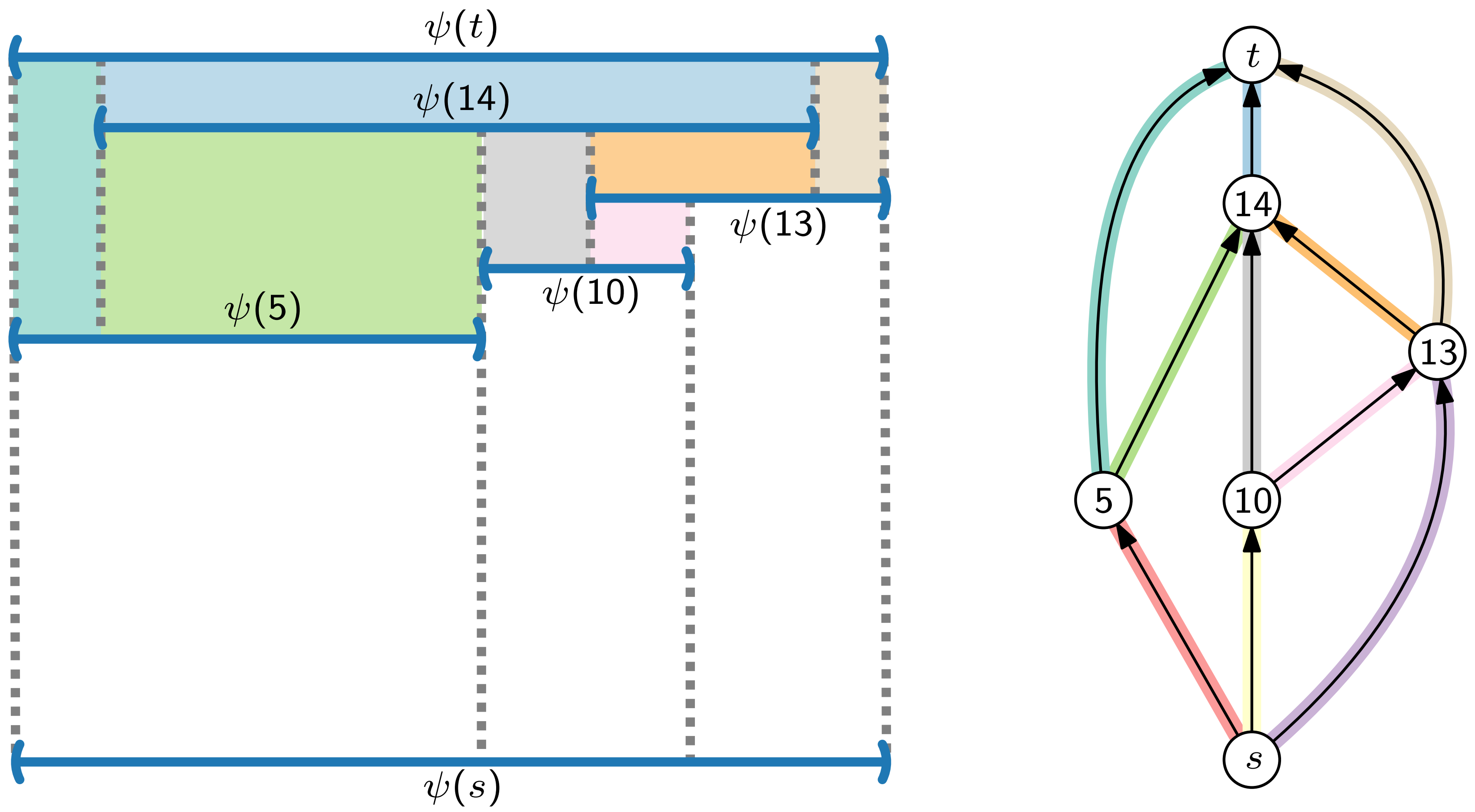


R-Nodes

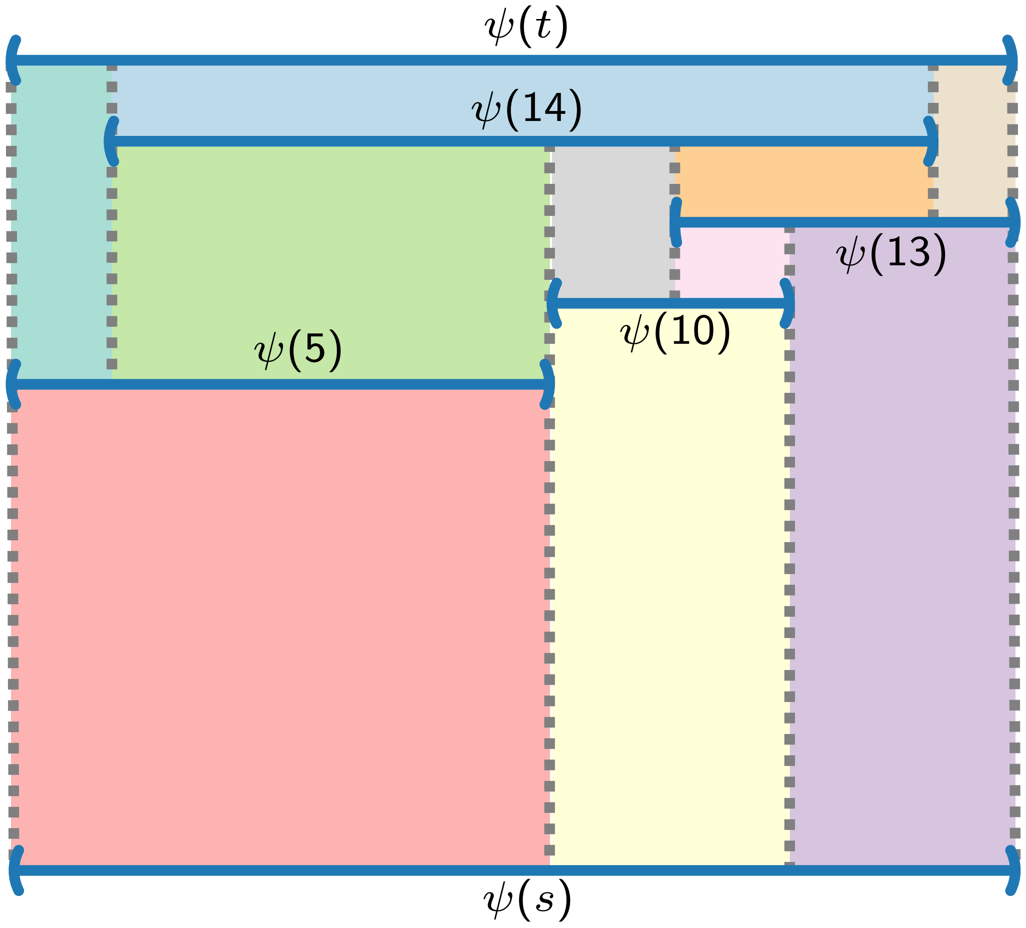


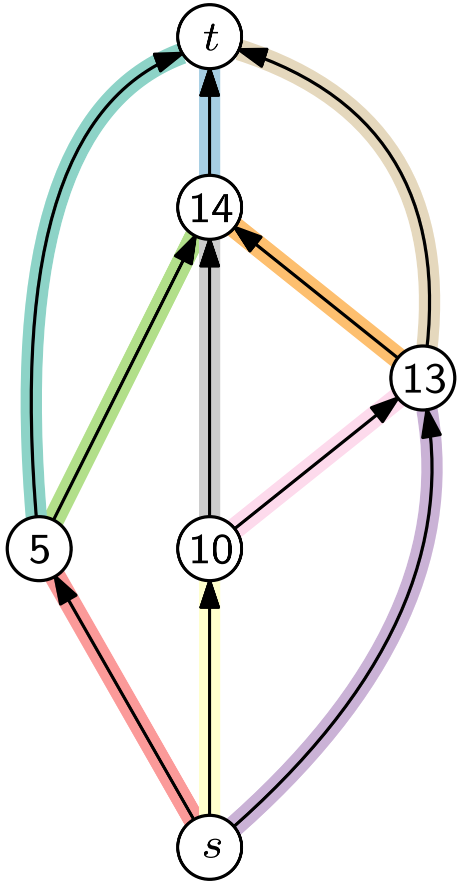


R-Nodes

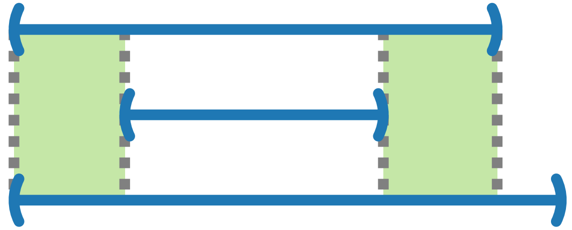


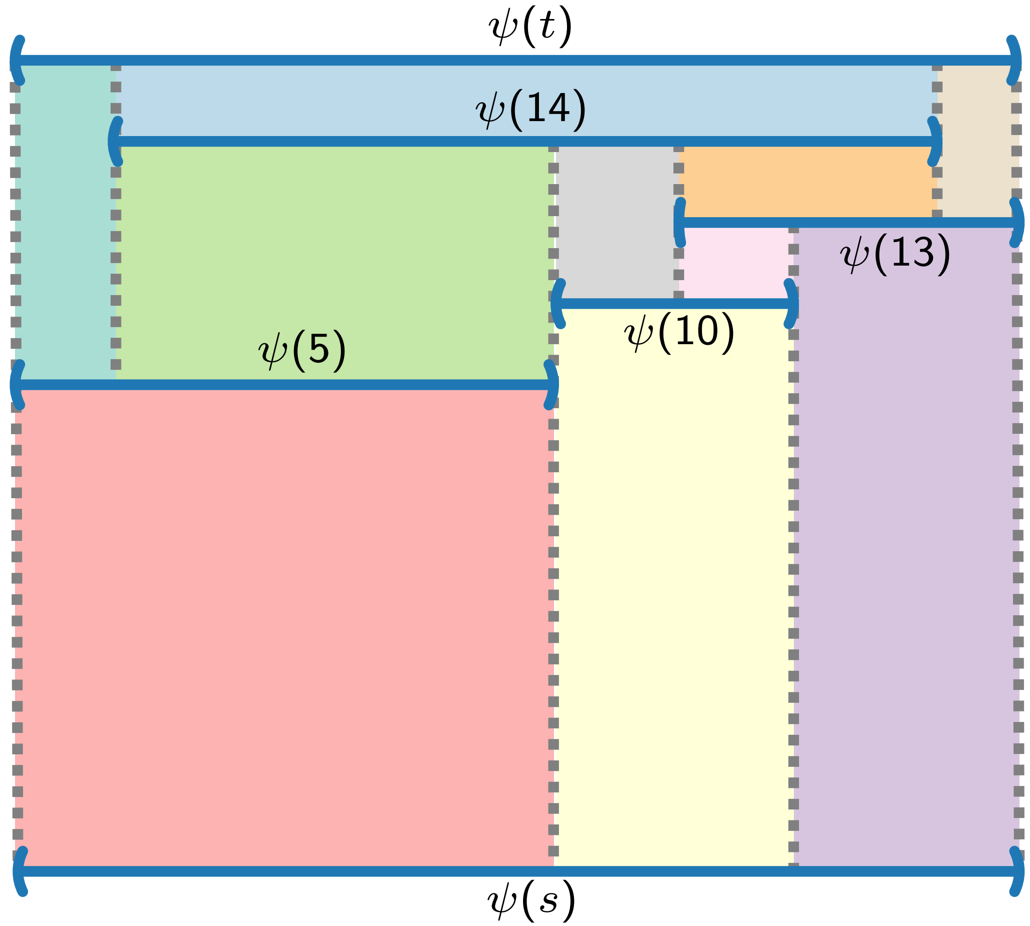
R-Nodes

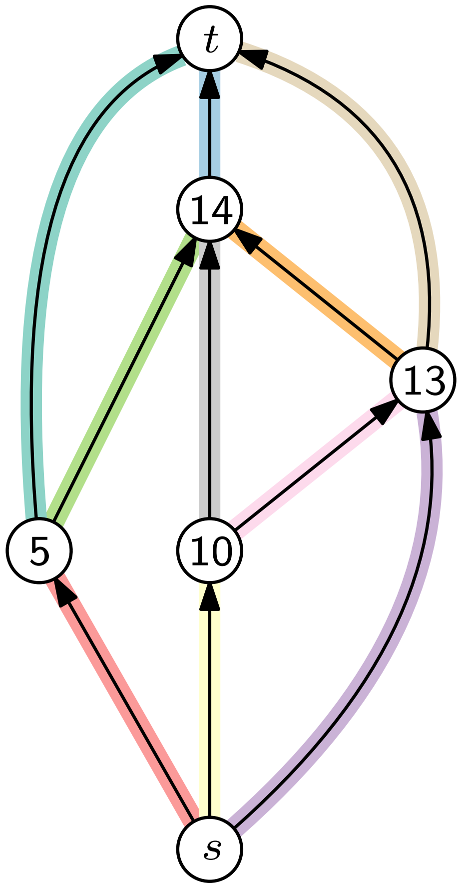




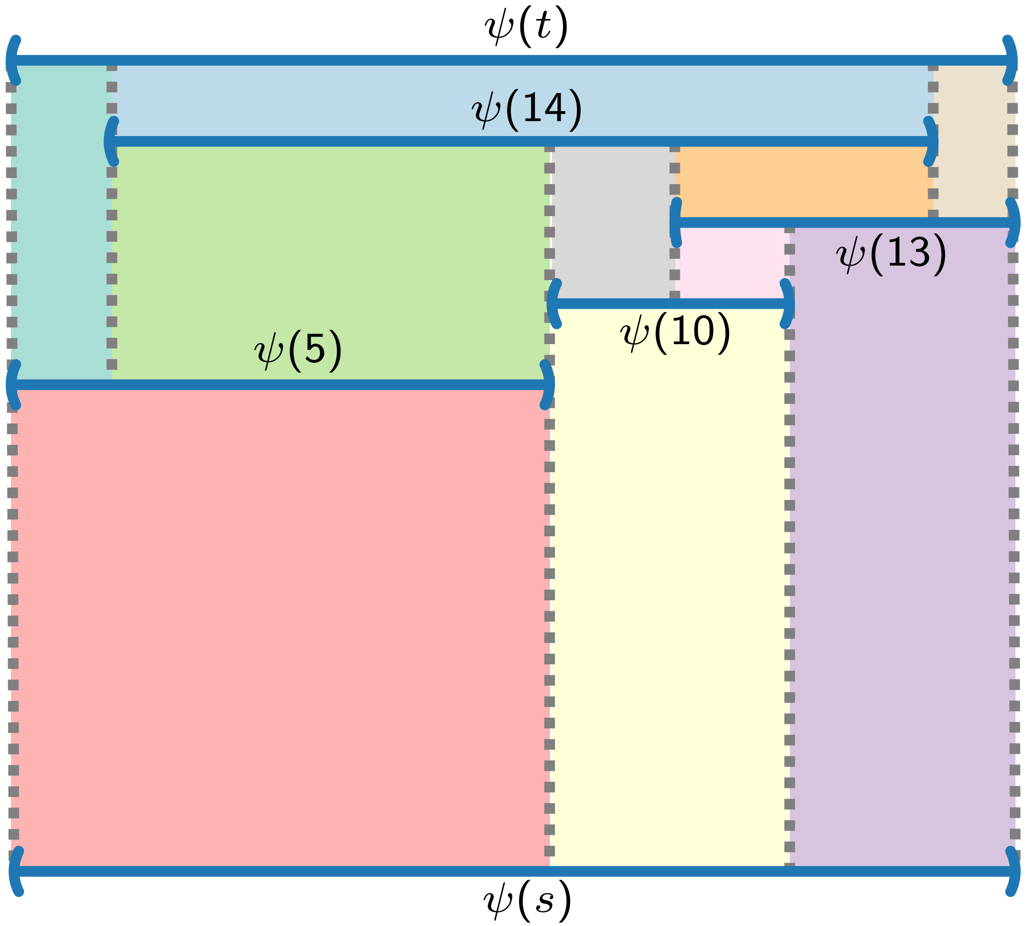
R-Nodes

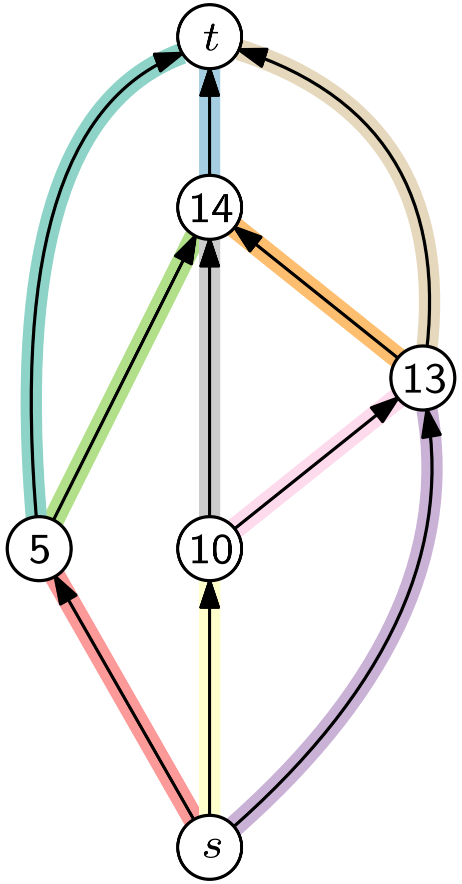


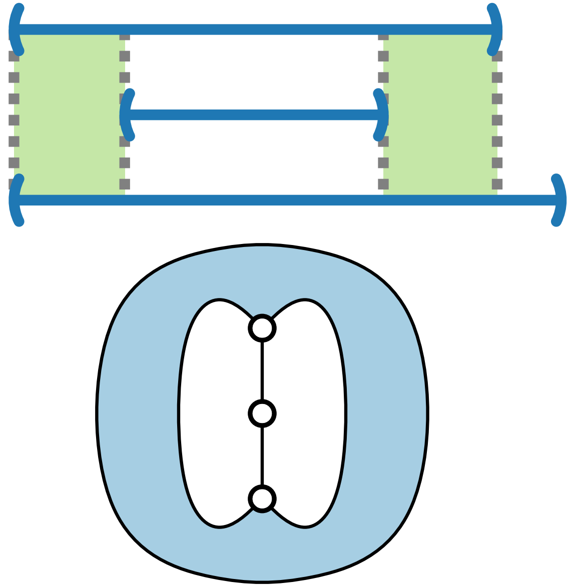




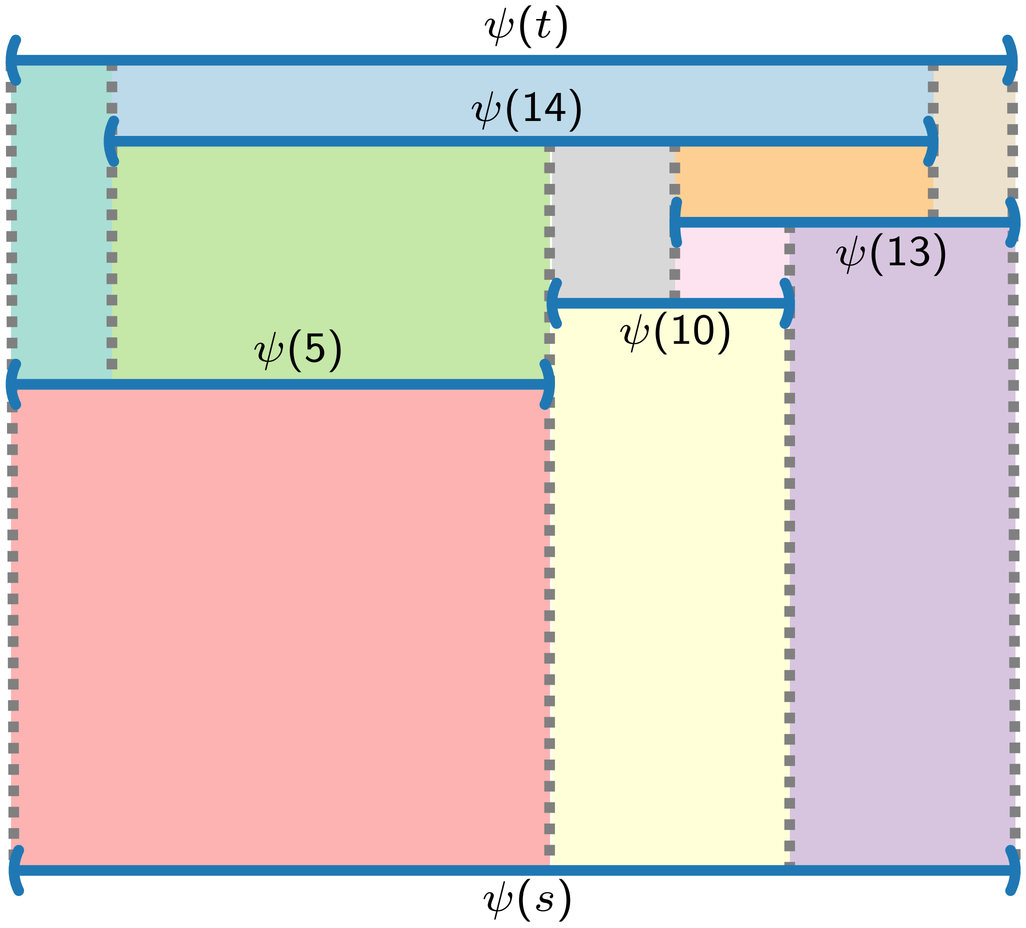
R-Nodes



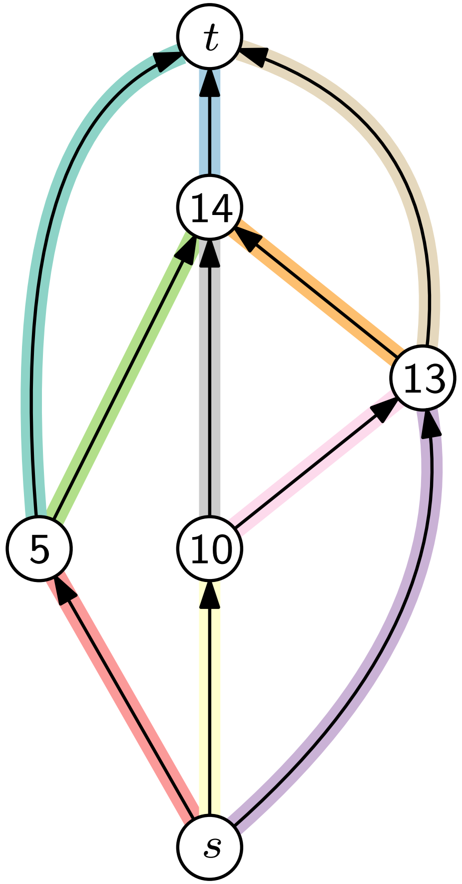




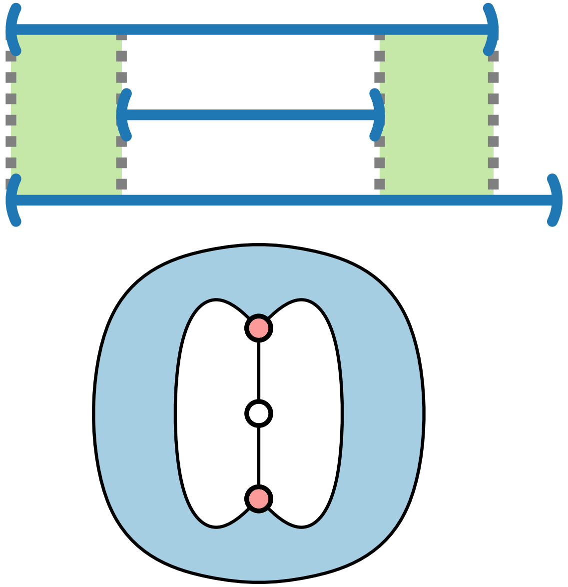
R-Nodes



$\psi(t)$
$\psi(14)$
$\psi(13)$
$\psi(10)$
$\psi(5)$
$\psi(s)$

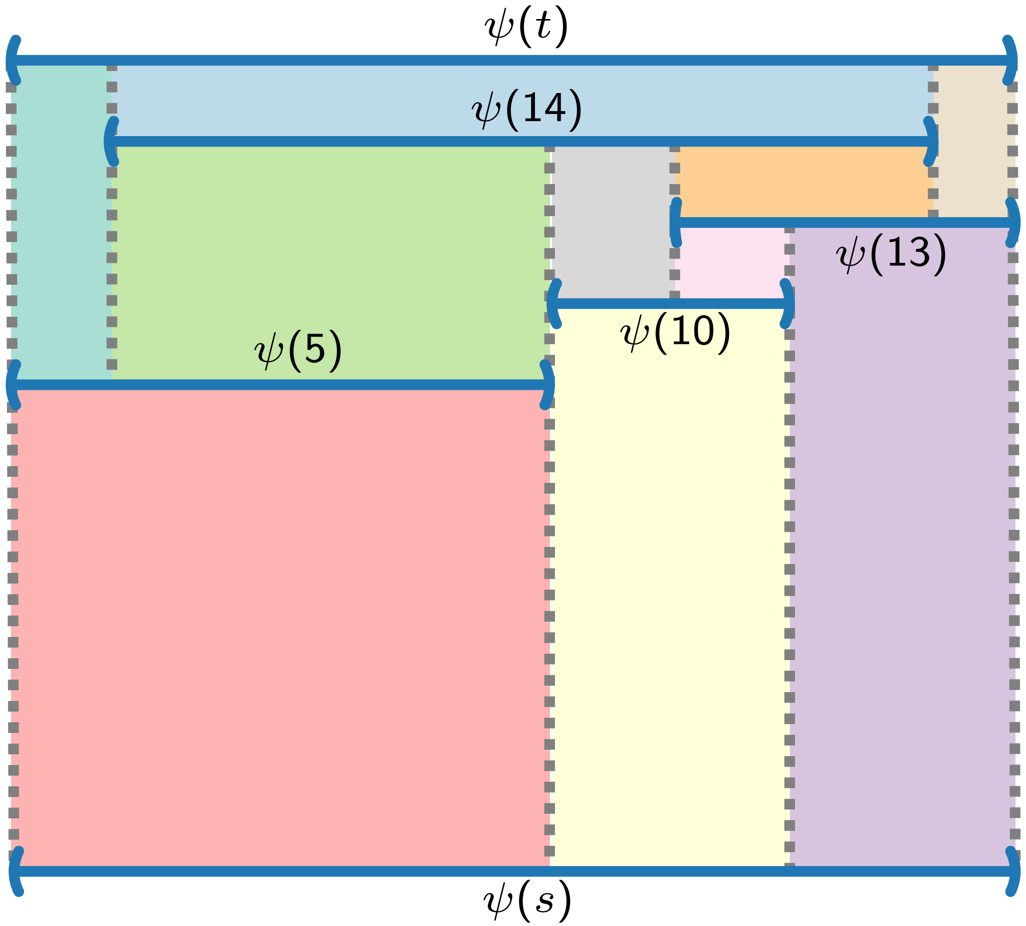


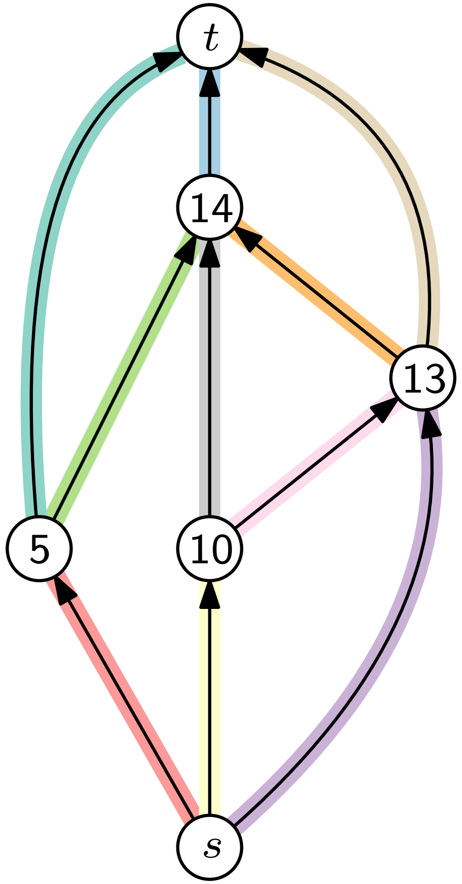
t
14
13
5
10
s

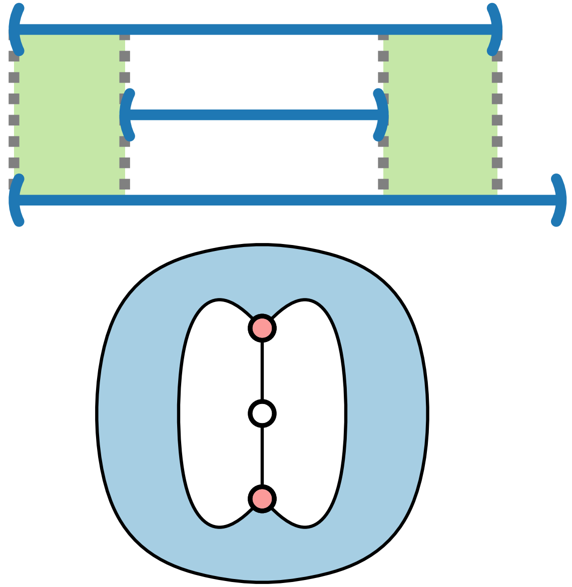


Separation pair!

R-Nodes



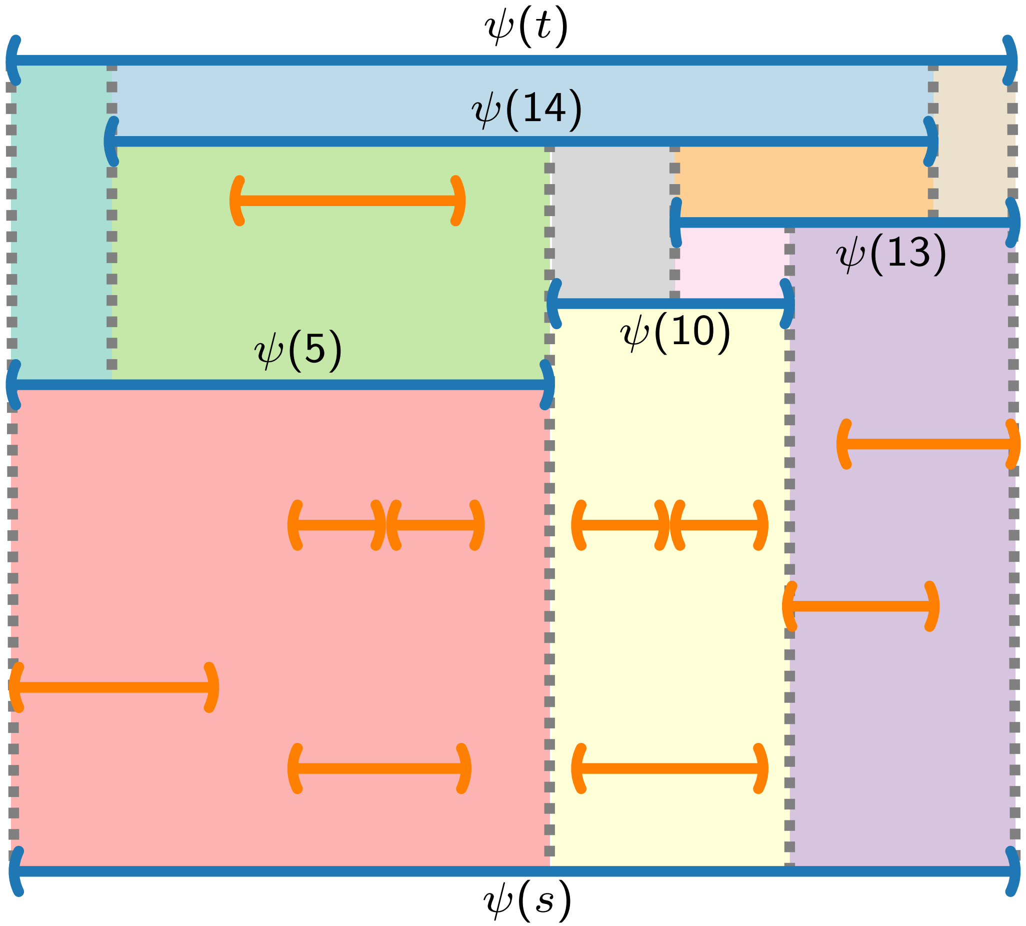


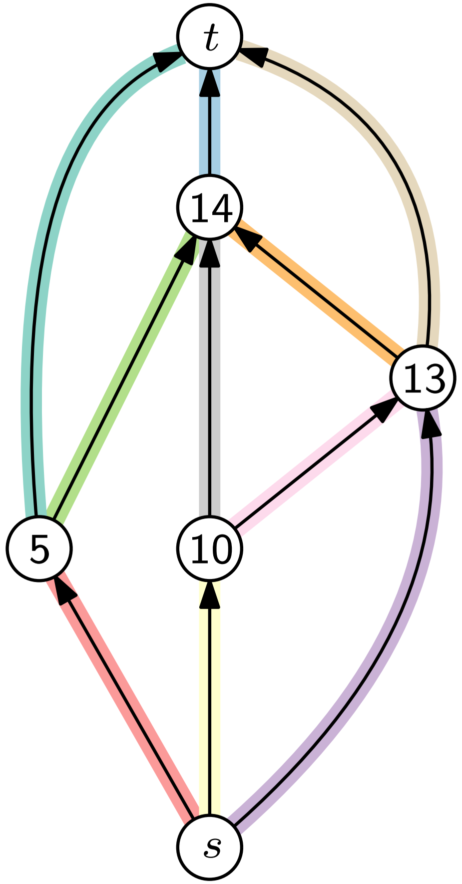


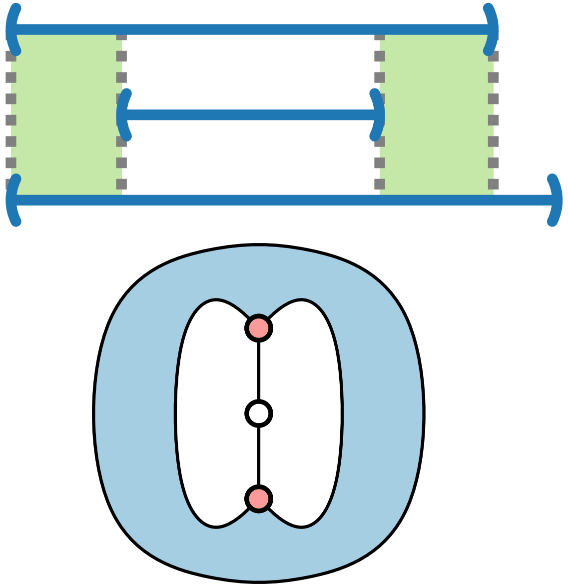
Separation pair!

($\nexists$ in **R**-component.)

R-Nodes





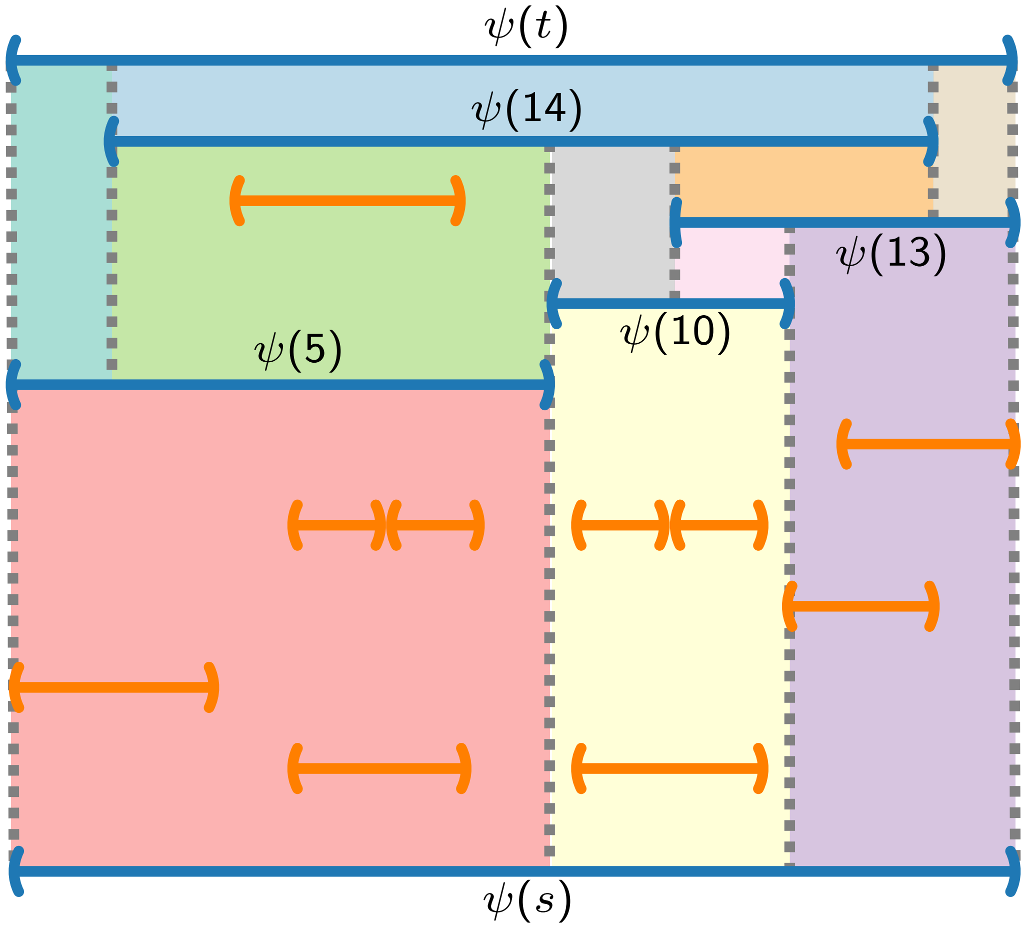


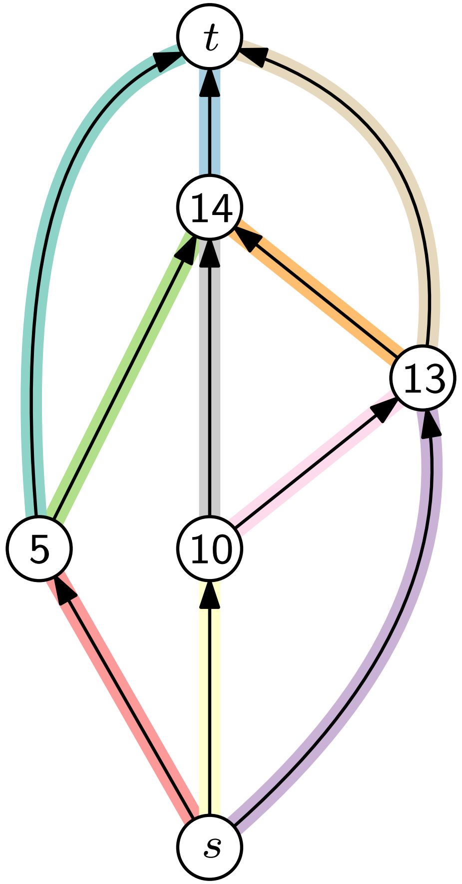
Separation pair!

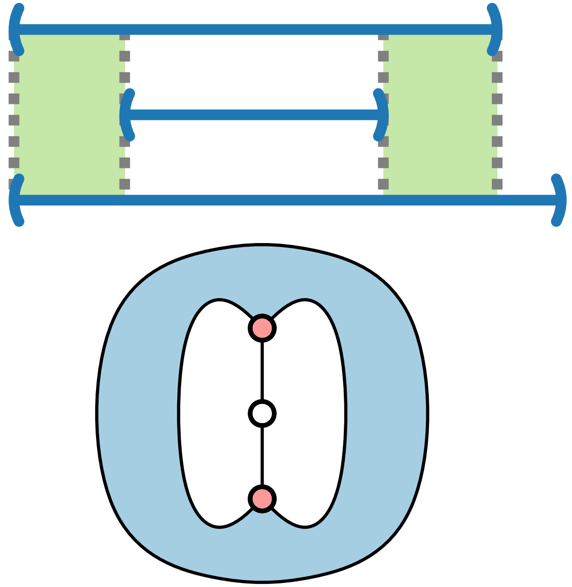
($\nexists$ in **R**-component.)

R-Nodes

- For each child (edge) e:





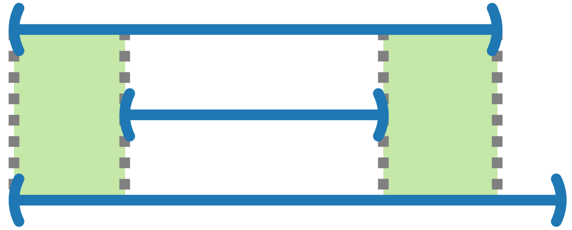


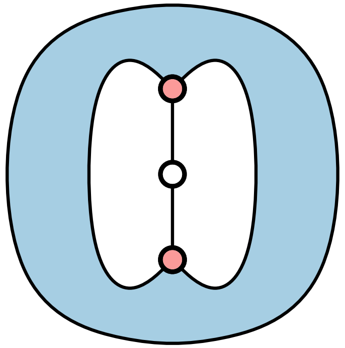
Separation pair!

($\nexists$ in **R**-component.)

R-Nodes

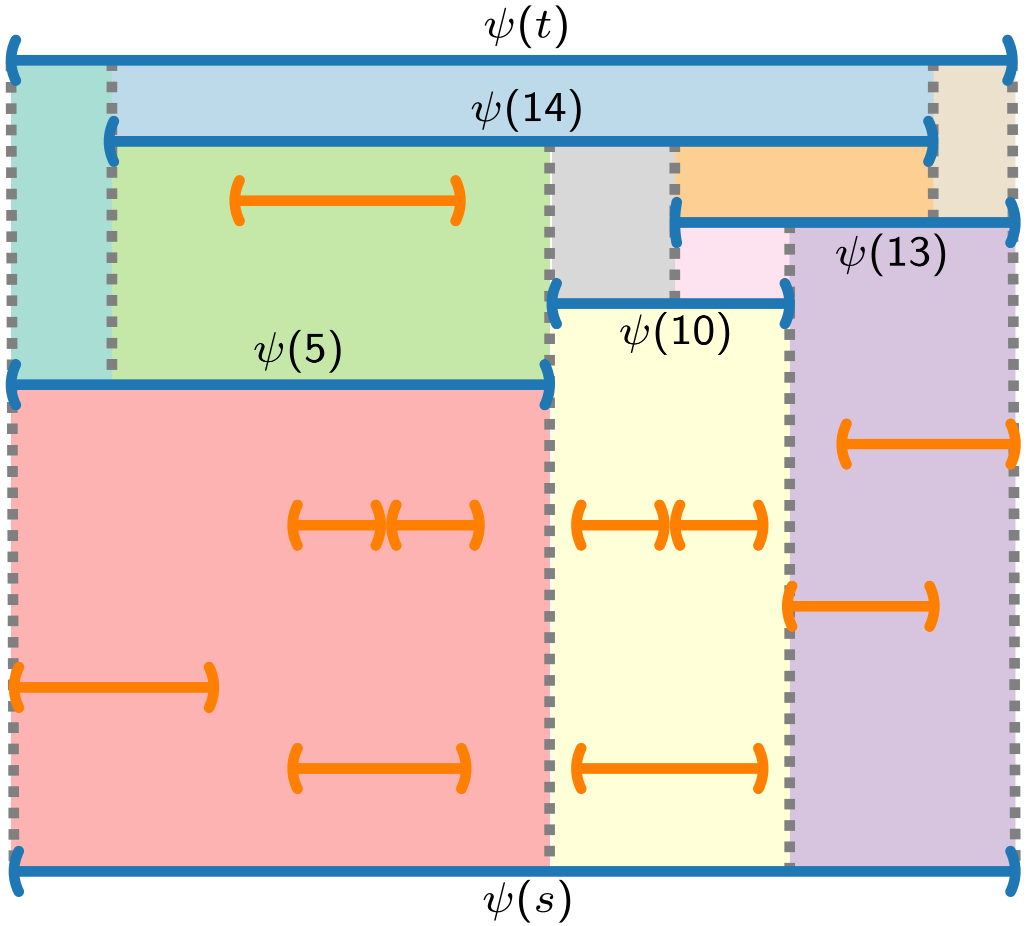
- For each child (edge) e:
  - Find all types of {**FF, FL, LF, LL**} that admit a drawing.

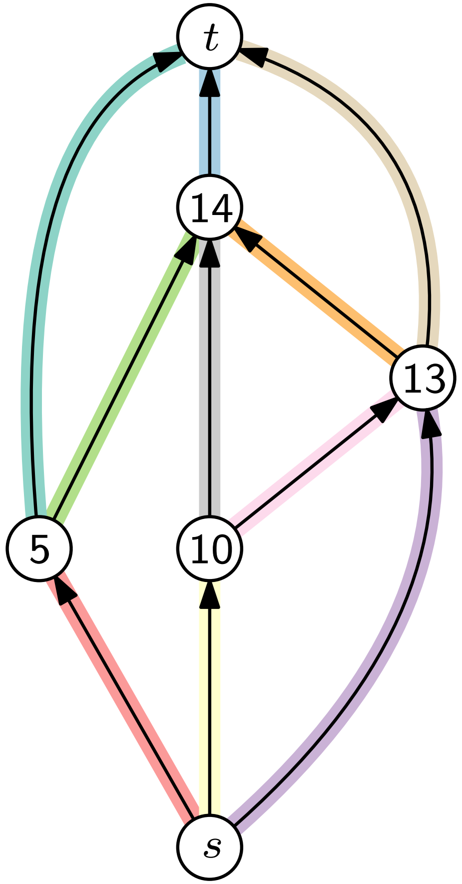




Separation pair!

($\nexists$ in **R**-component.)





R-Nodes with 2-SAT Formulation

- For each child (edge) e:
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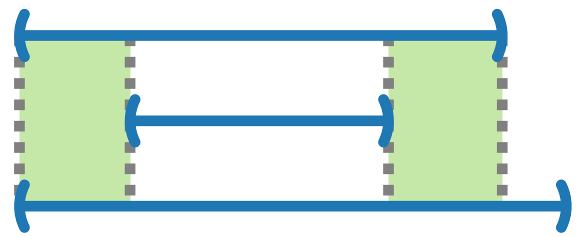
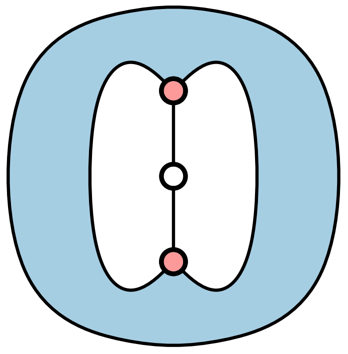
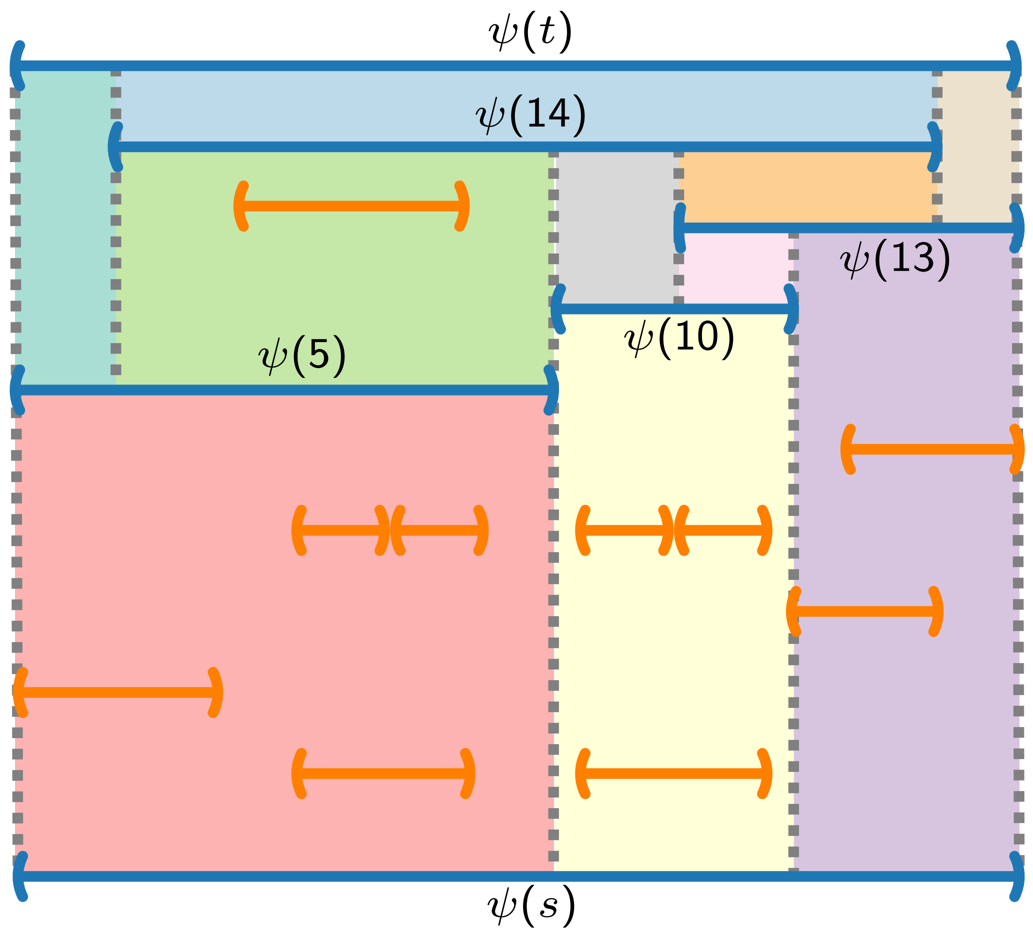
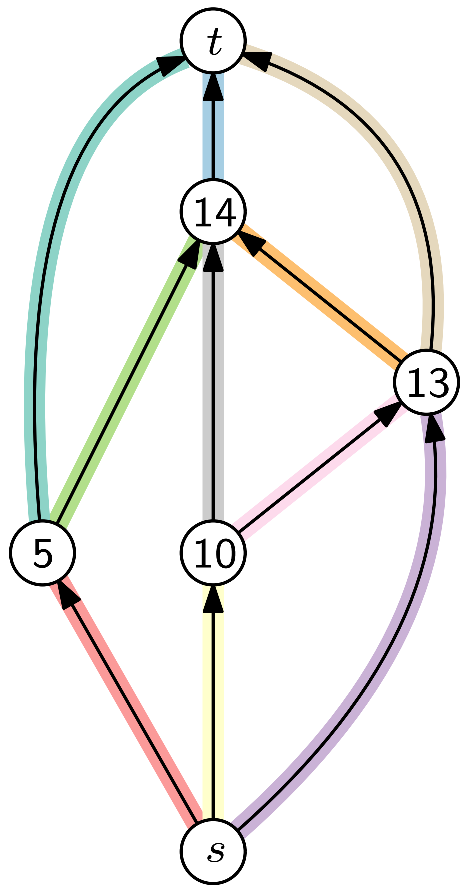


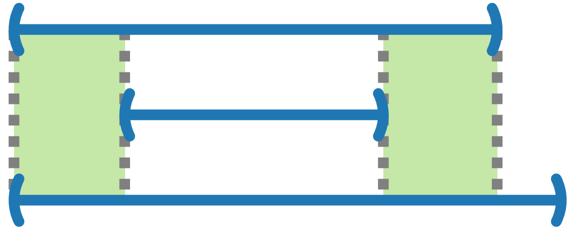


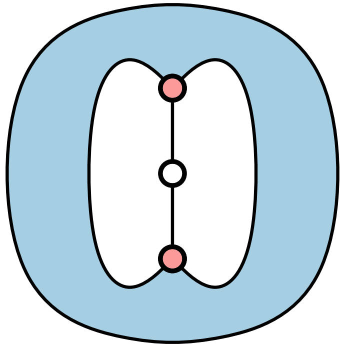




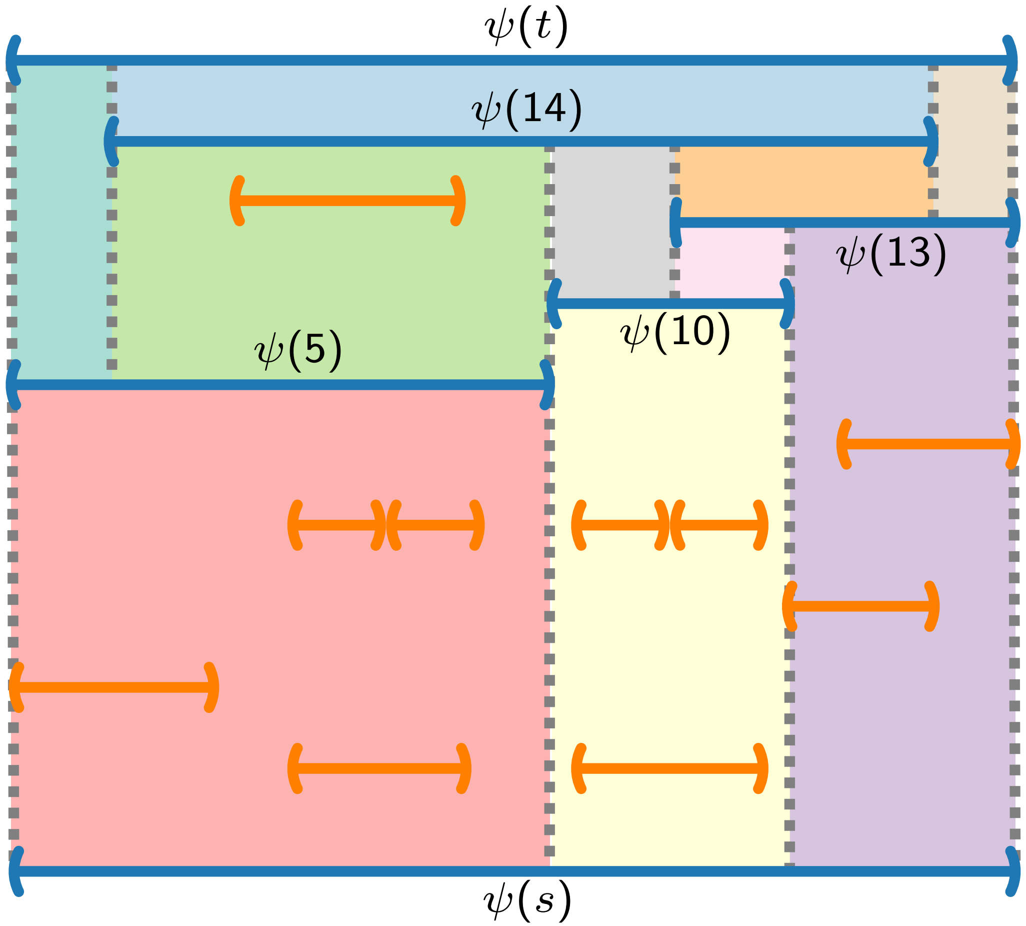
R-Nodes with 2-SAT Formulation

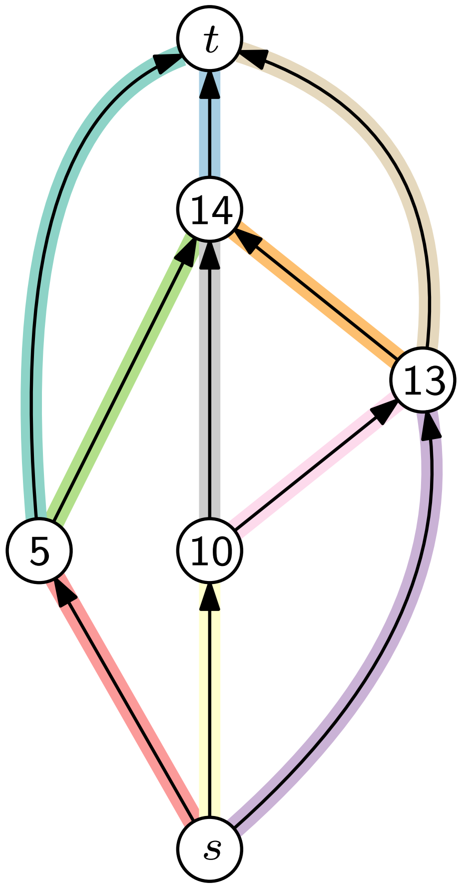
- For each child (edge) e:
  - Find all types of {**FF, FL, LF, LL**} that admit a drawing.
  - Use two variables (l_e and r_e) to encode the type of its tile (**F** $= 0$).





Separation pair!
($\nexists$ in **R**-component.)





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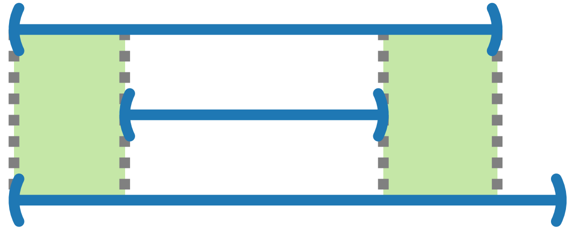

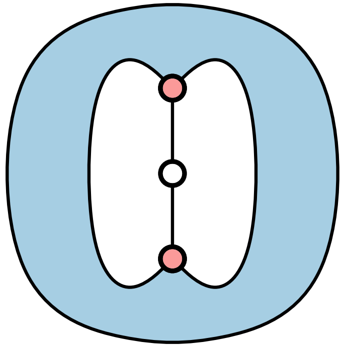

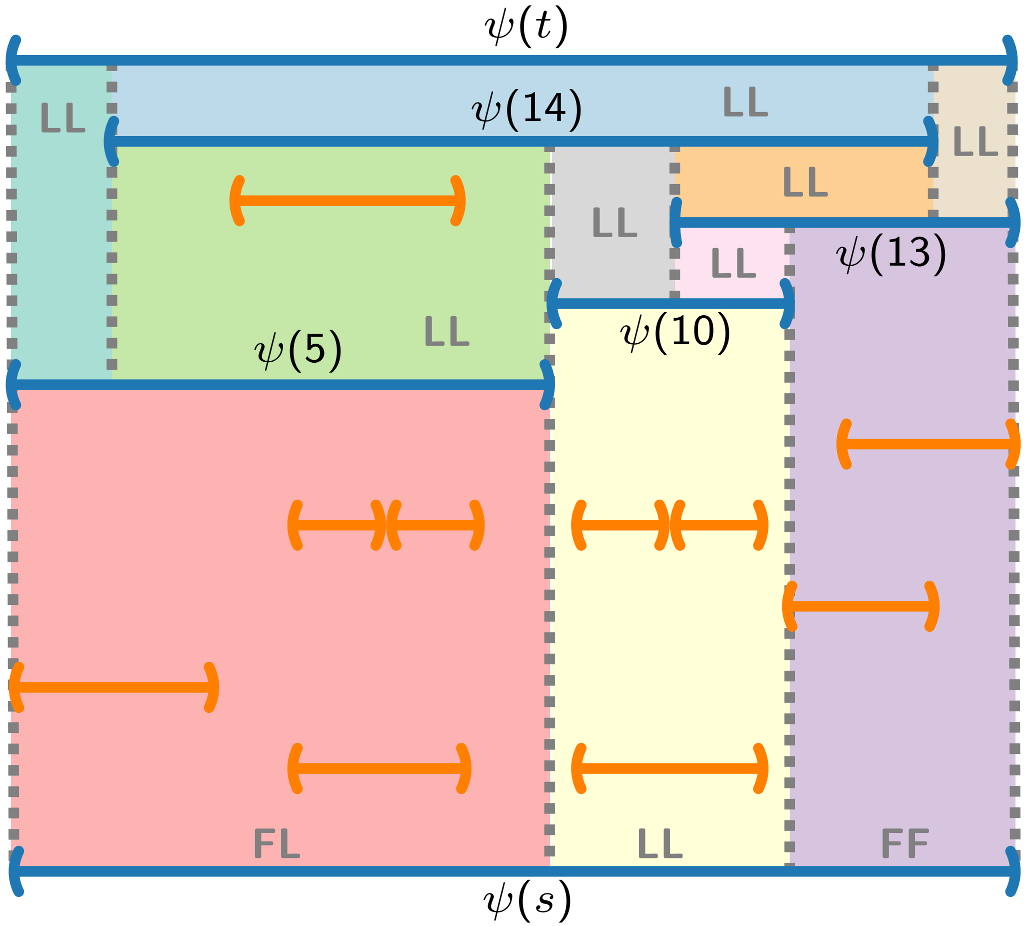

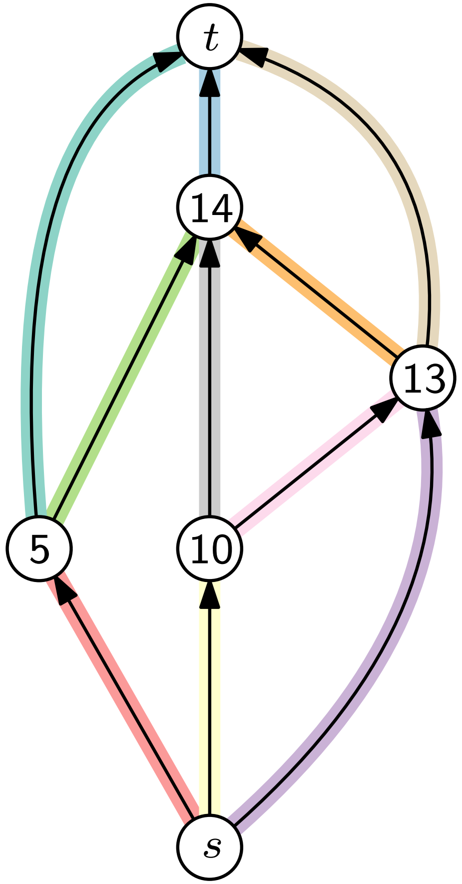

R-Nodes with 2-SAT Formulation

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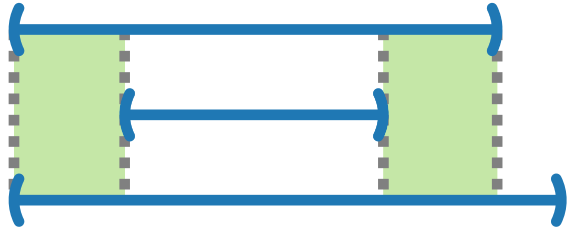
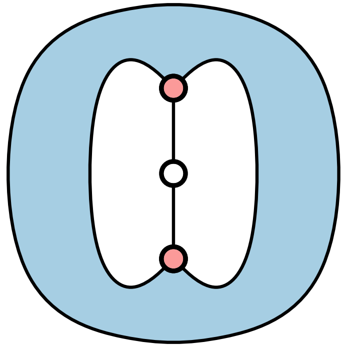
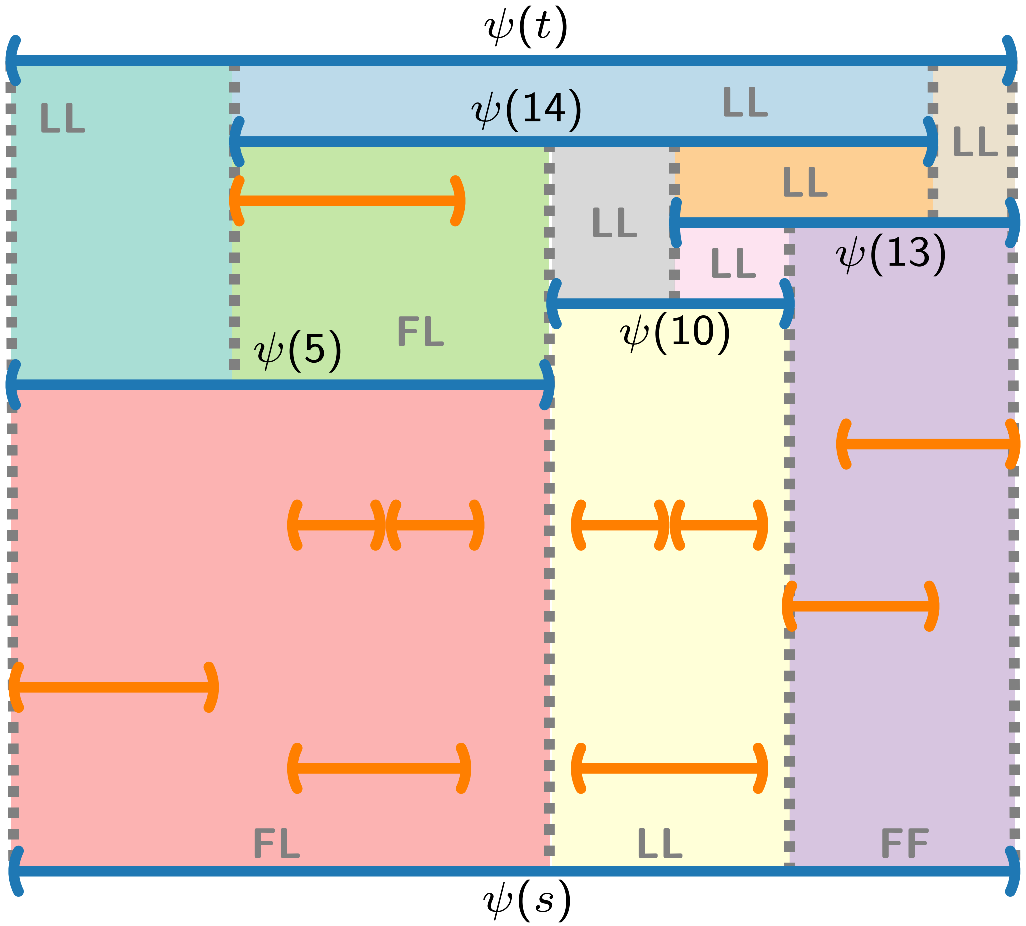
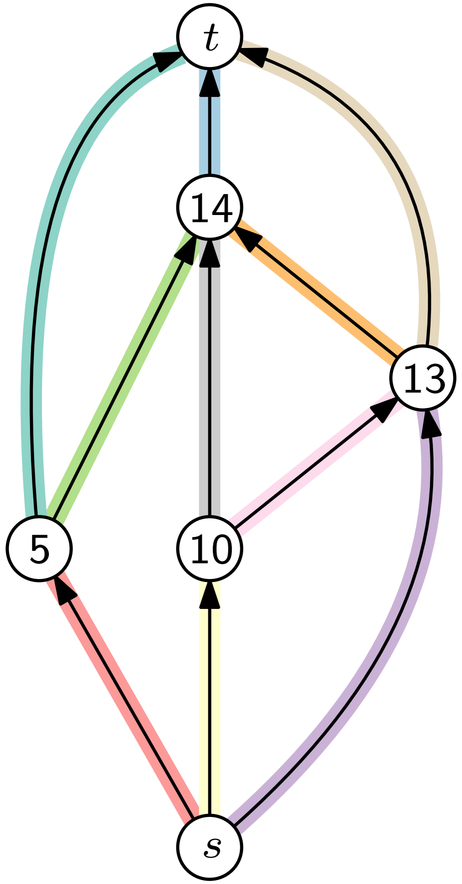


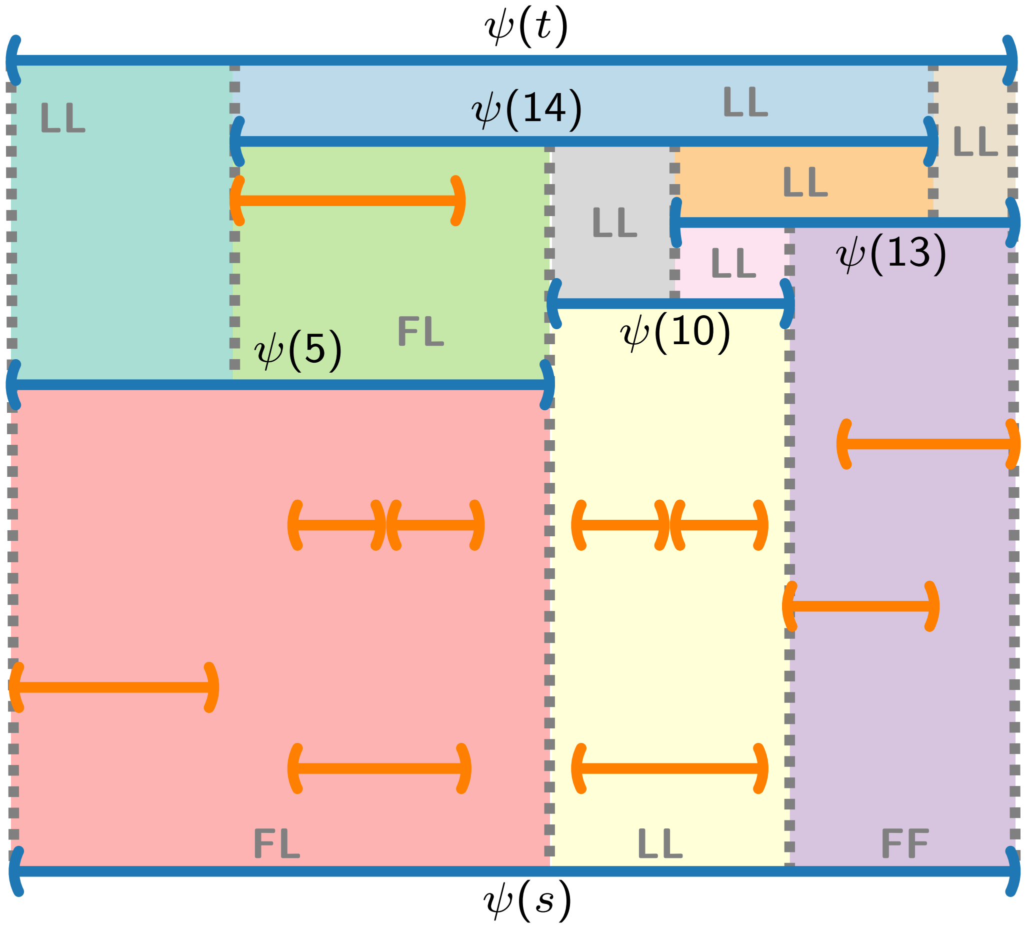


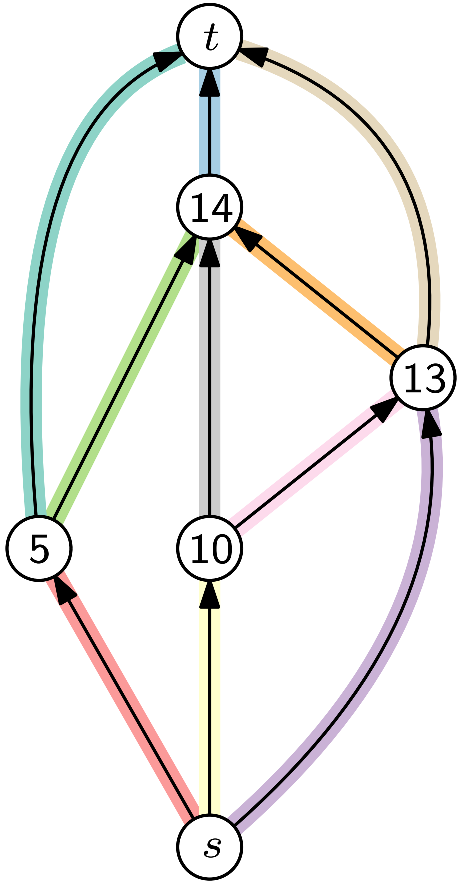


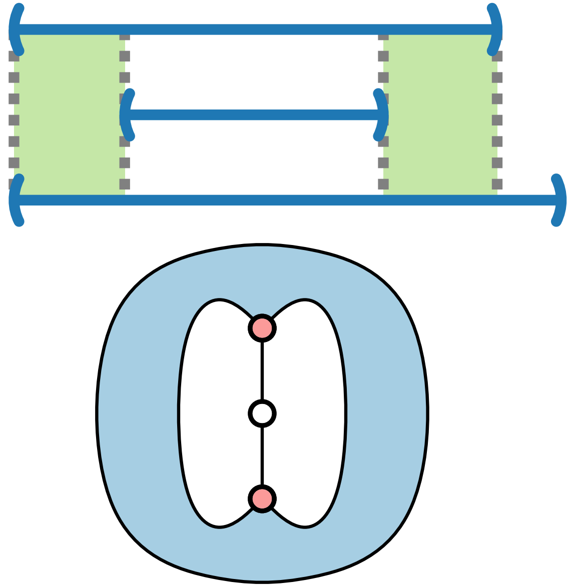


R-Nodes with 2-SAT Formulation

- For each child (edge) e:
  - Find all types of {**FF, FL, LF, LL**} that admit a drawing.
  - Use two variables (l_e and r_e) to encode the type of its tile (**F** $= 0$).
  - Add *consistency clauses*







Separation pair!

($\nexists$ in **R**-component.)

R-Nodes with 2-SAT Formulation

- For each child (edge) e:
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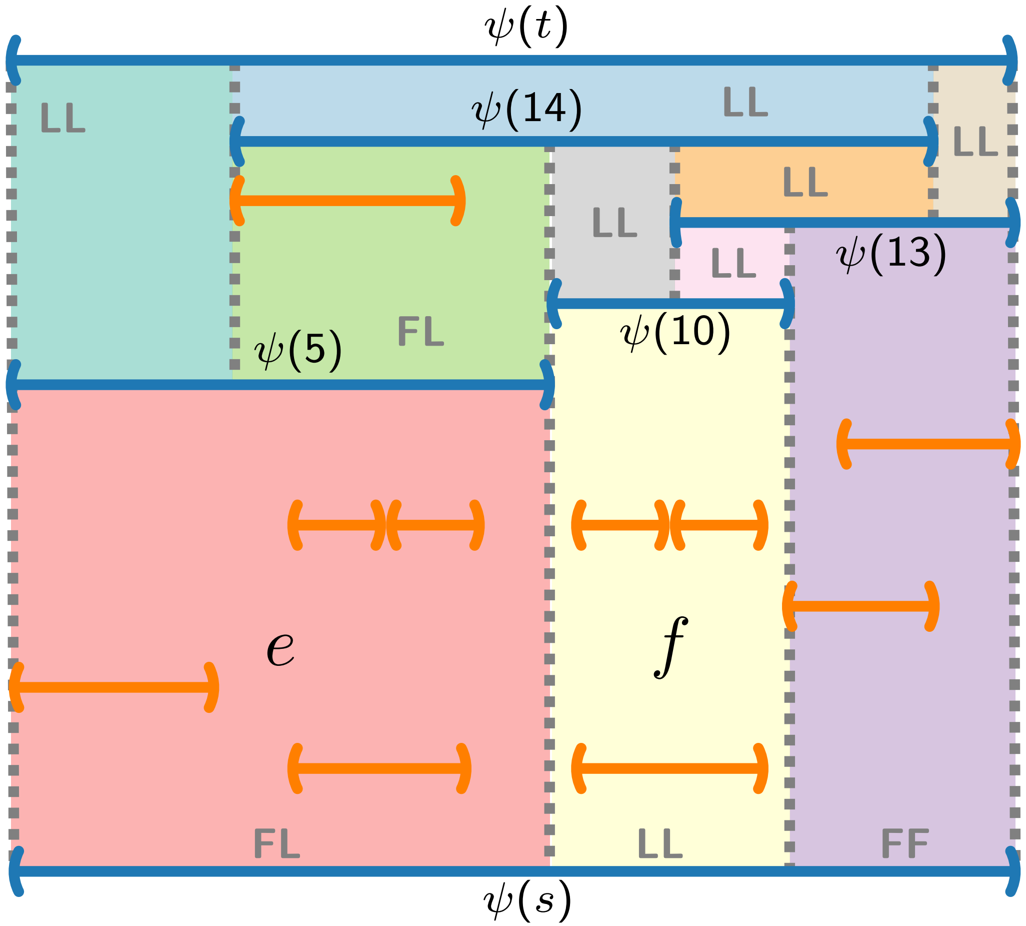

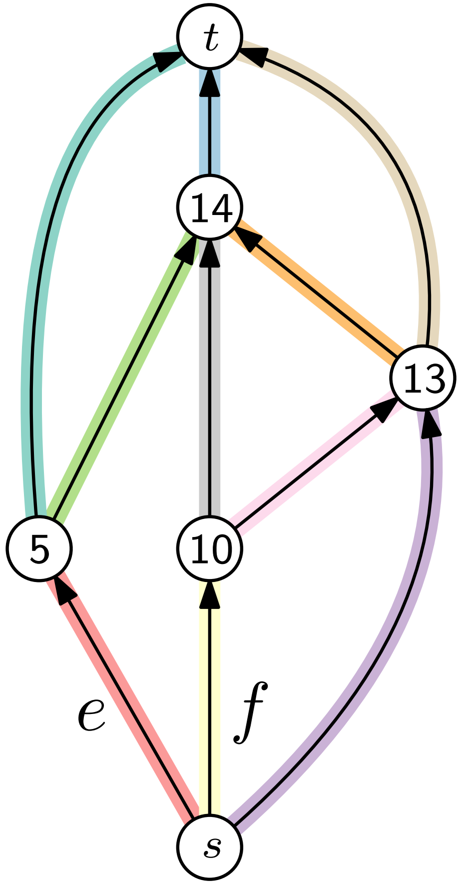

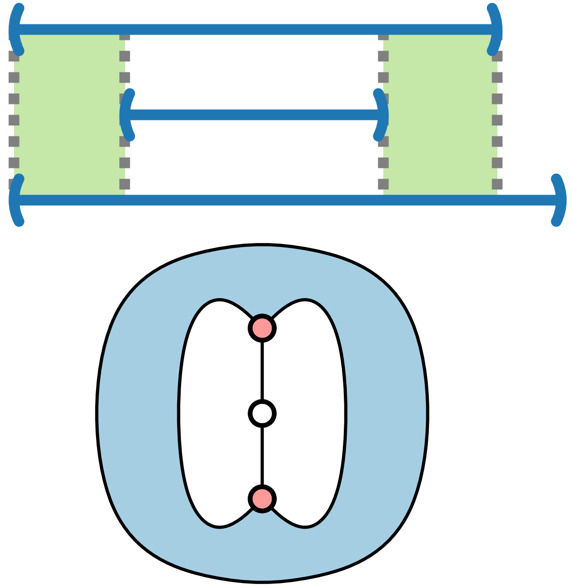

Separation pair!

($\nexists$ in **R**-component.)

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- For each child (edge) e:
  - Find all types of {**FF**, **FL**, **LF**, **LL**} that admit a drawing.
  - Use two variables (l_e and r_e) to encode the type of its tile (**F** $= 0$).
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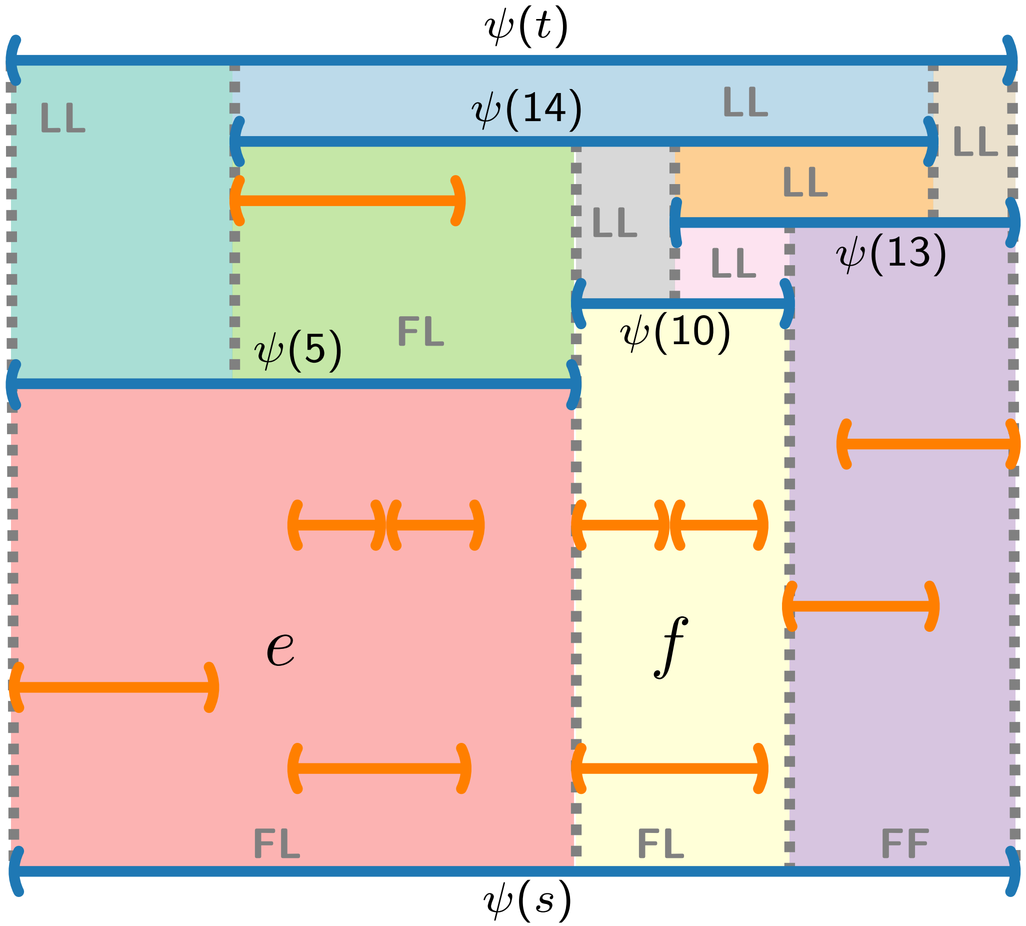

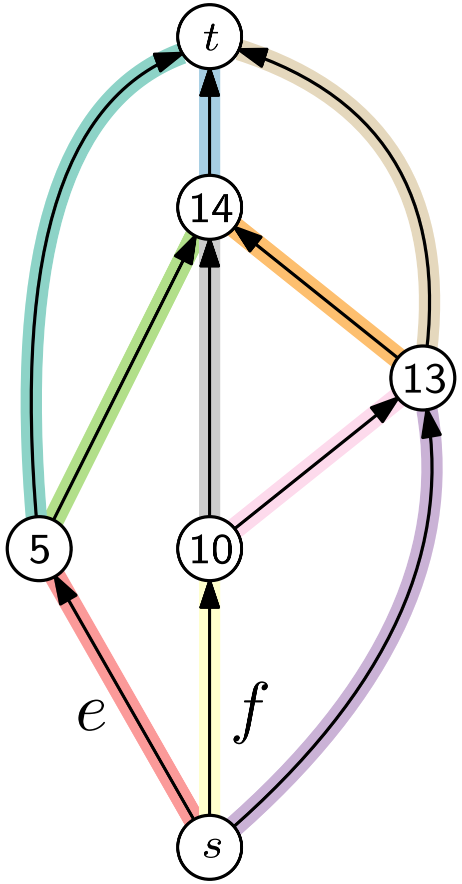

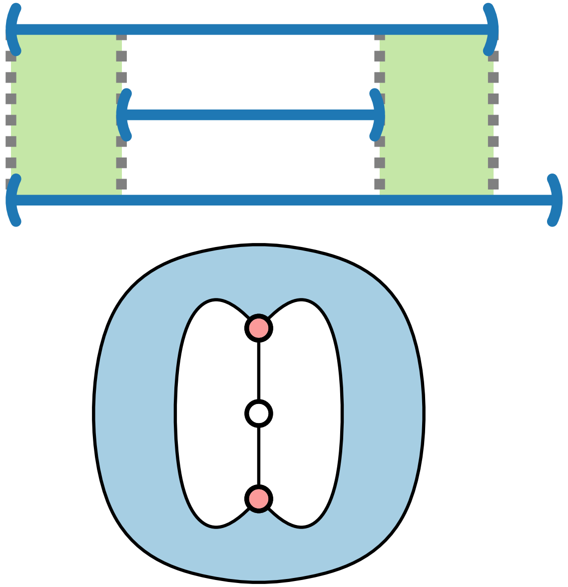

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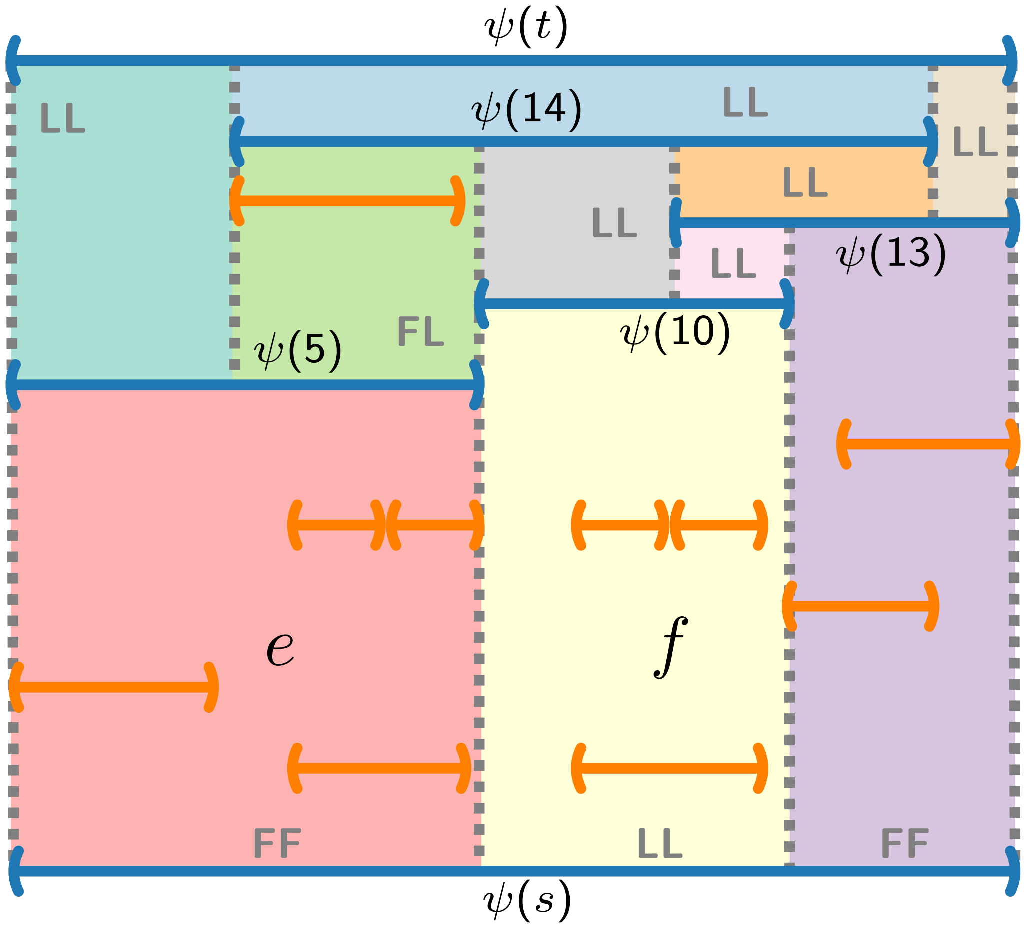

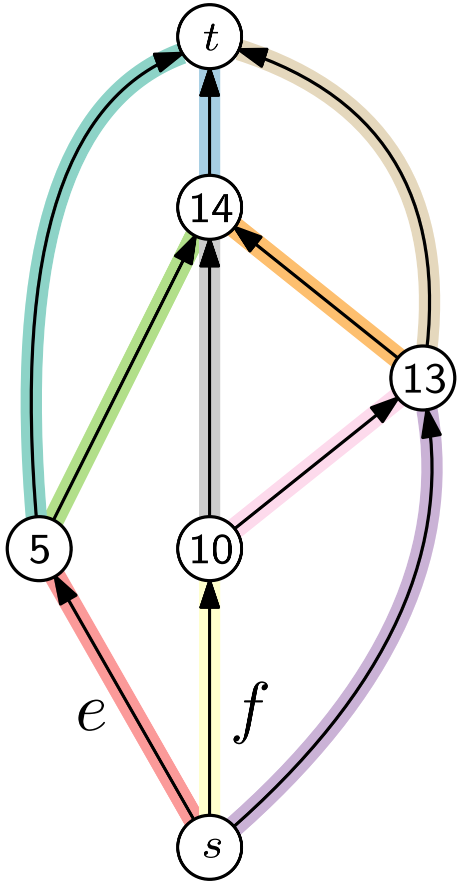

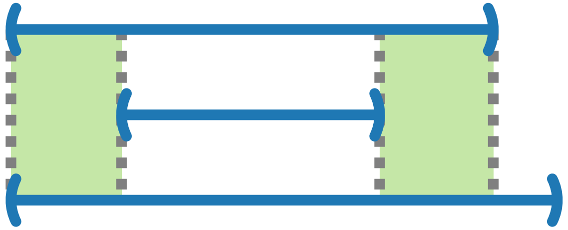

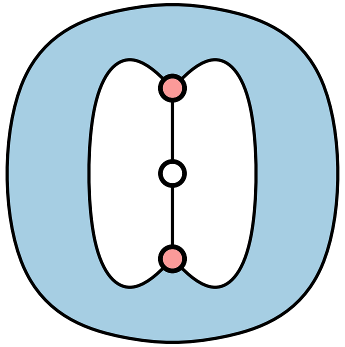

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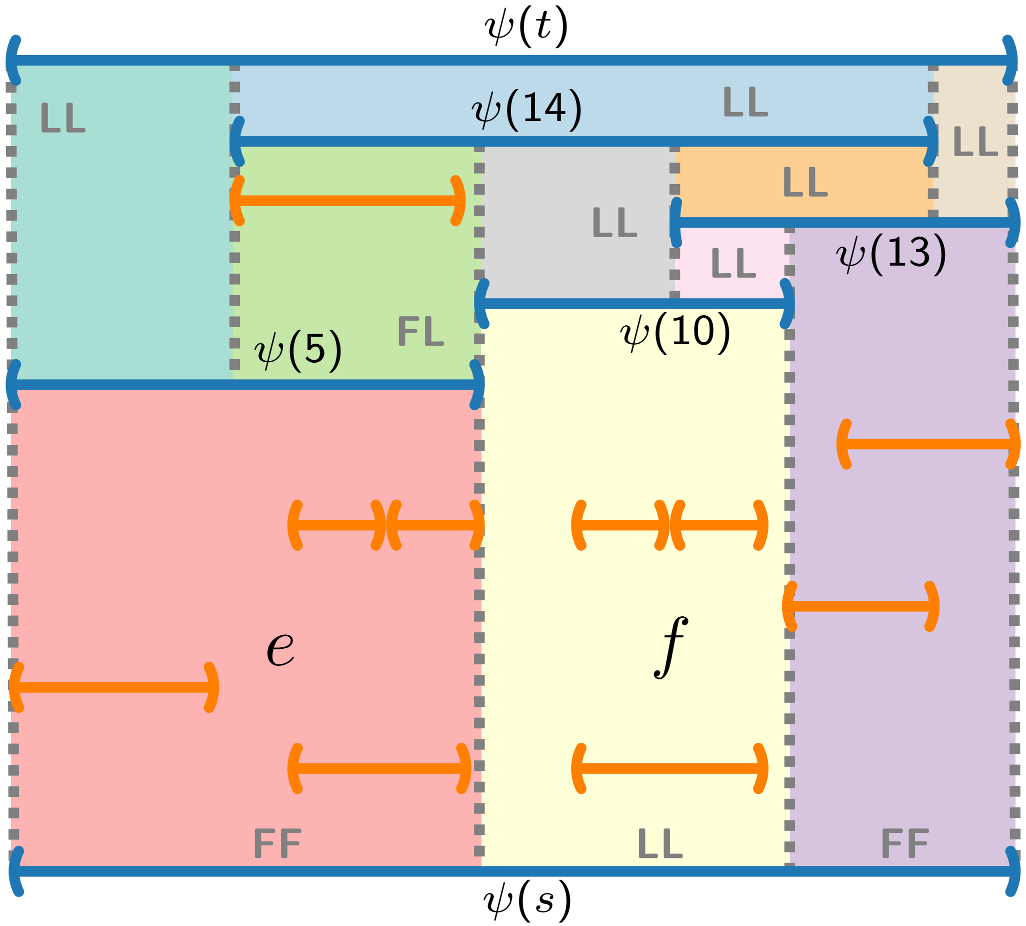
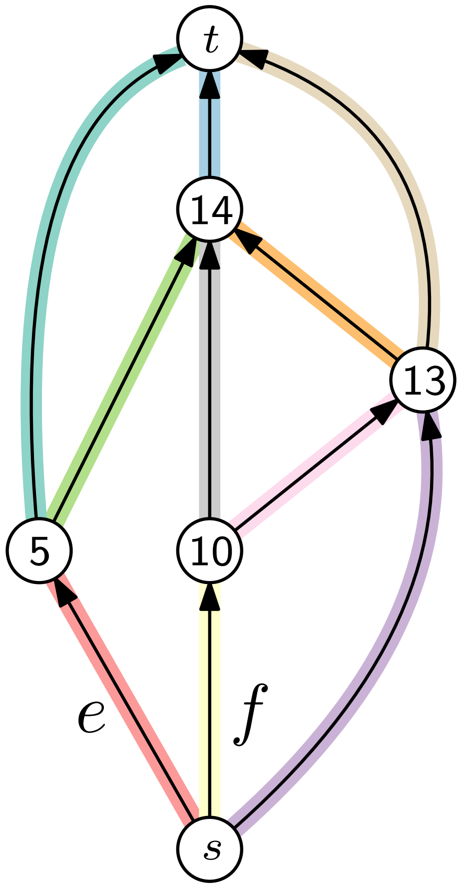
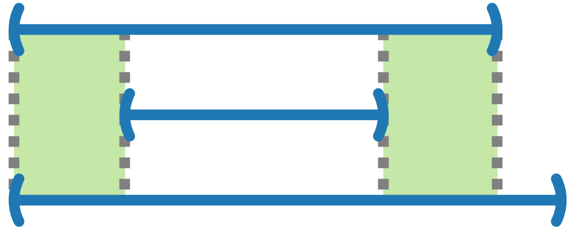
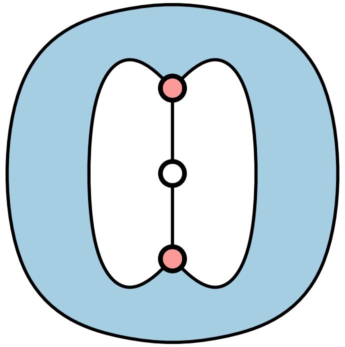
  - Add *consistency clauses*: e.g., $\neg(\neg r_e \wedge \neg l_f)$









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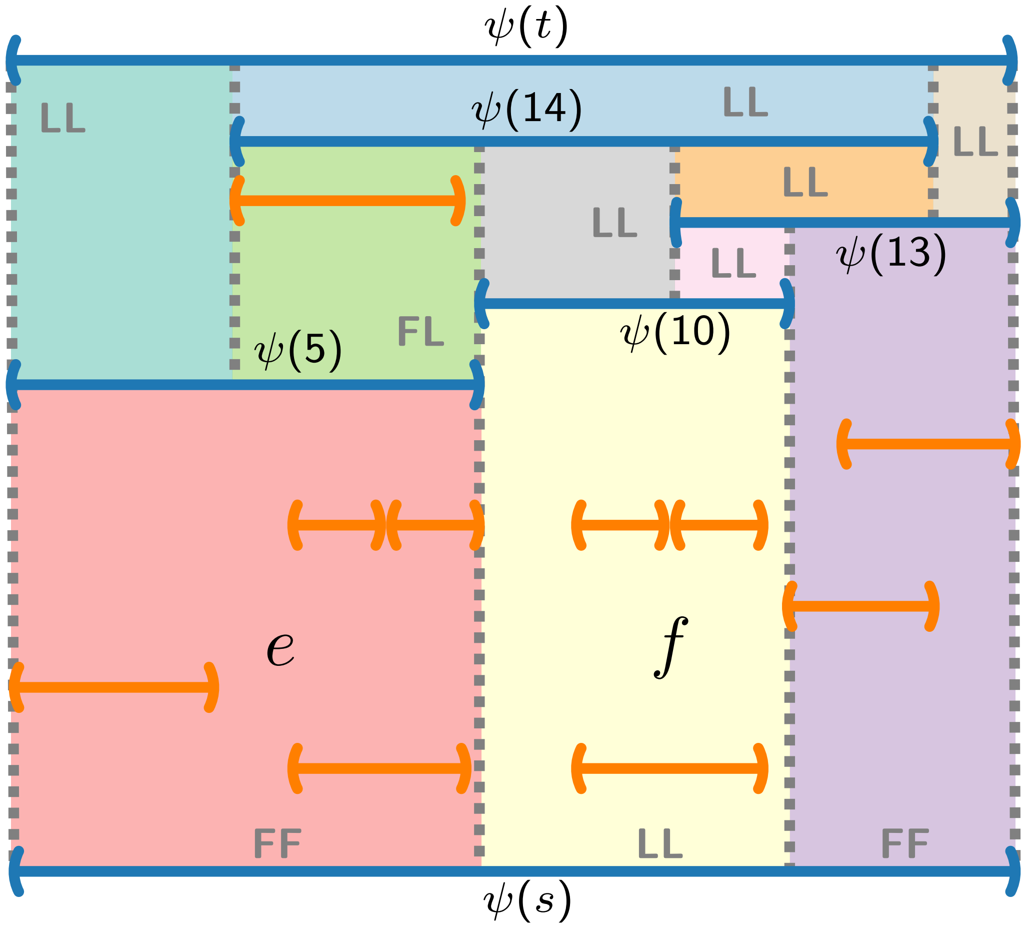
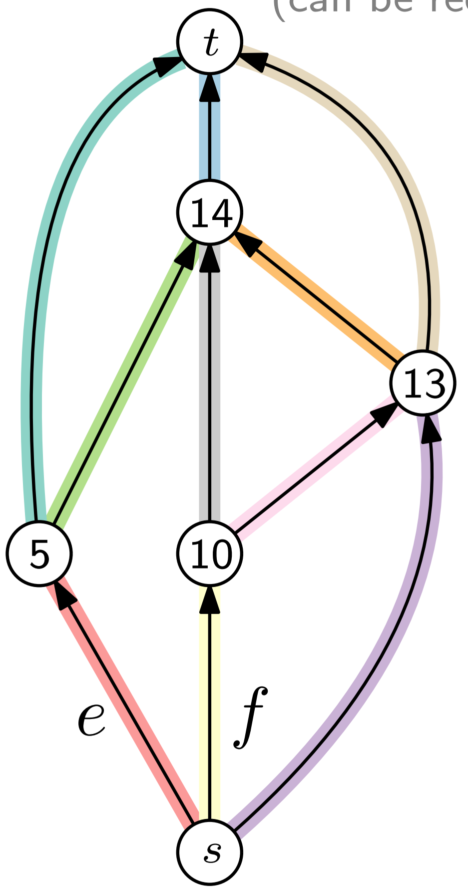
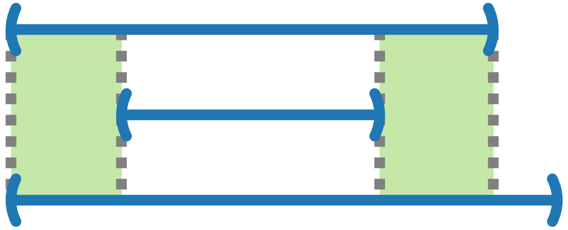
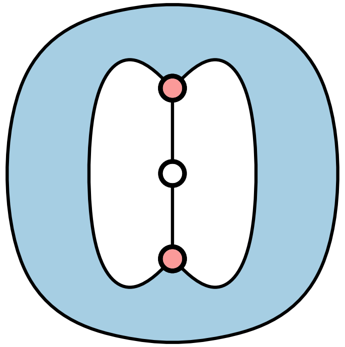
  - Add *consistency clauses*: e.g., $\neg(\neg r_e \wedge \neg l_f)$ $\rightarrow O(n^2)$ many.

(can be reduced to $O(n \log^2 n)$)









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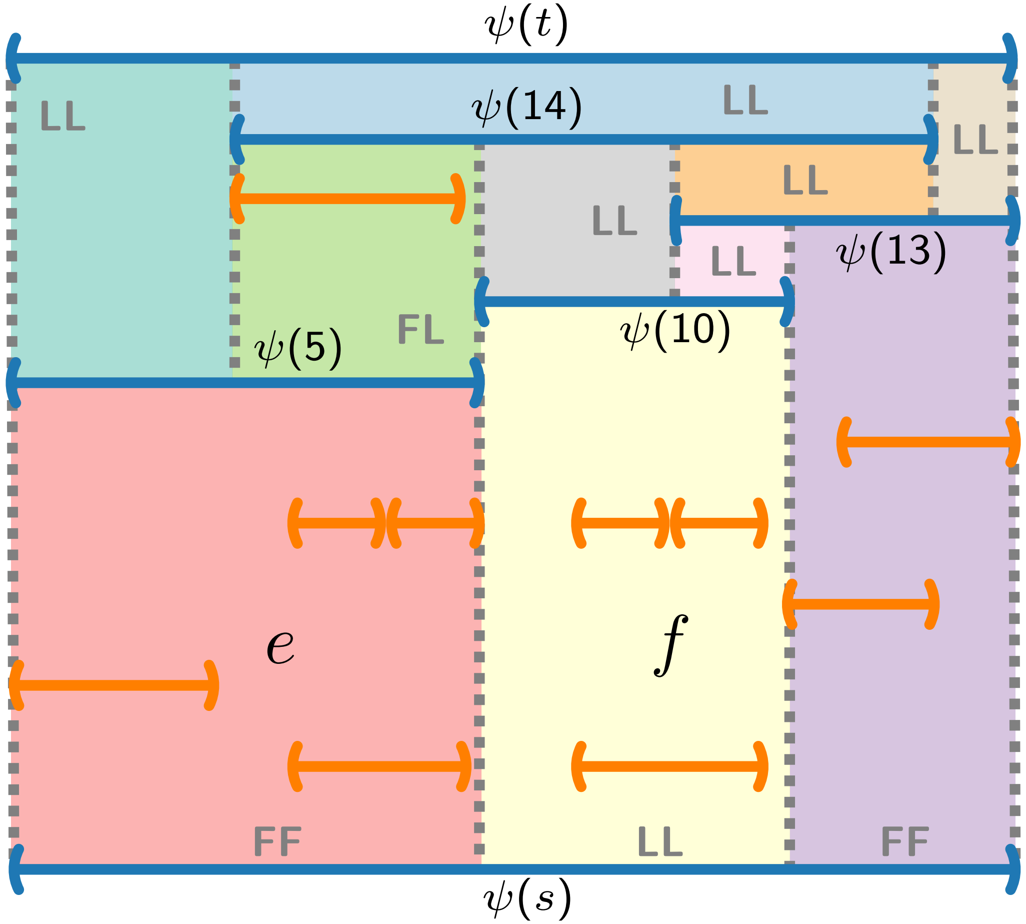

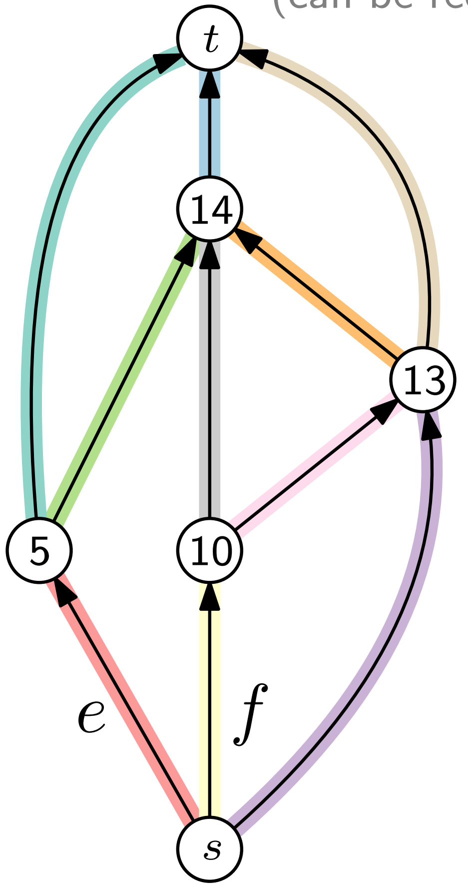

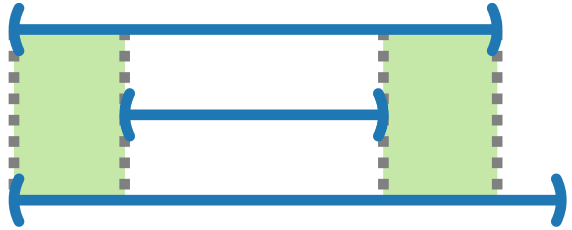

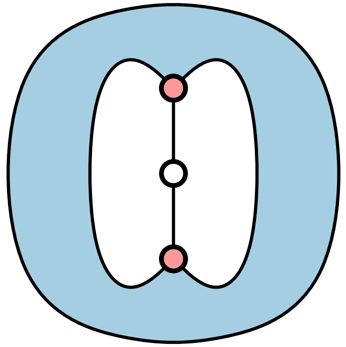

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- Finding a satisfying assignment of a 2-SAT formula can be done in linear time!

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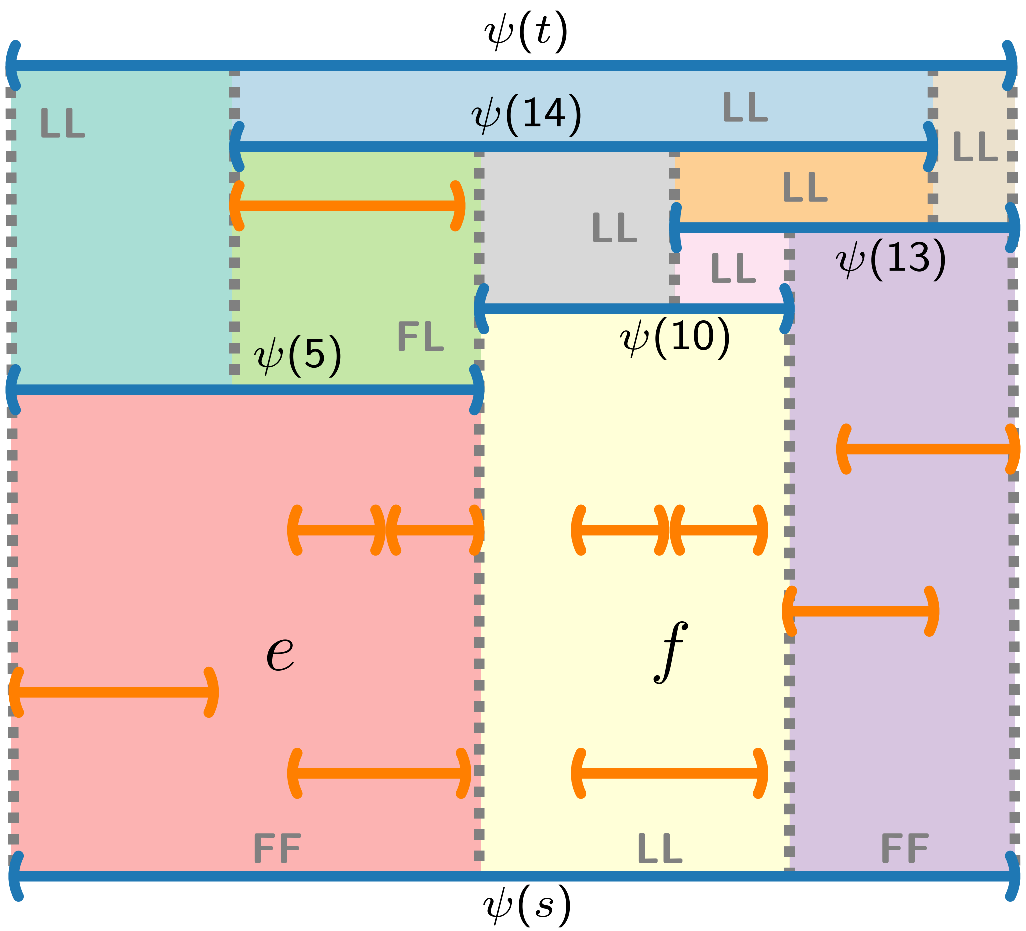

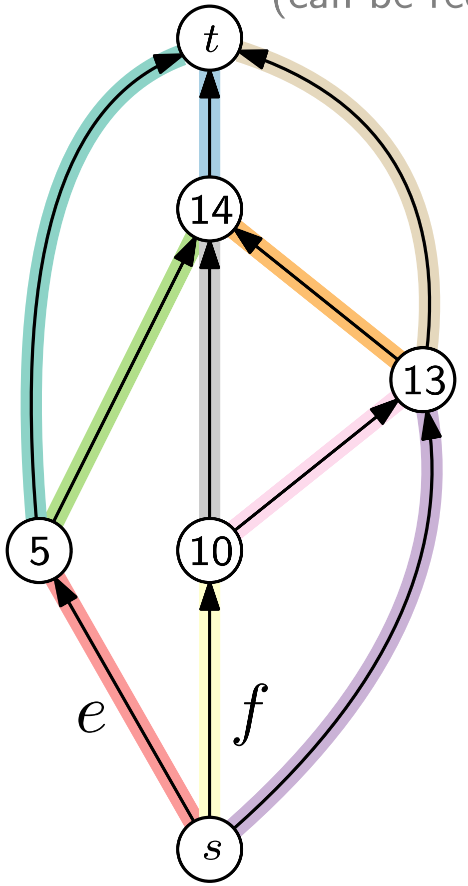

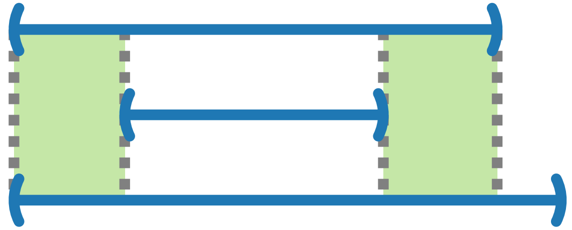

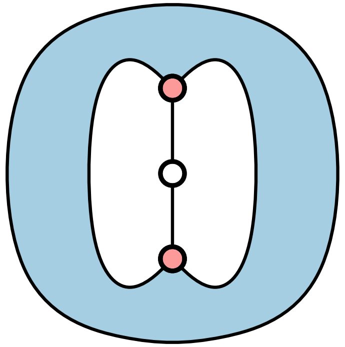

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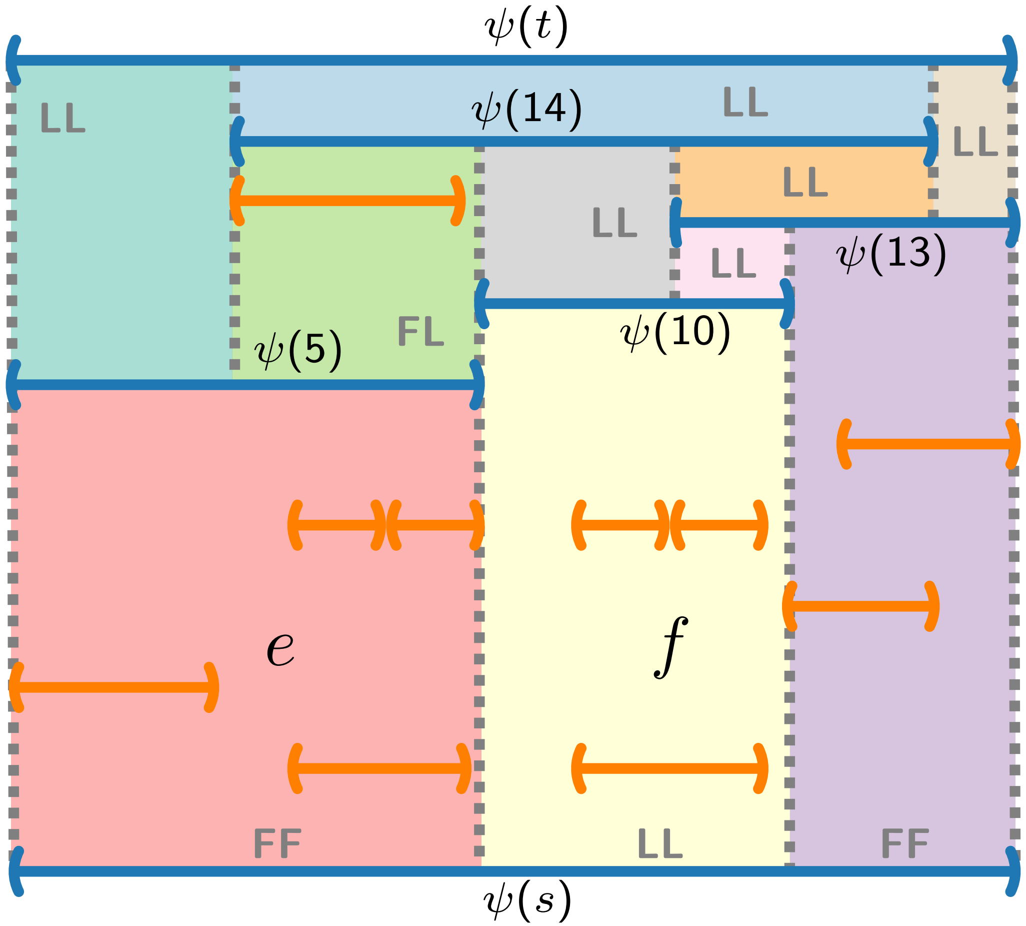

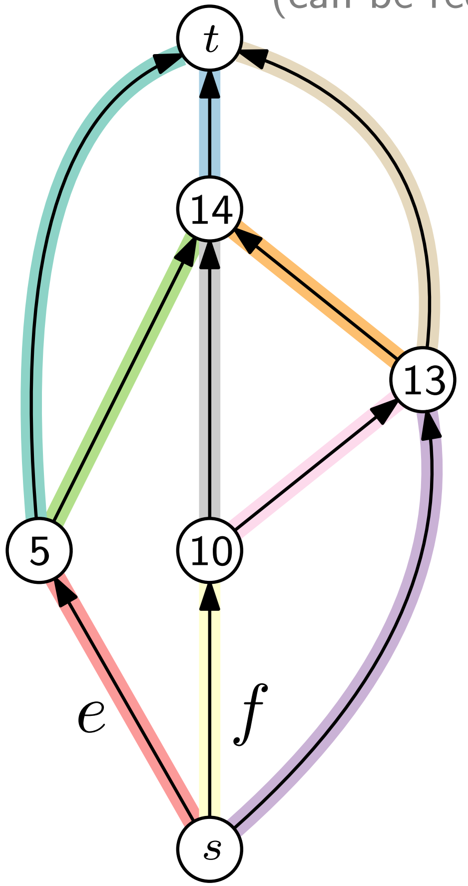

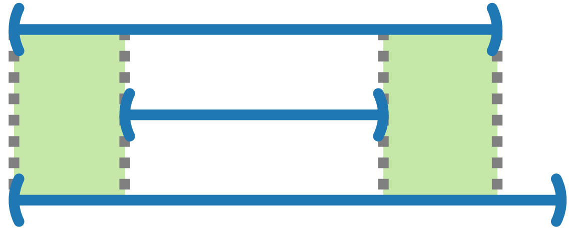

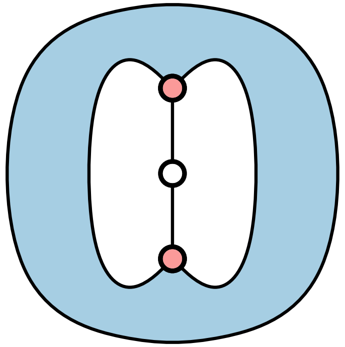

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Results and Outline

[Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18]

Theorem 1.
Rectangular ε-bar visibility representation extension can be solved in $\mathcal{O}(n \log^2 n)$ time *for st-graphs*.

- Dynamic program via SPQR-trees
- Easier version: $\mathcal{O}(n^2)$

Theorem 2.
ε-bar visibility representation extension is NP-complete.

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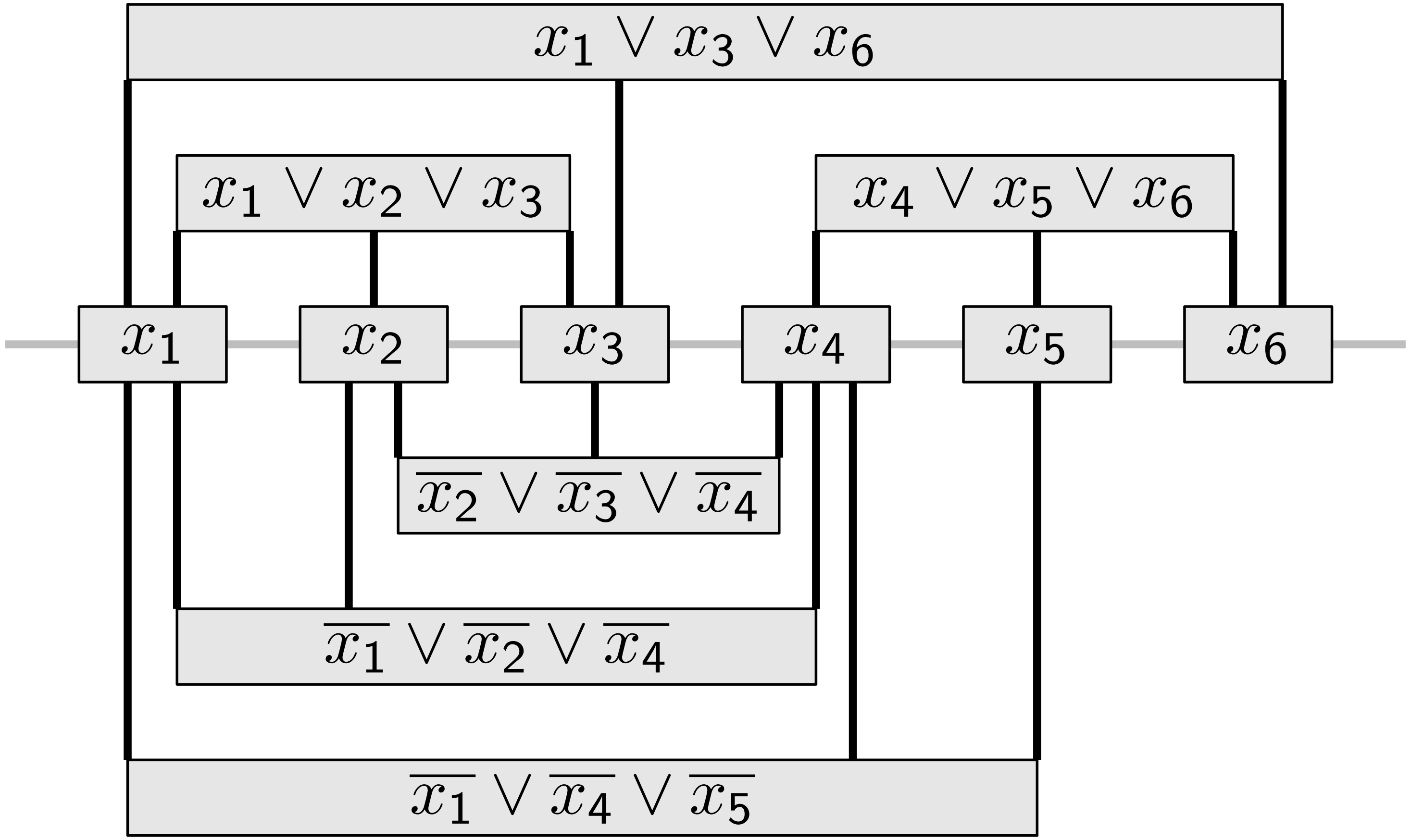

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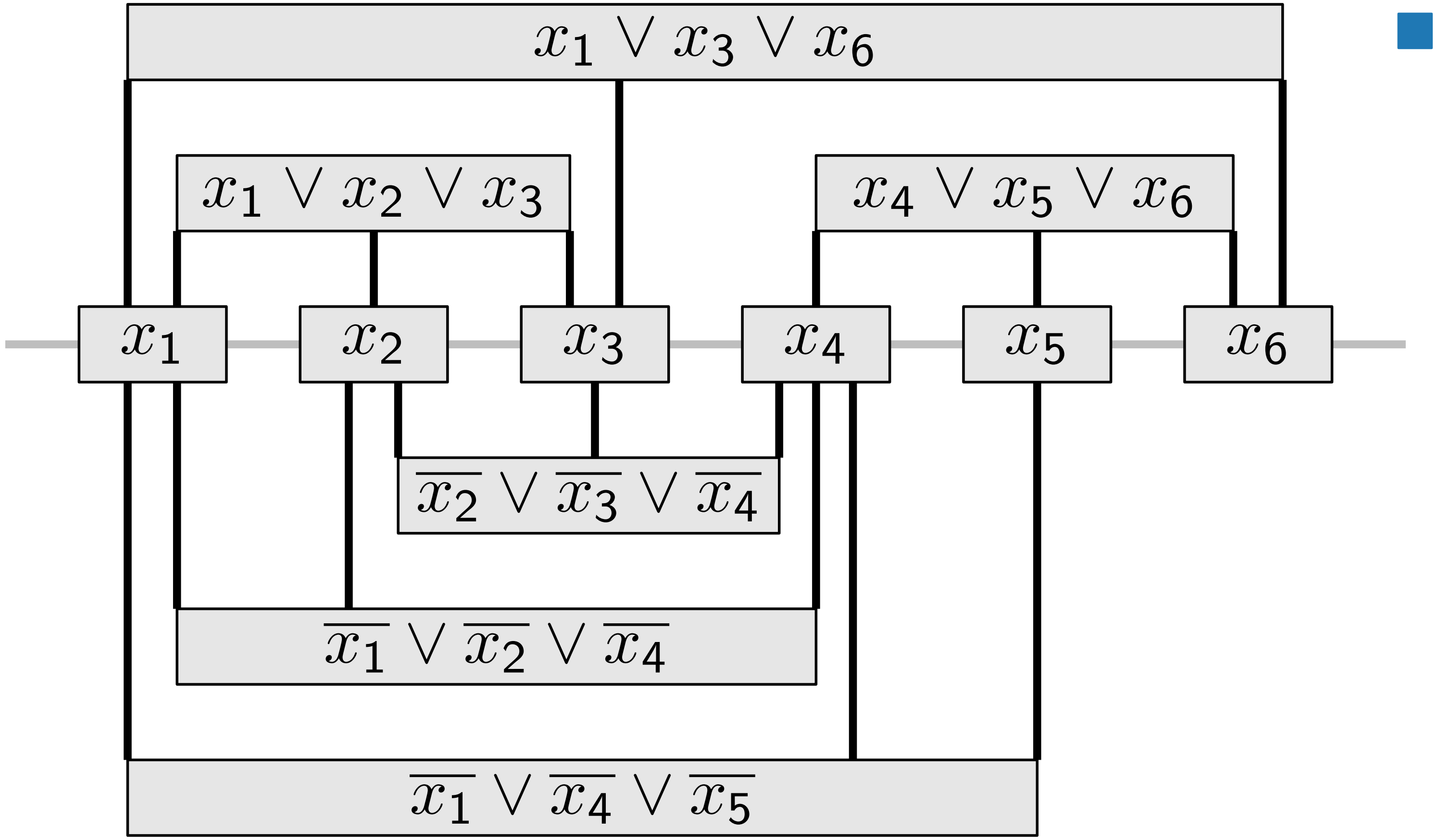

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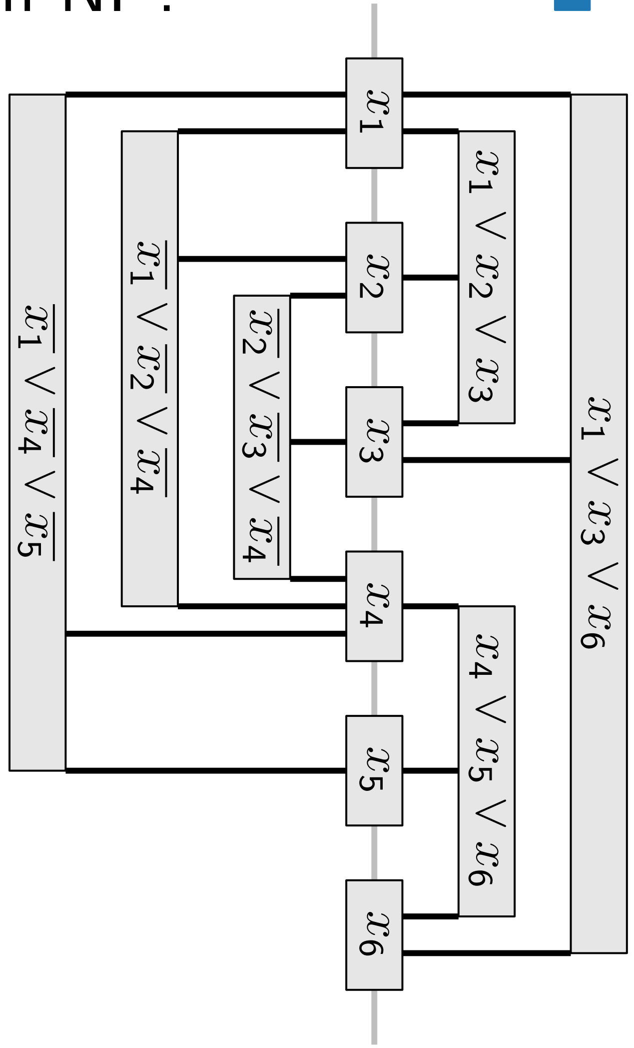

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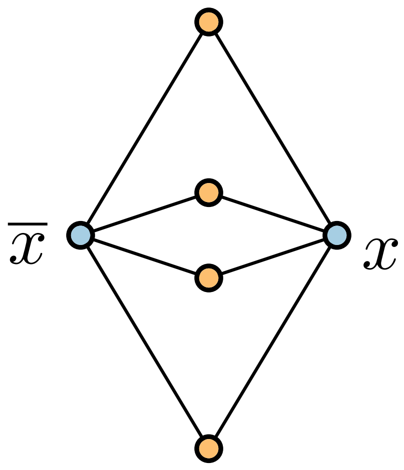

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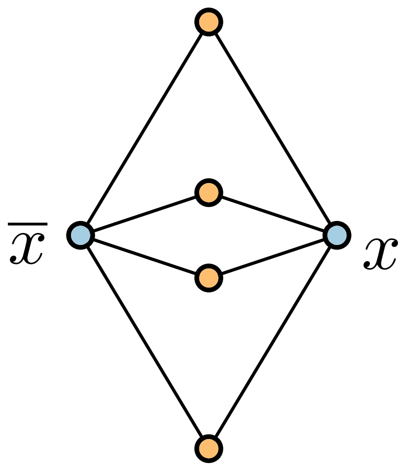

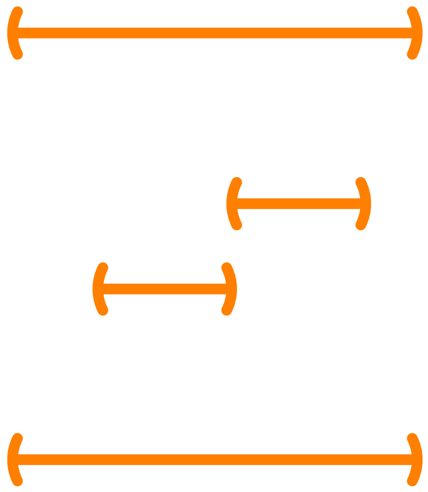

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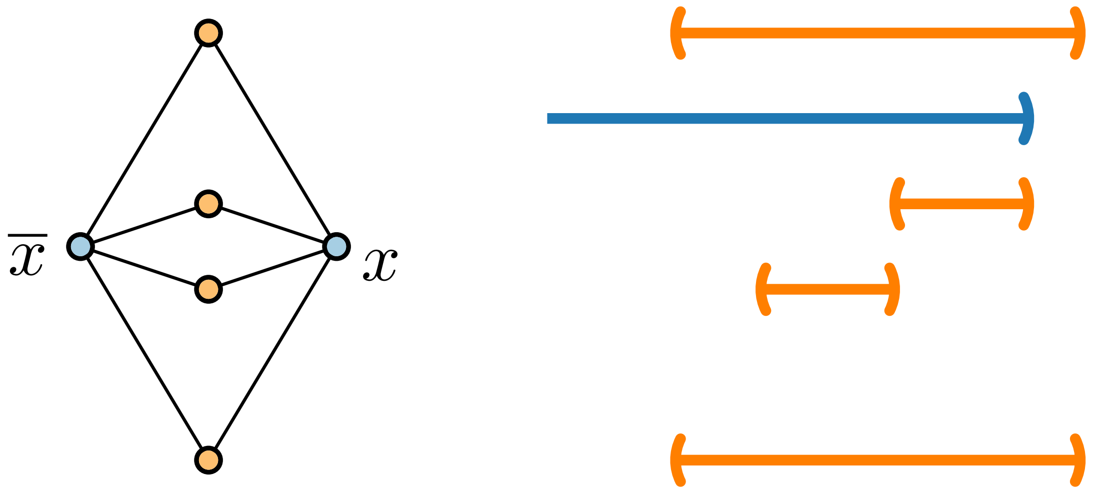

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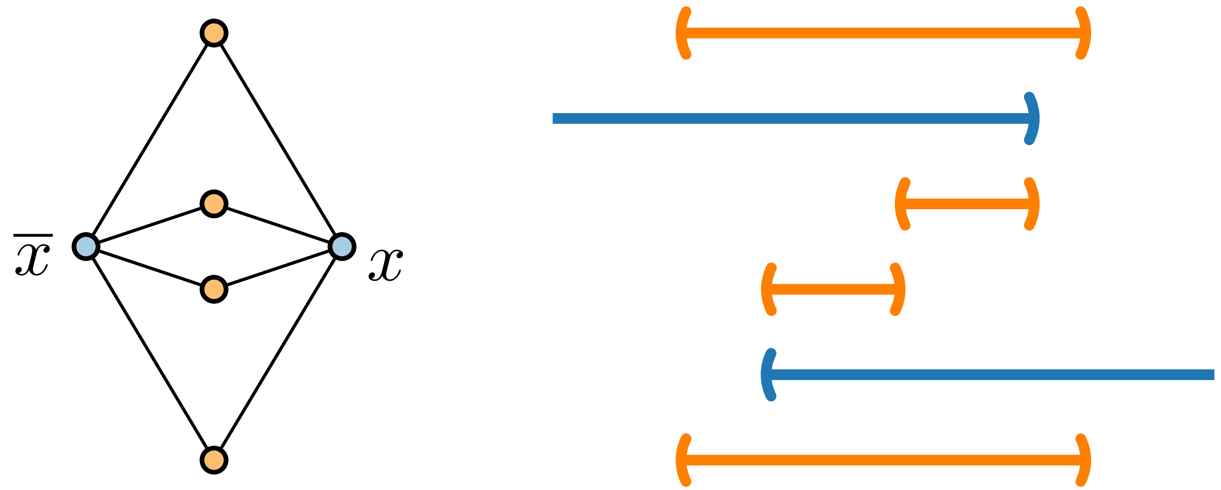

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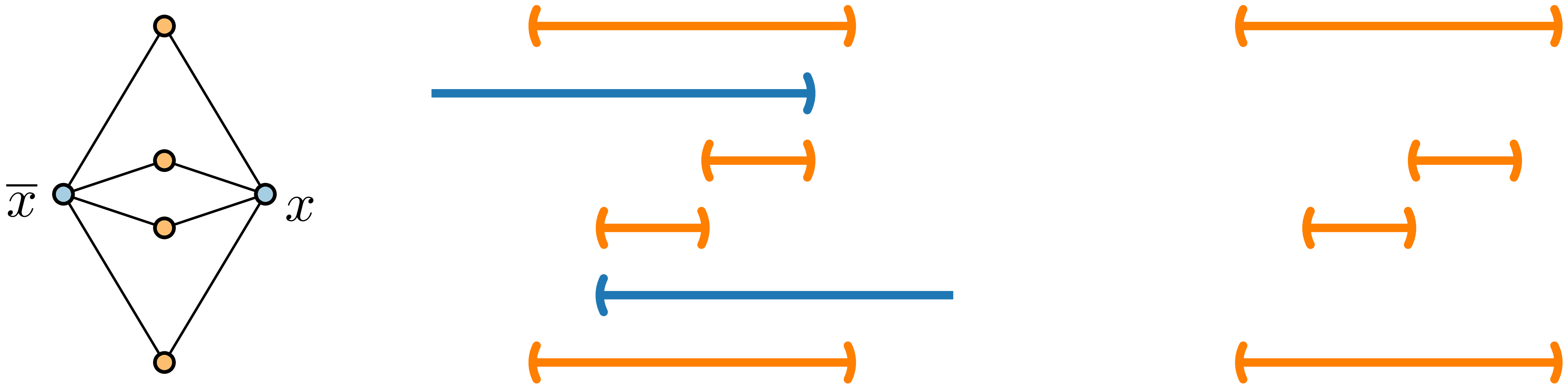

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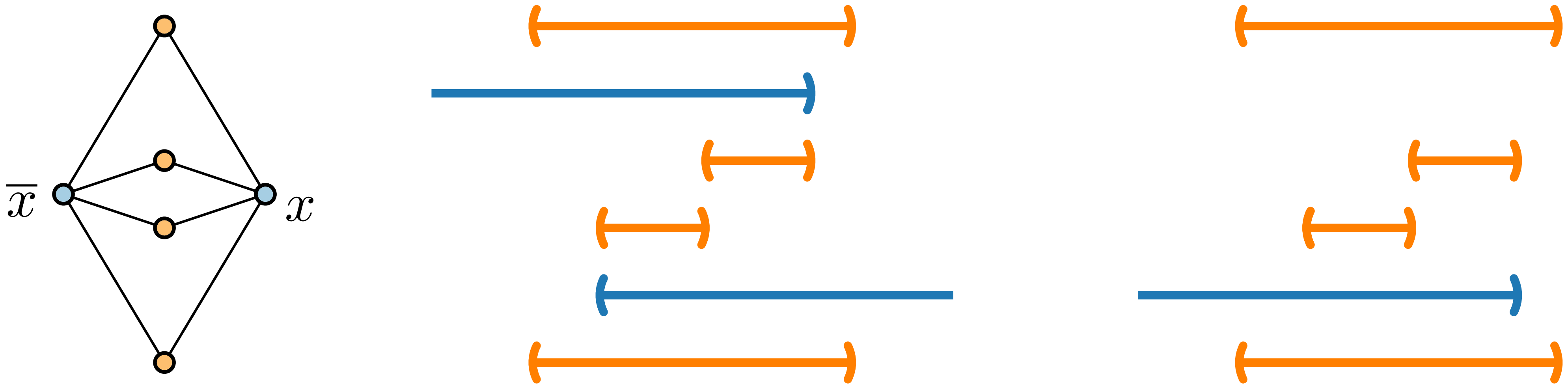

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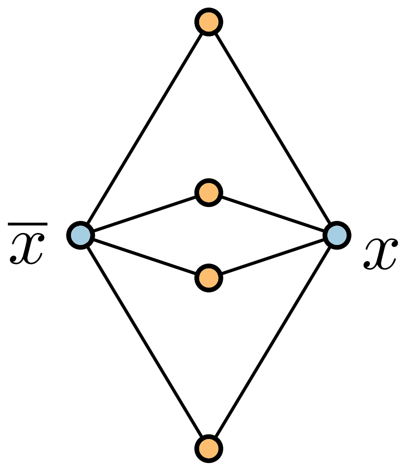

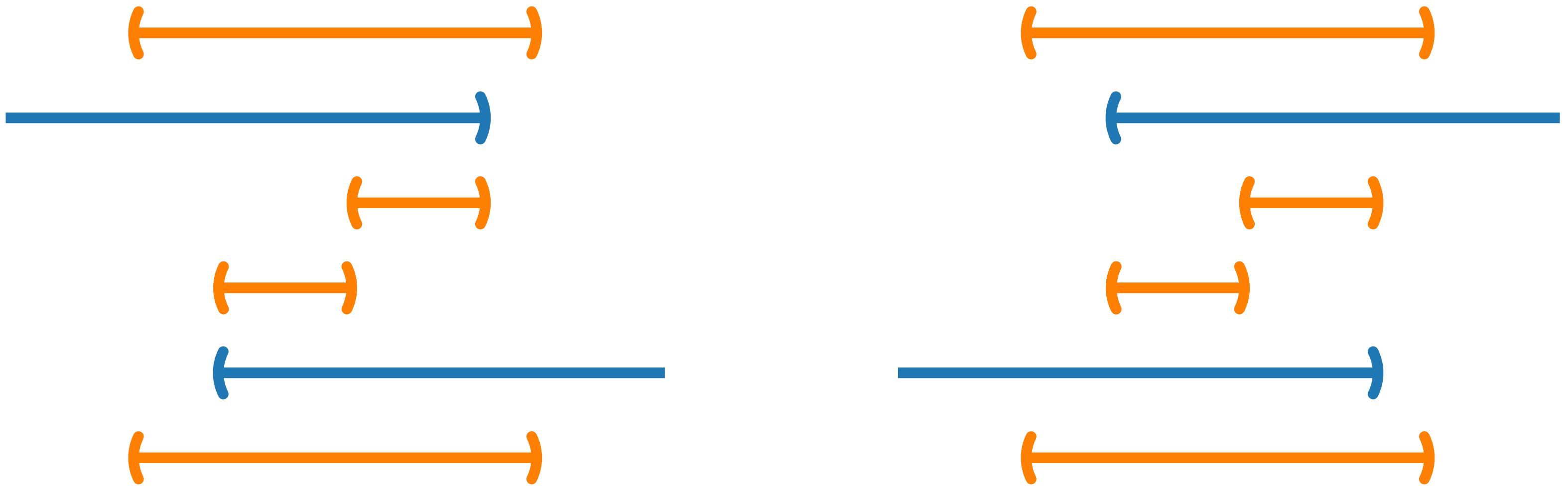

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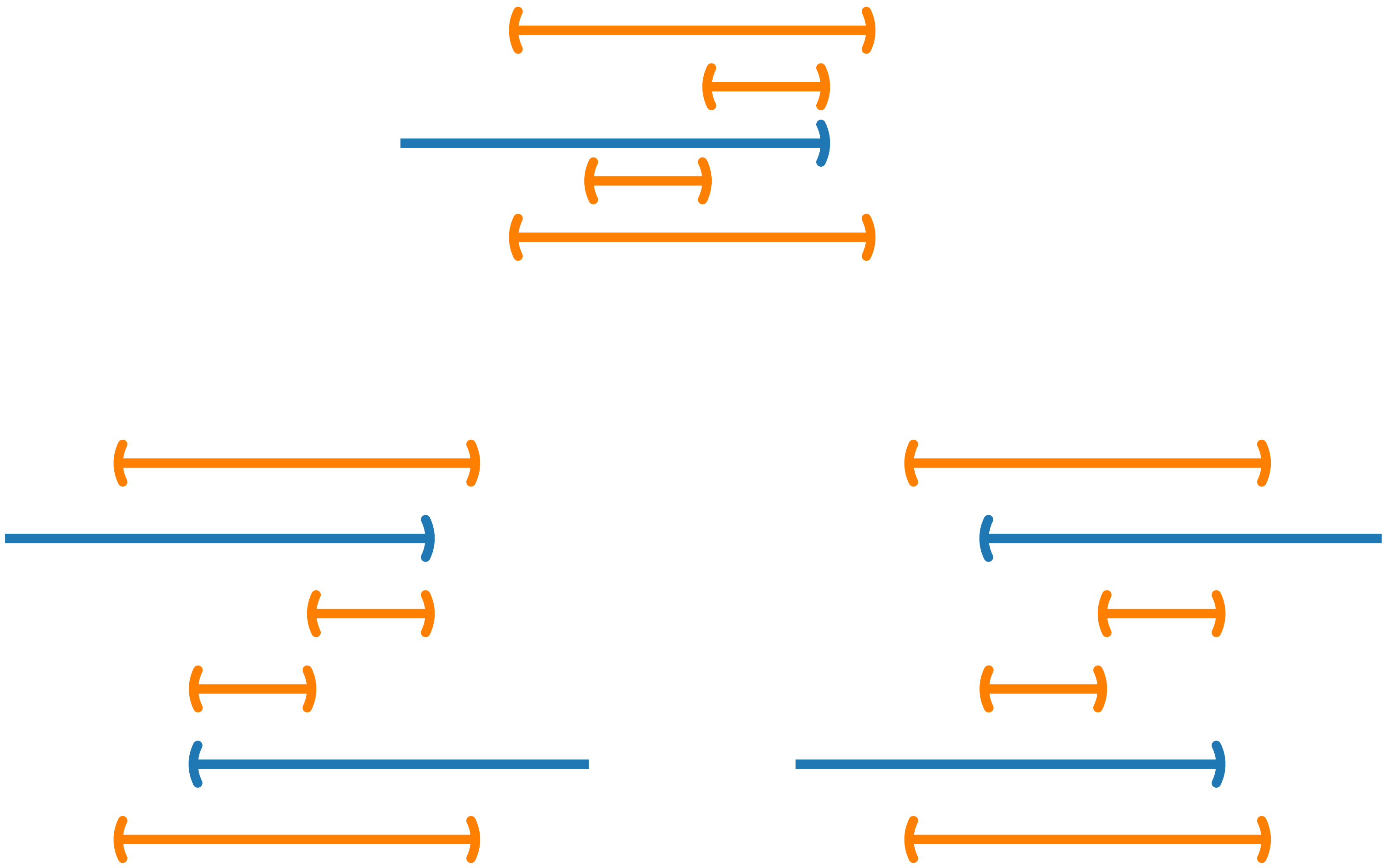

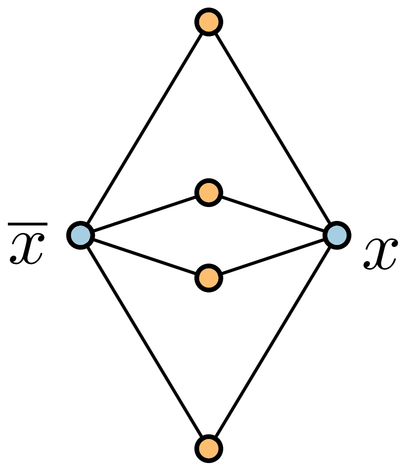

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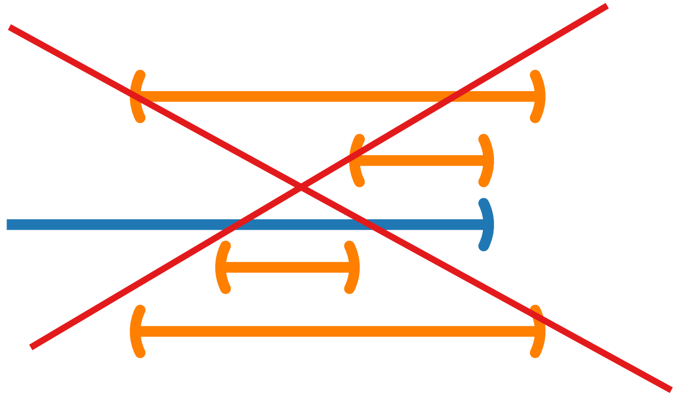

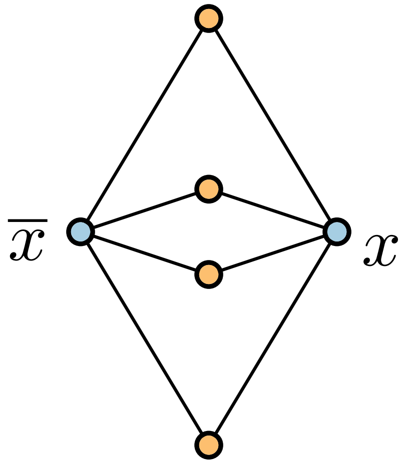

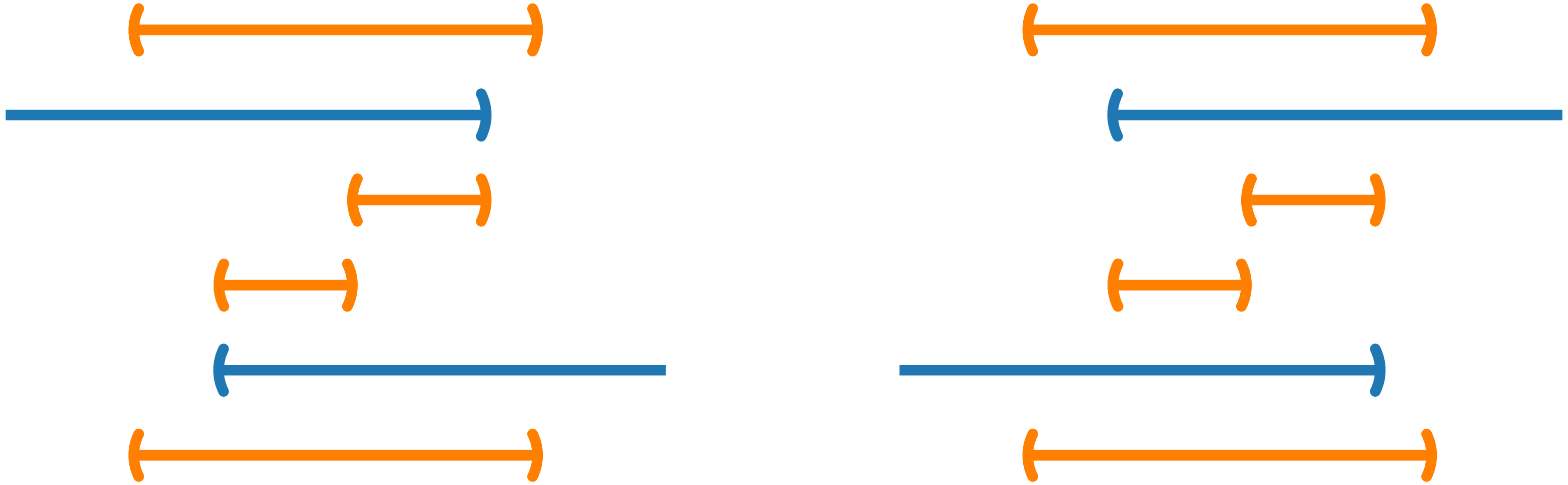

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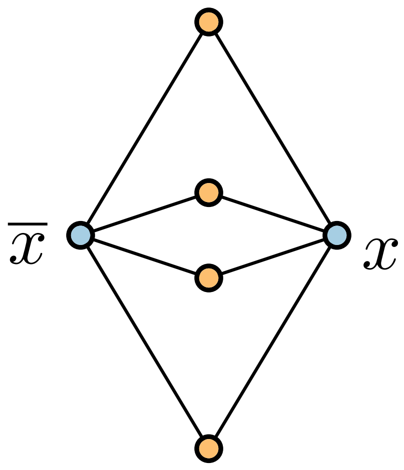

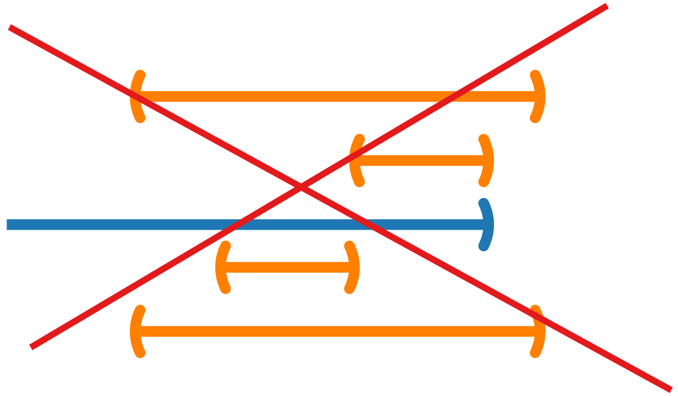

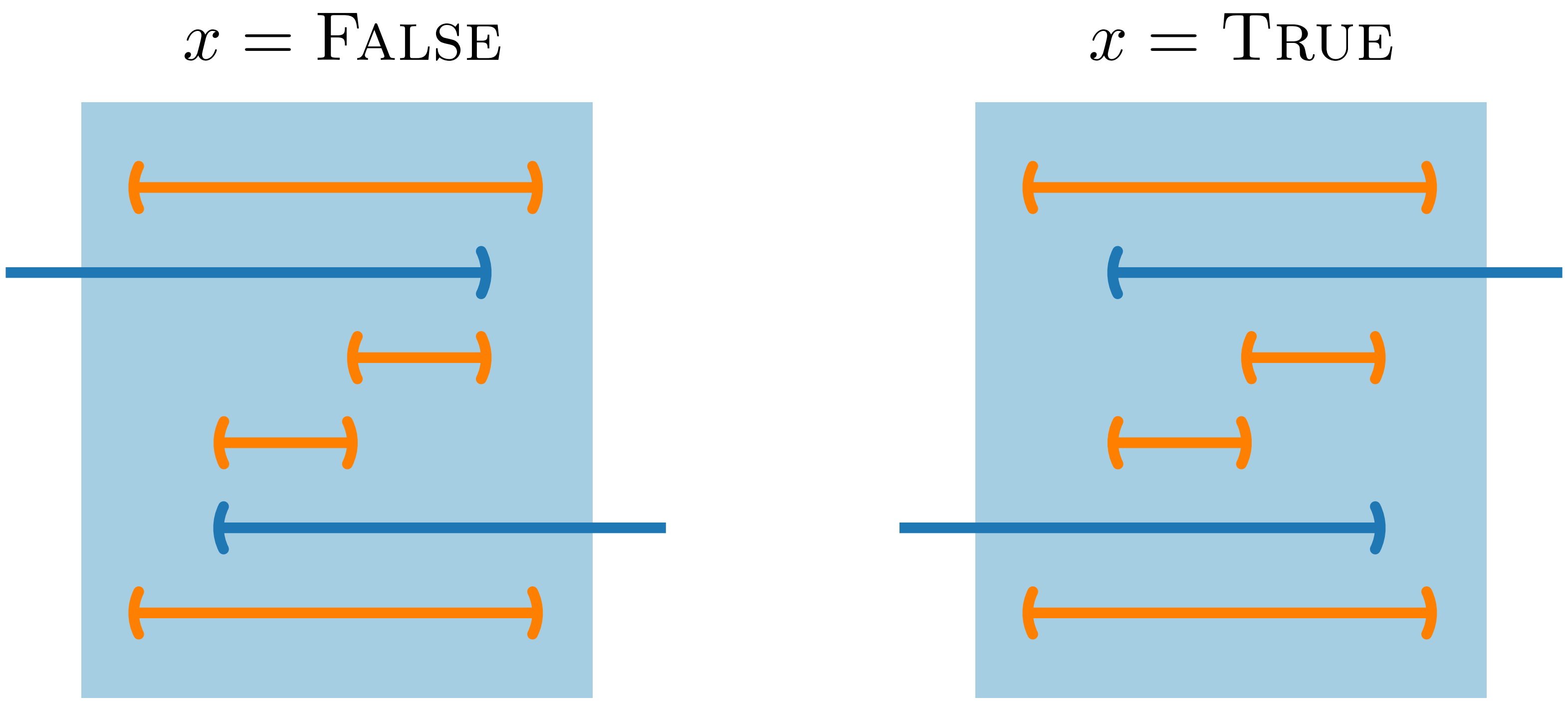

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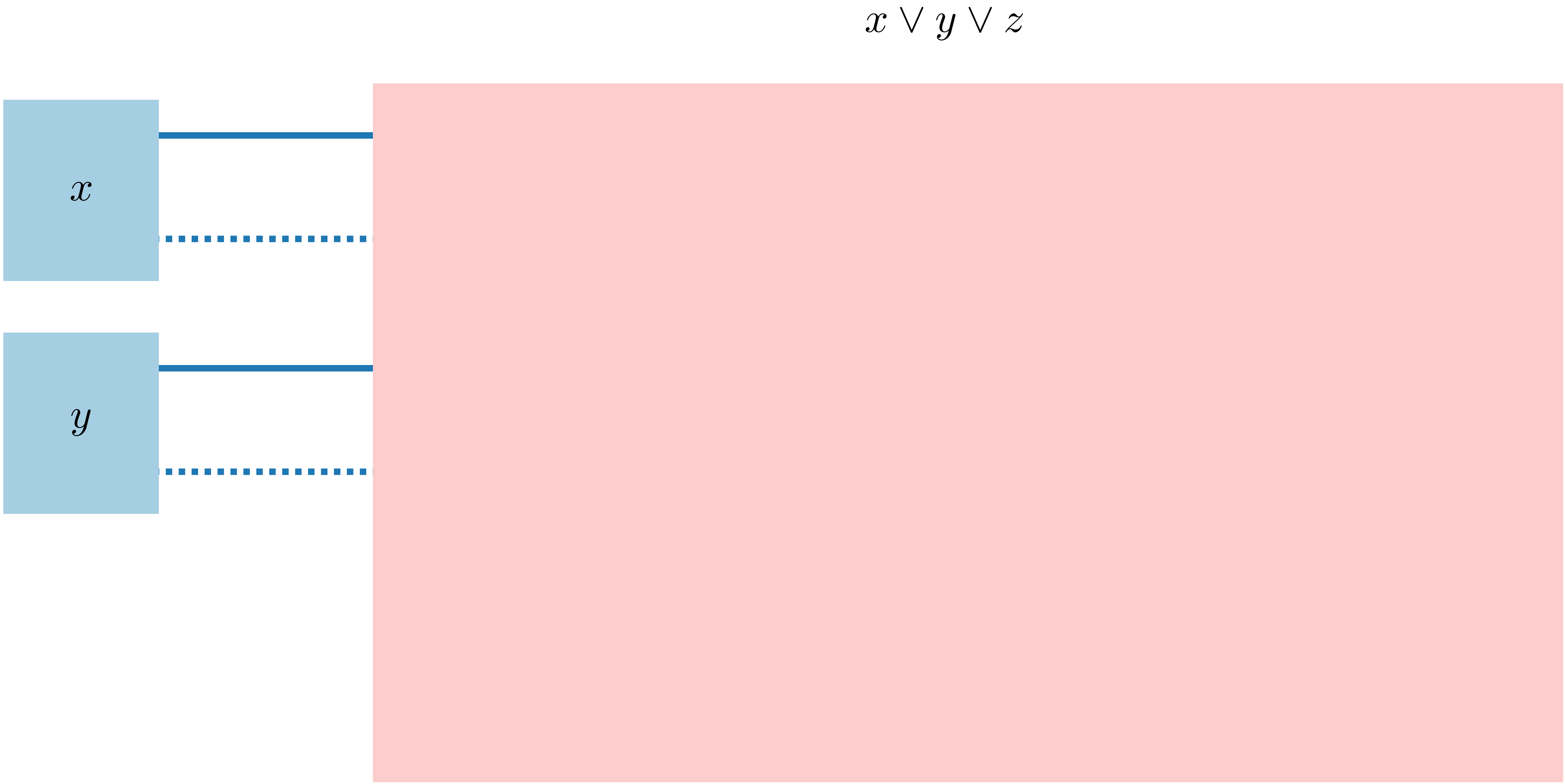

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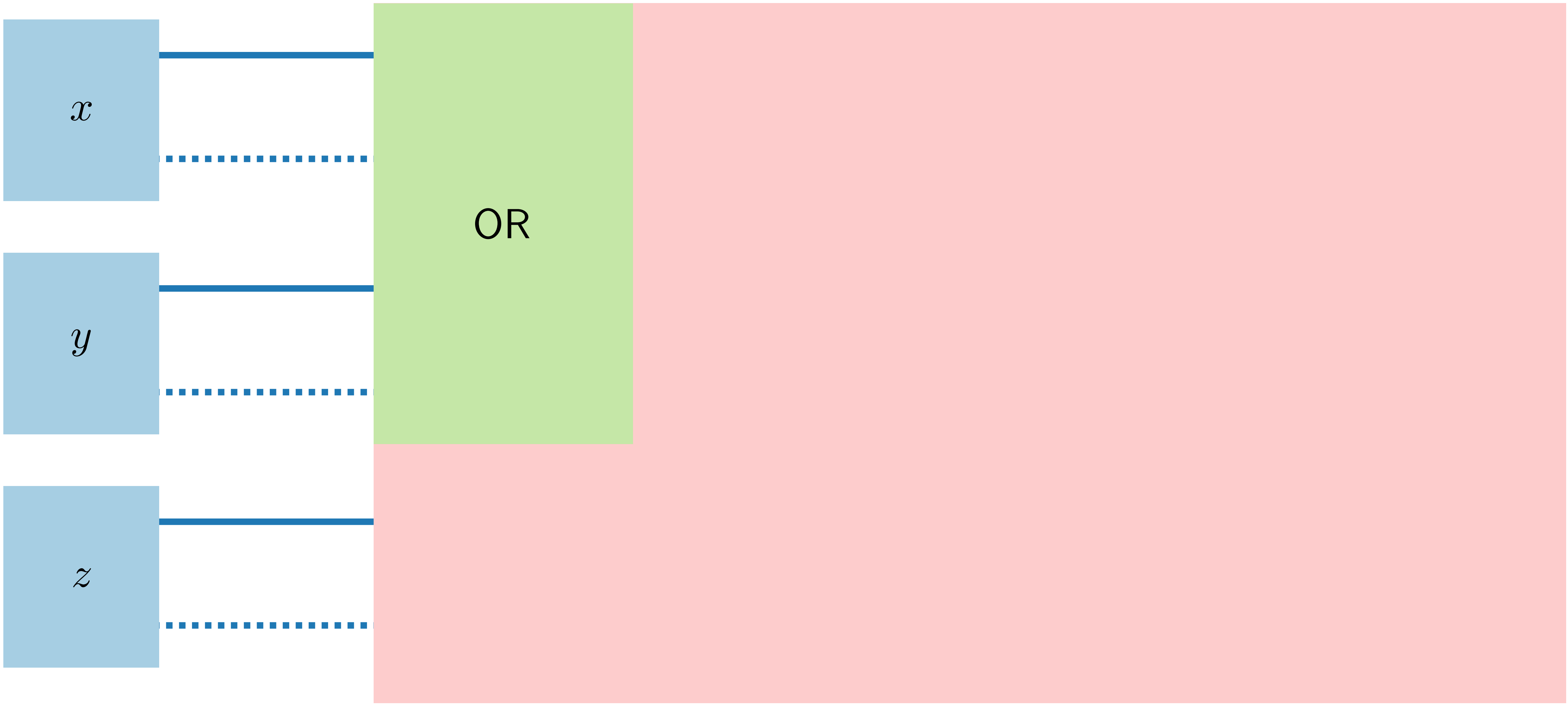

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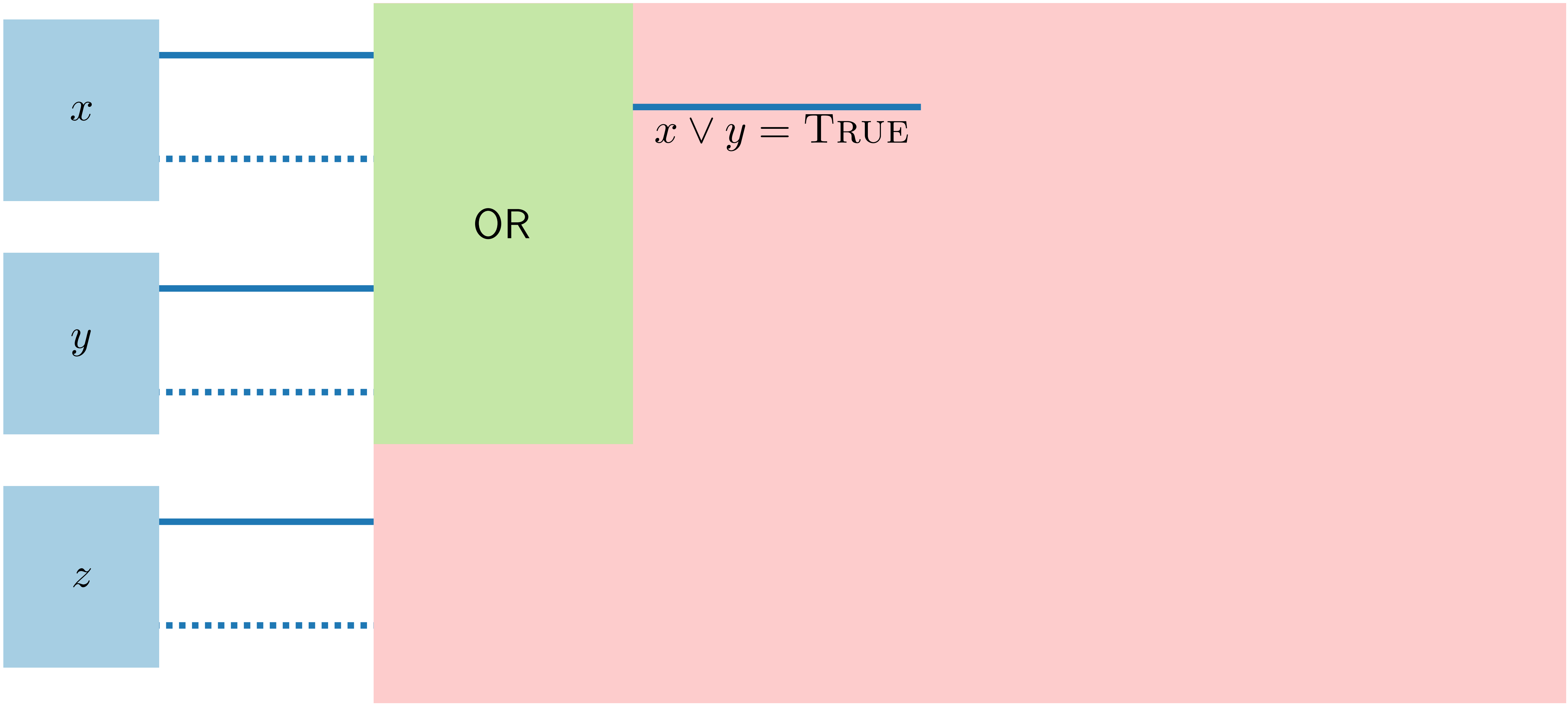

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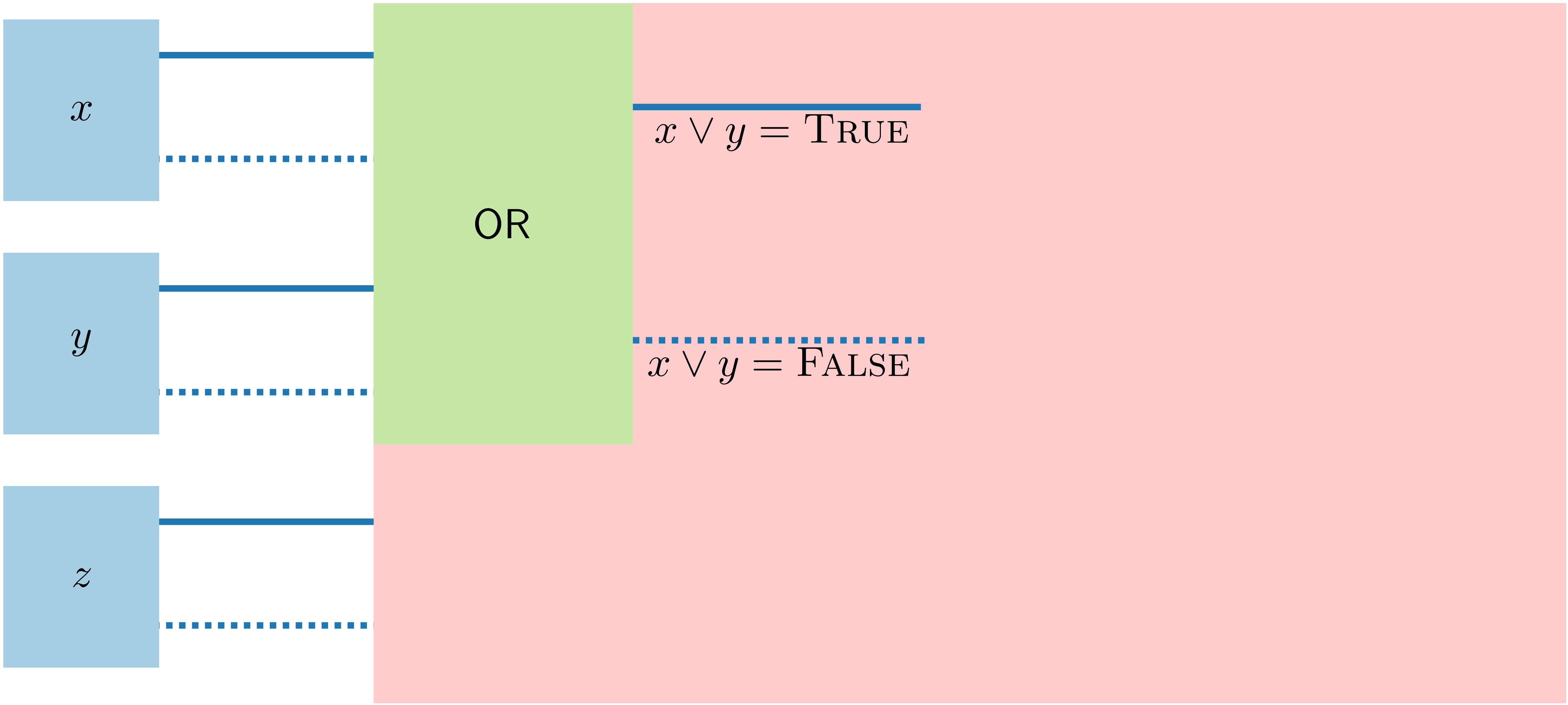

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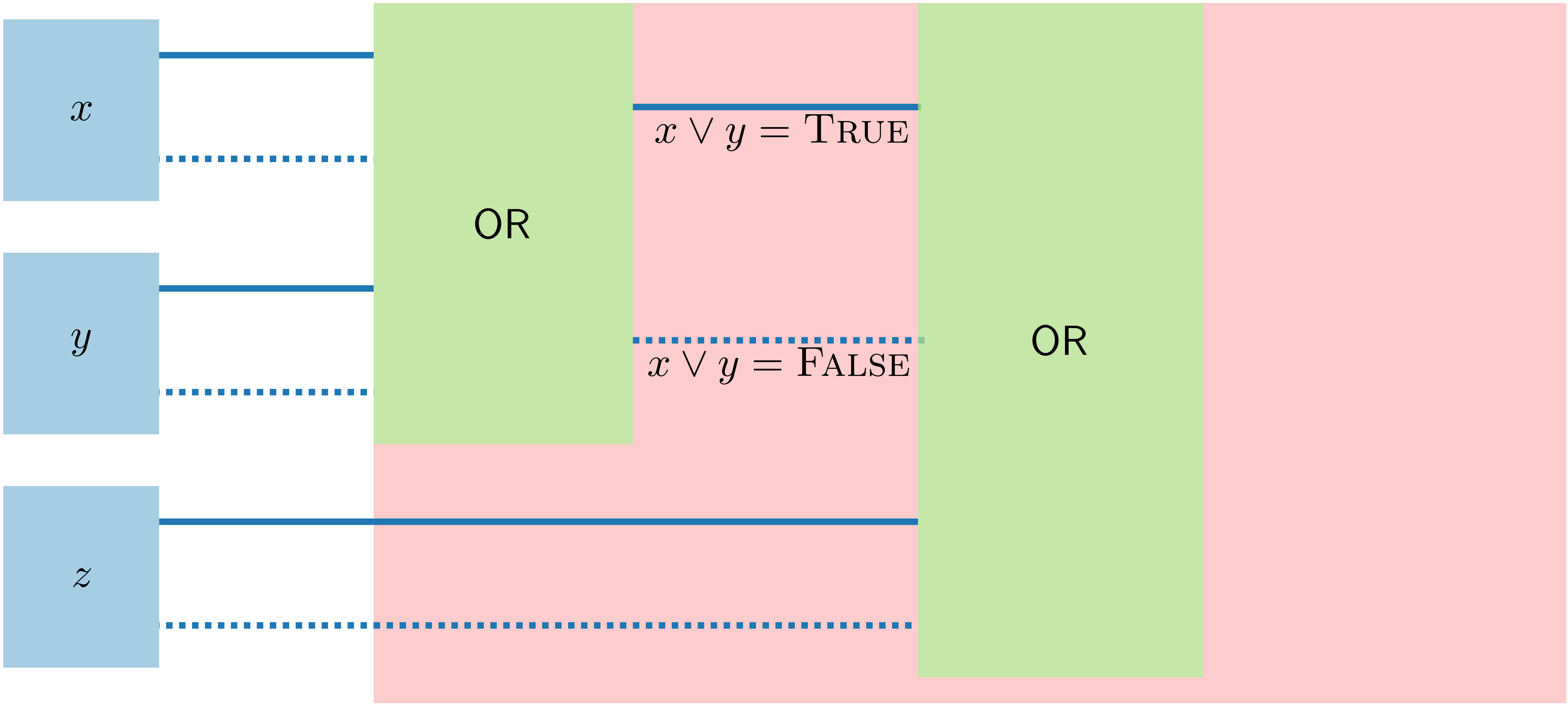

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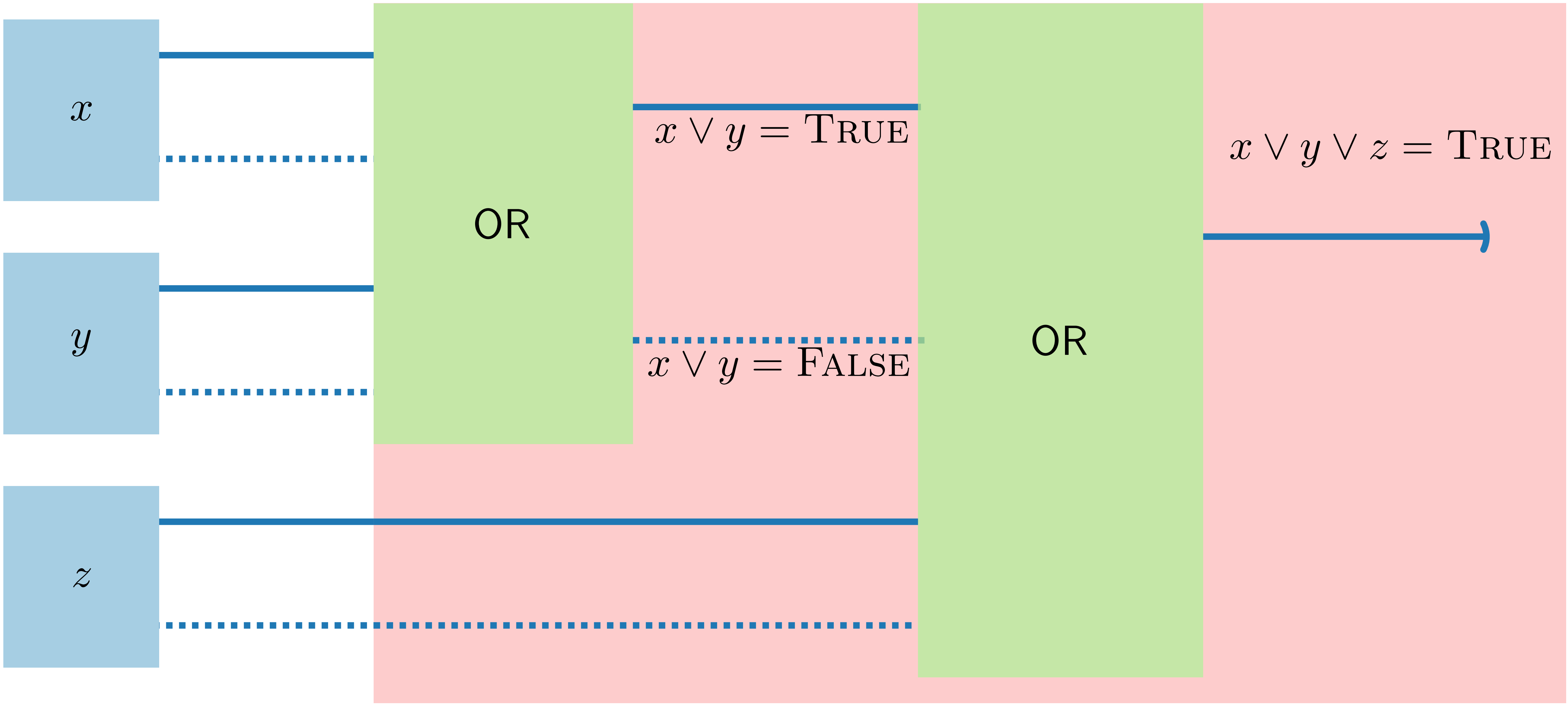

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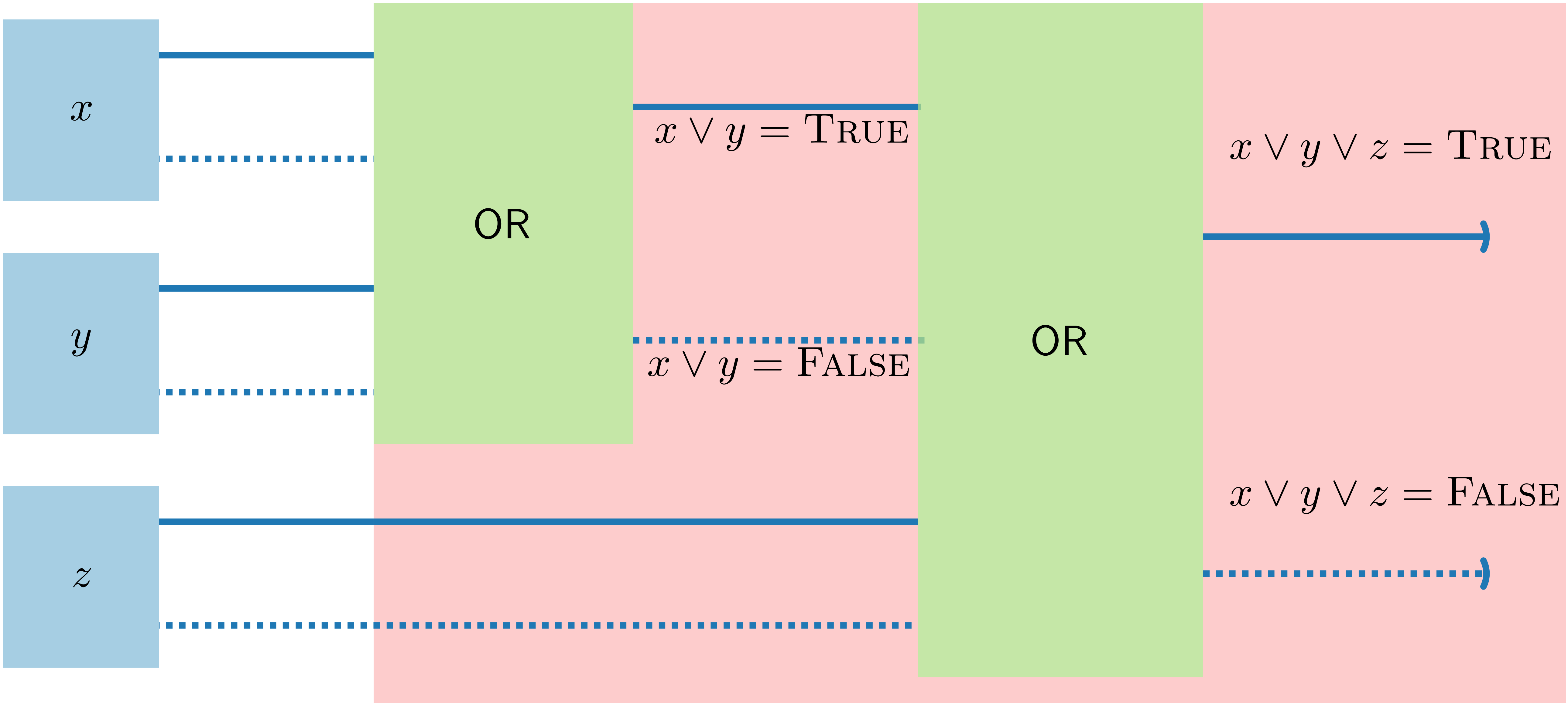

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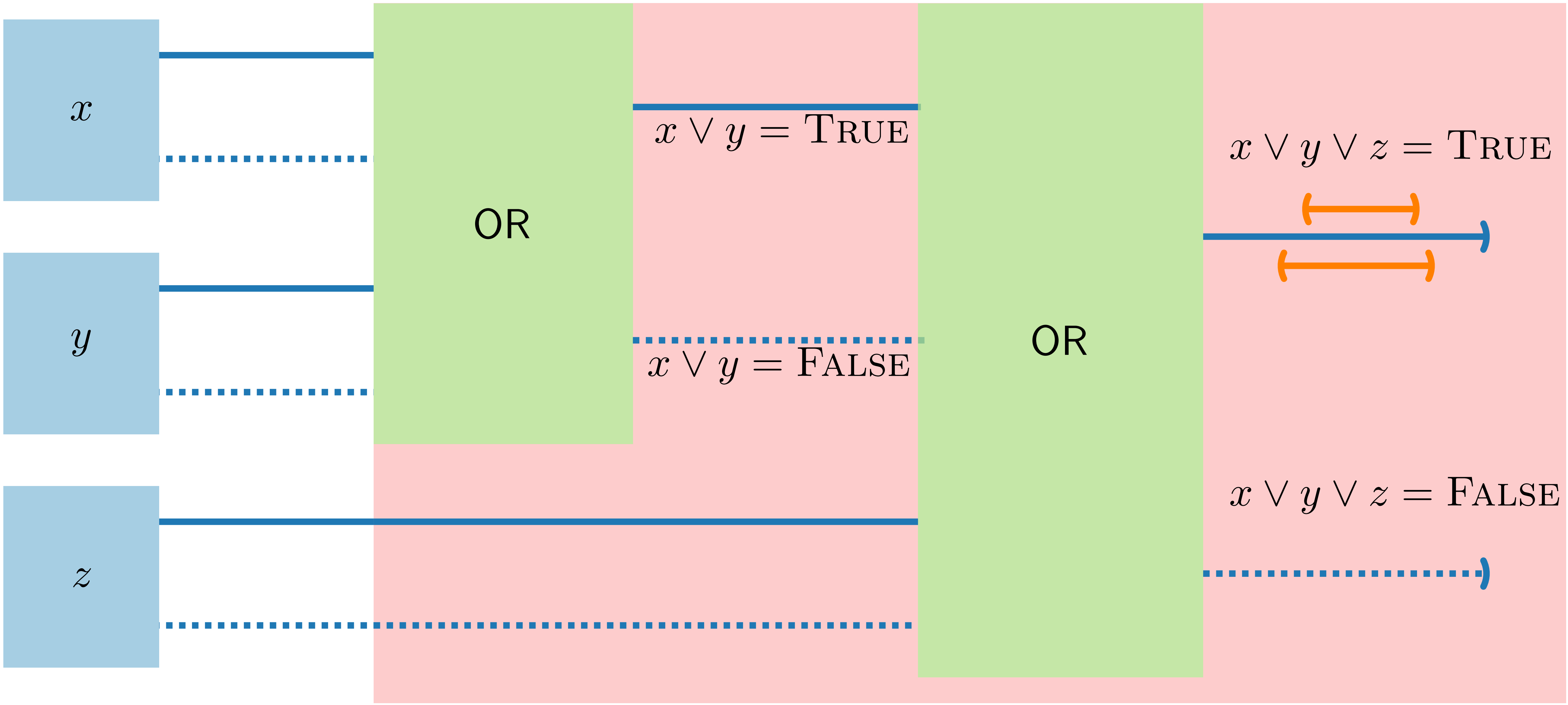

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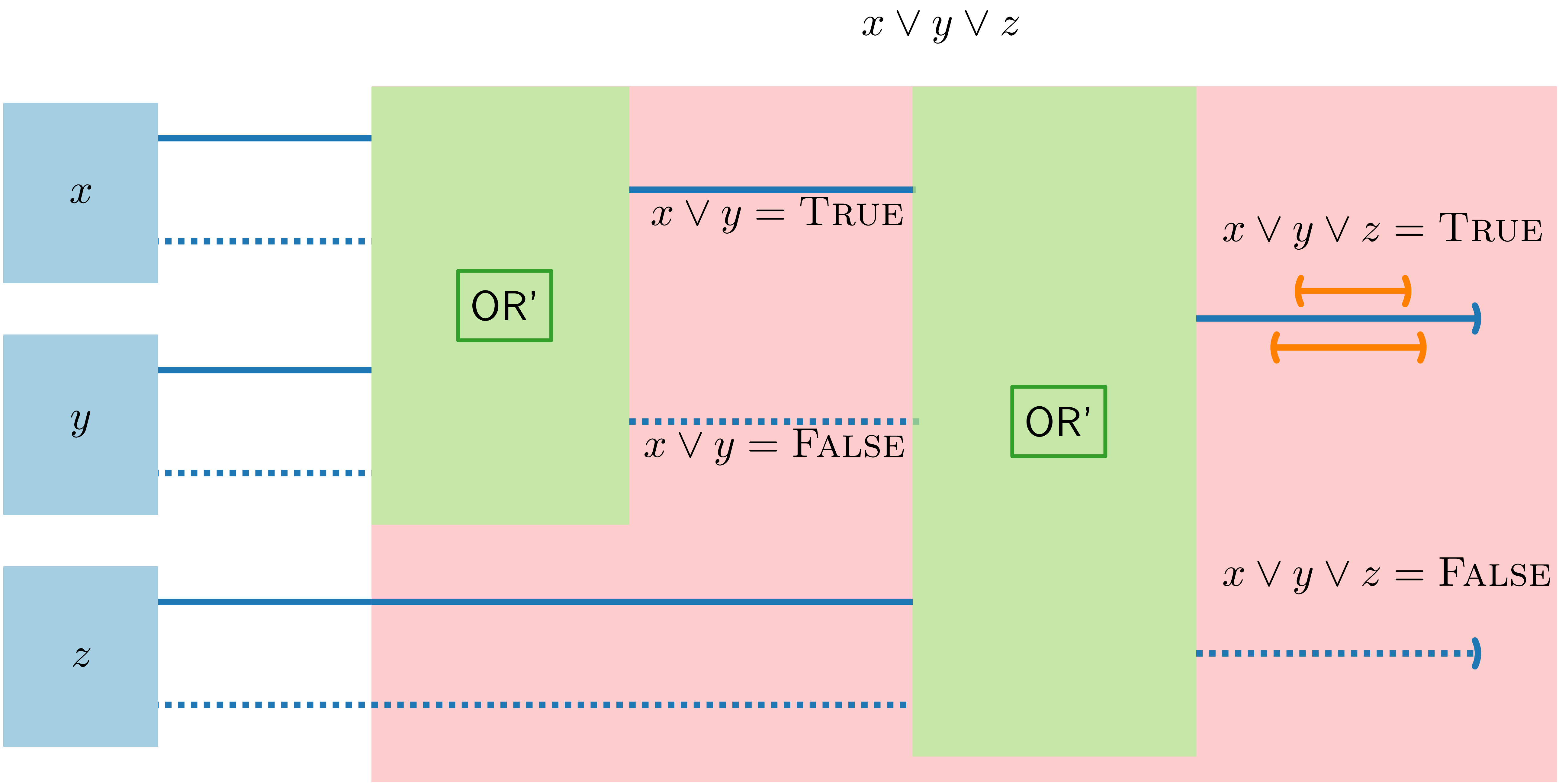

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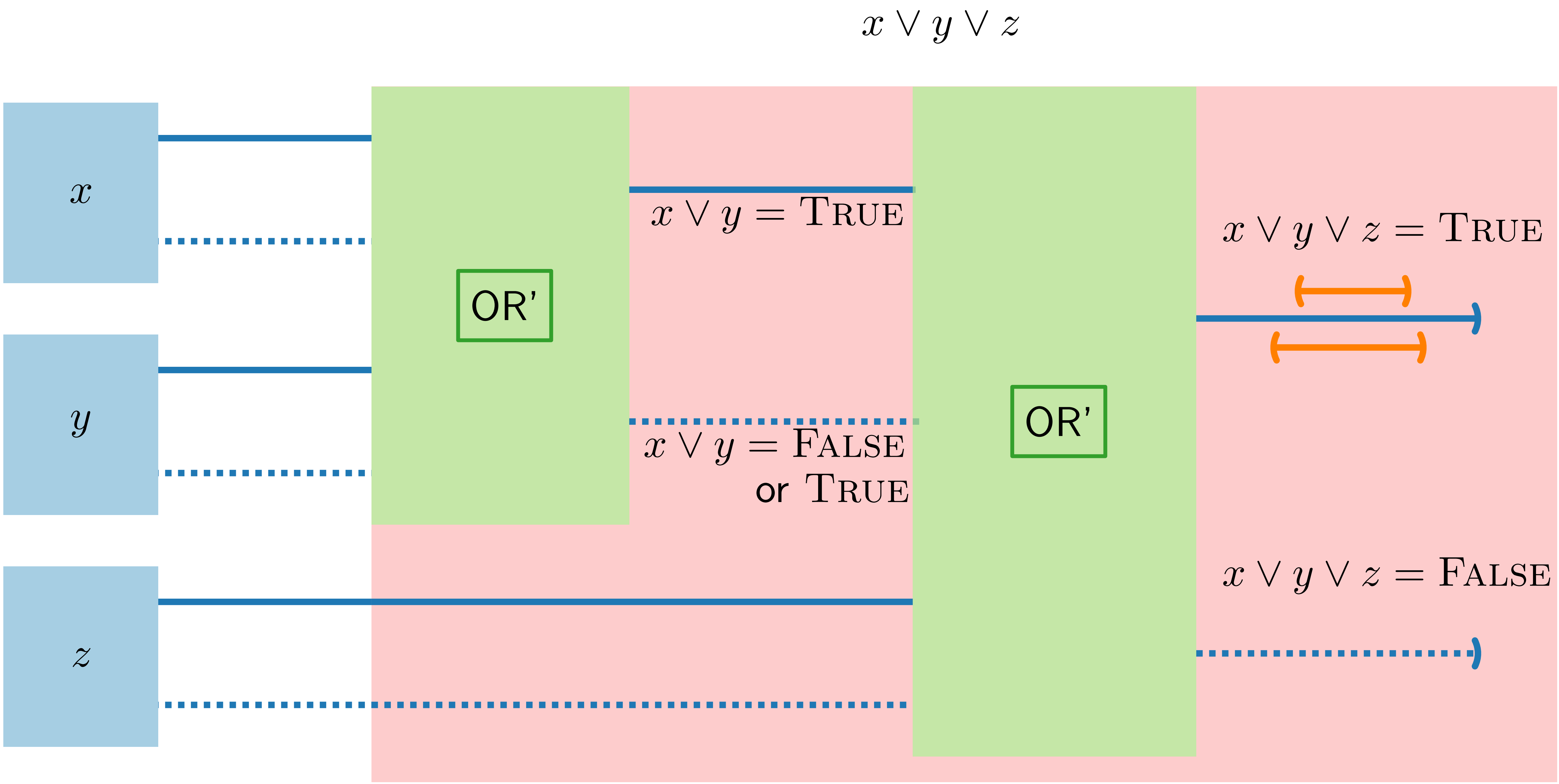

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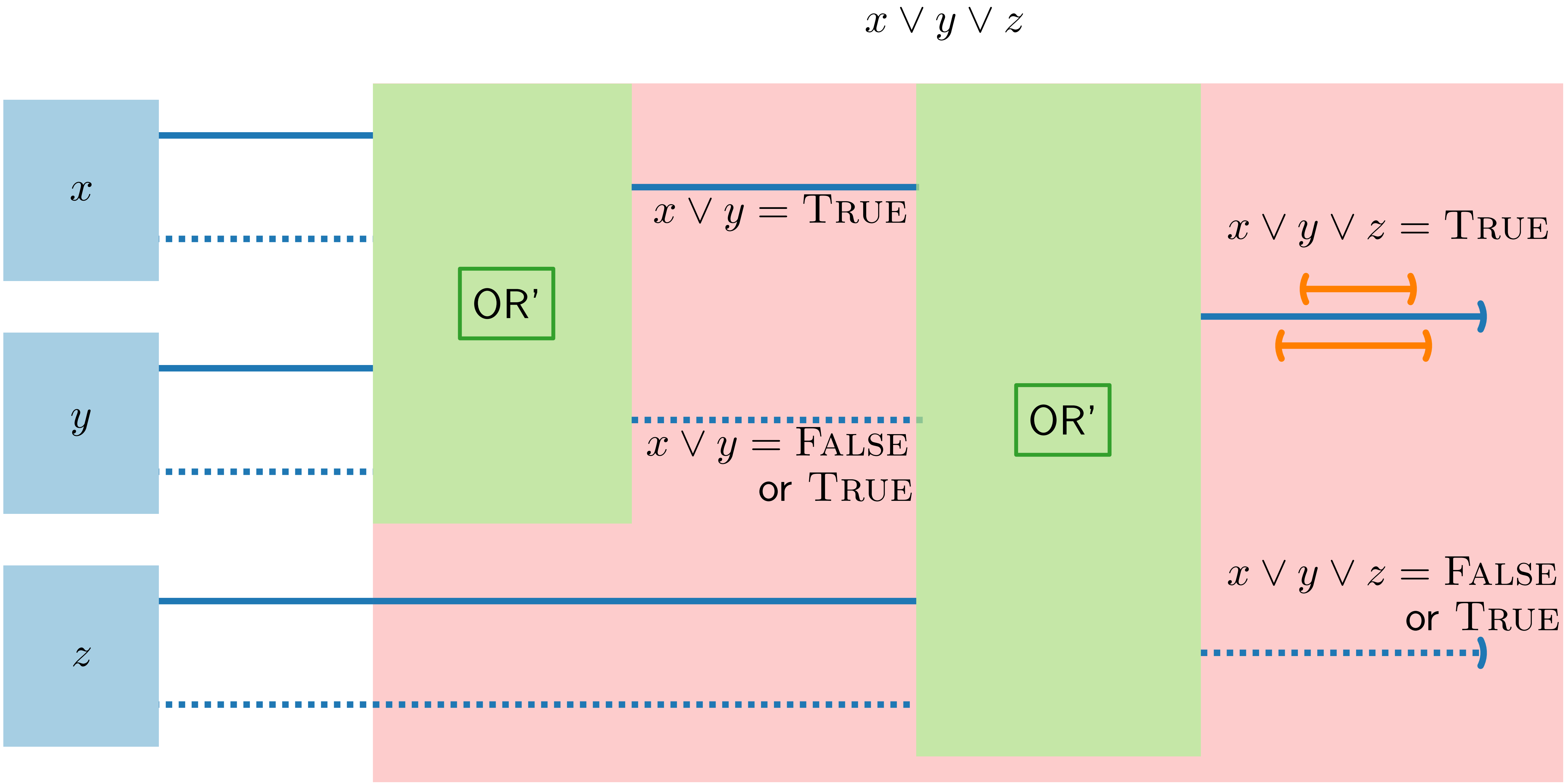

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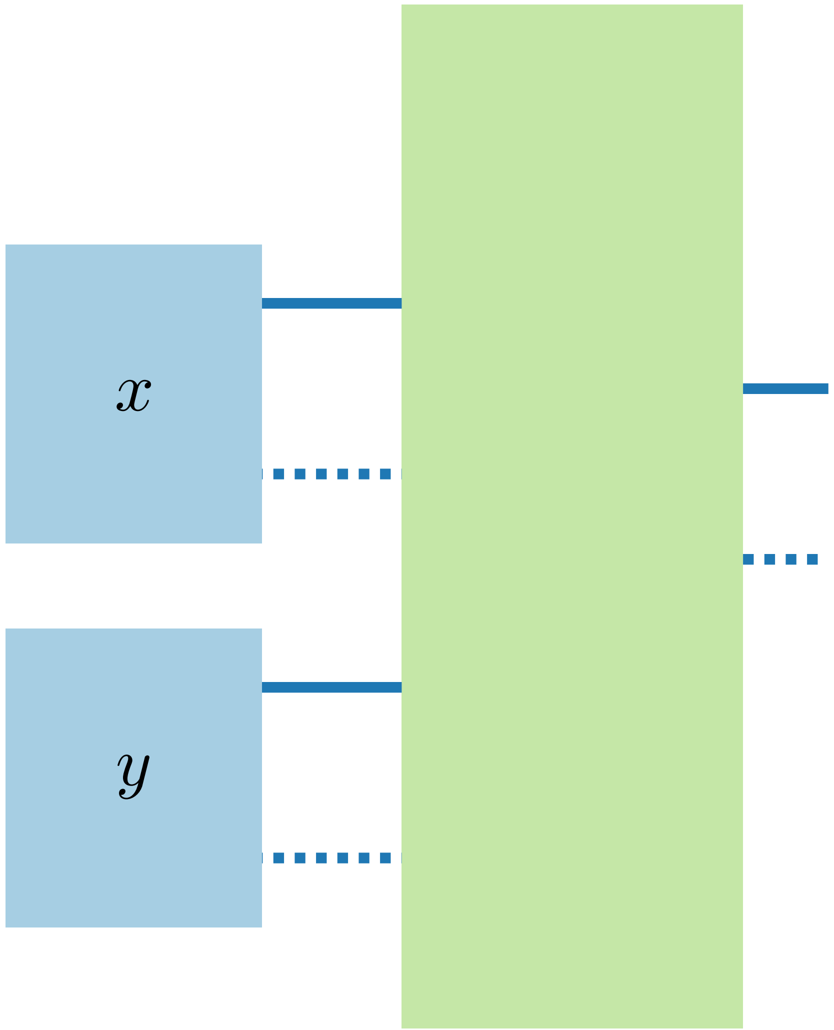

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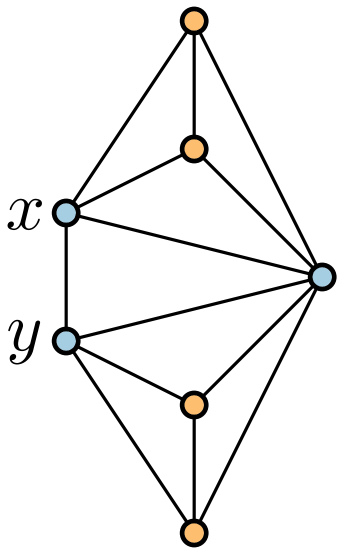

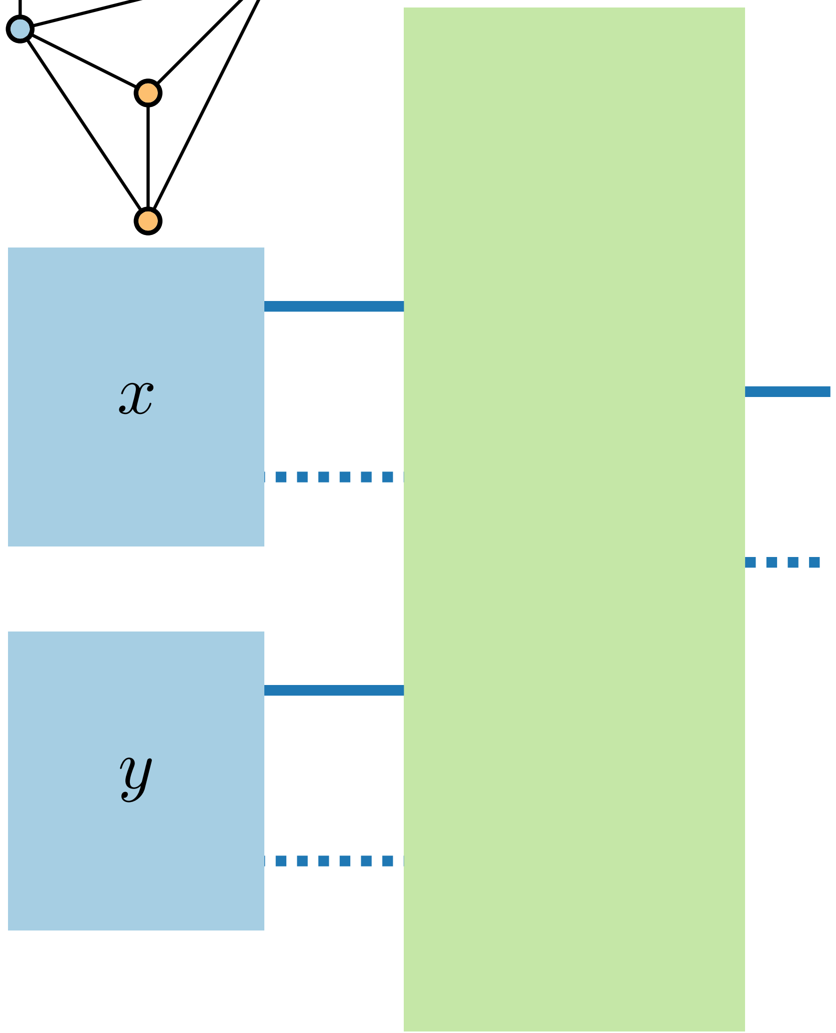

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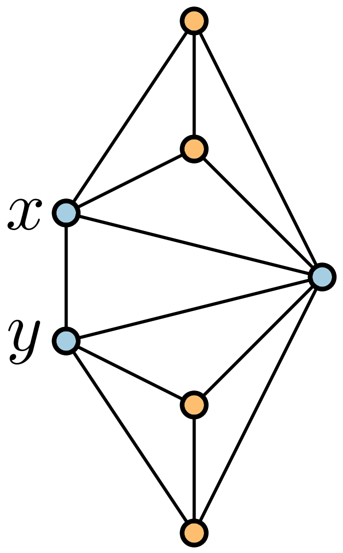

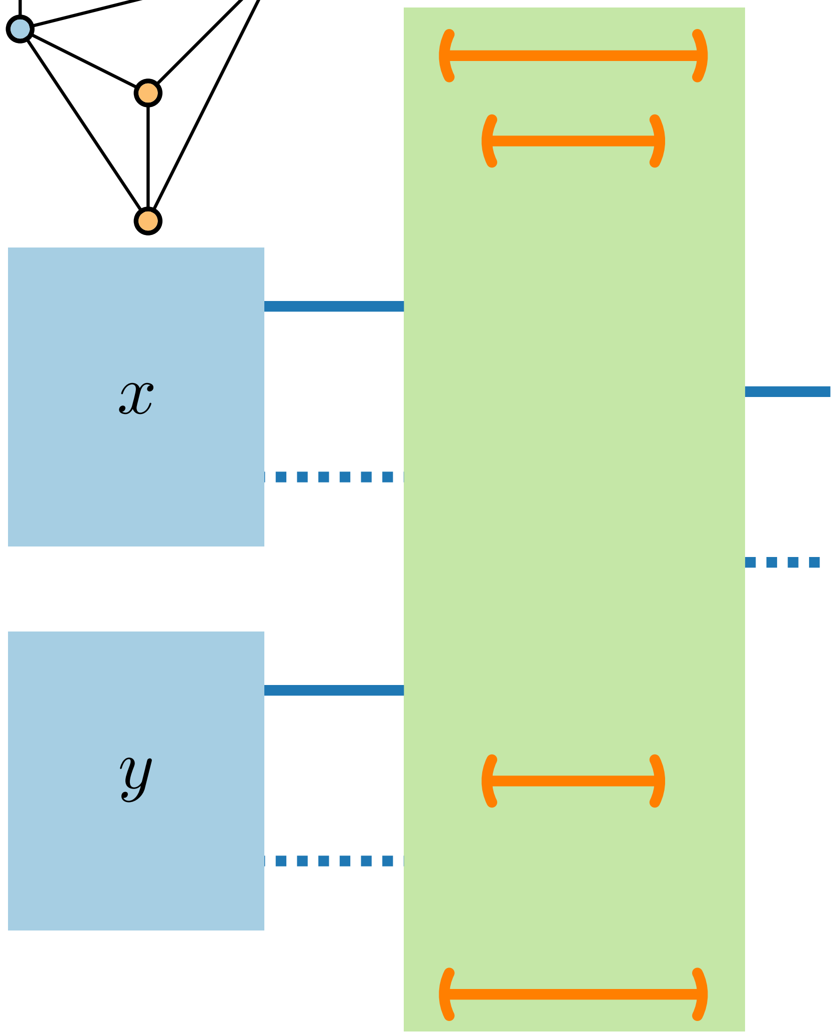

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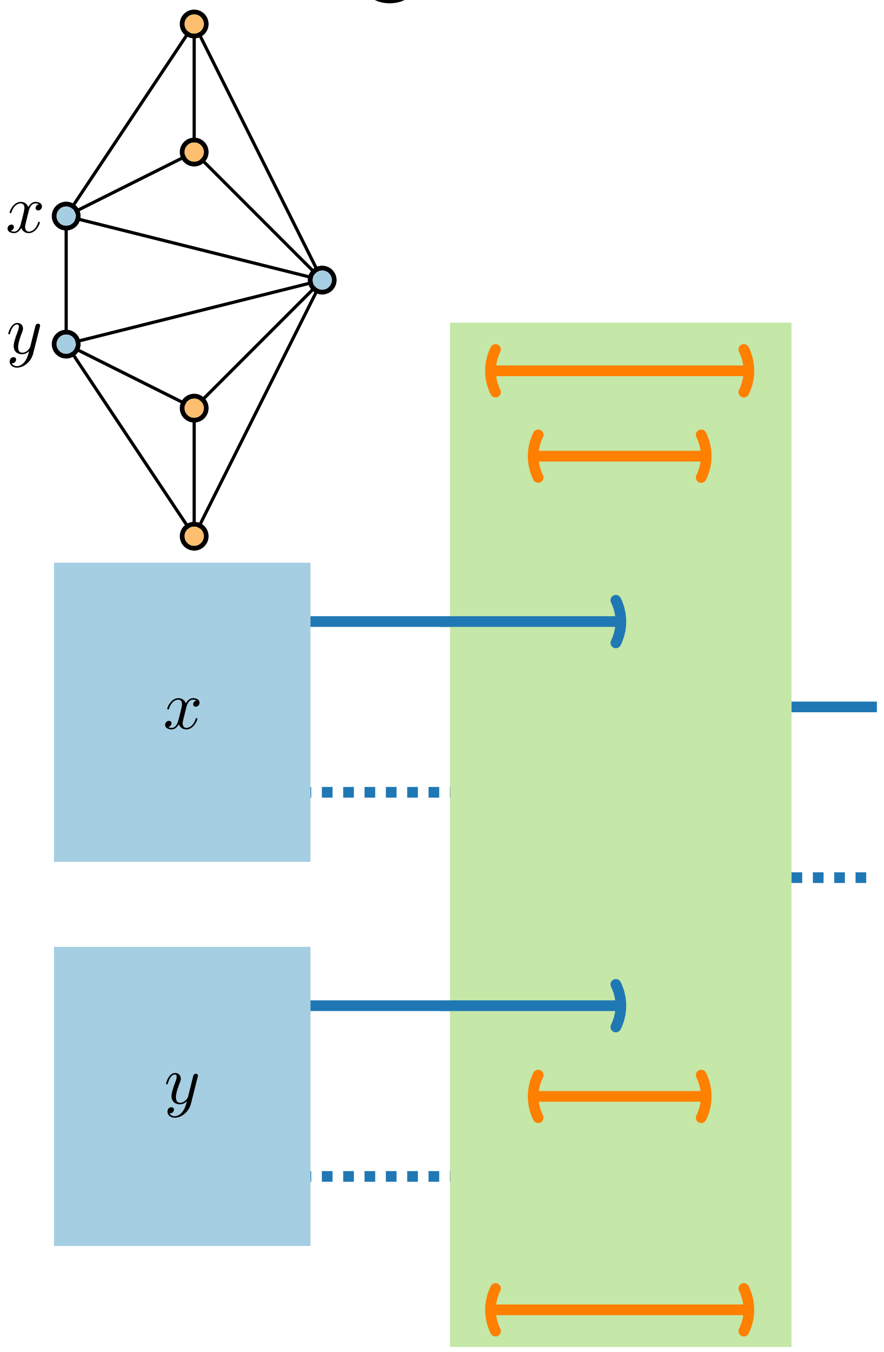

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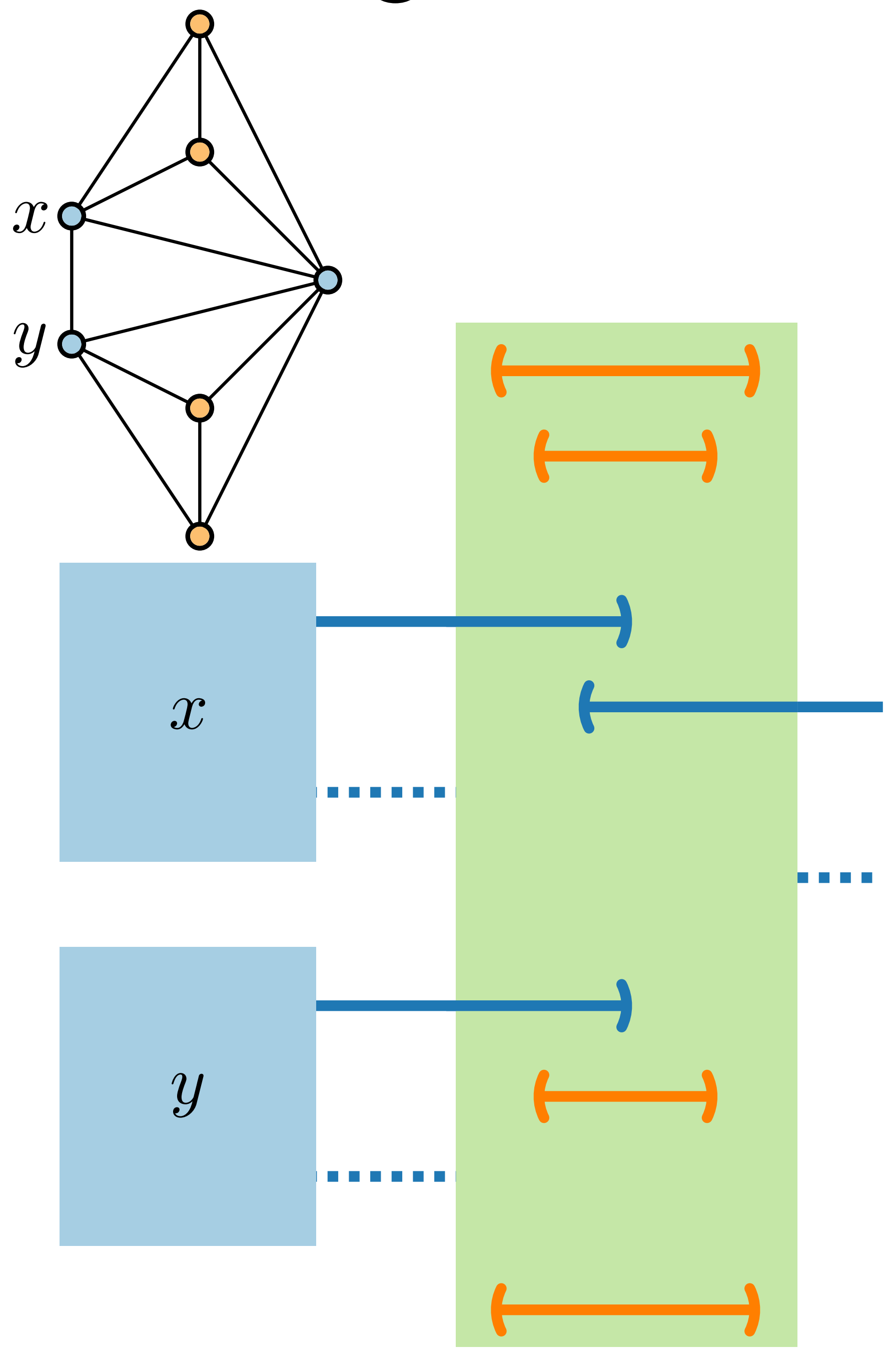

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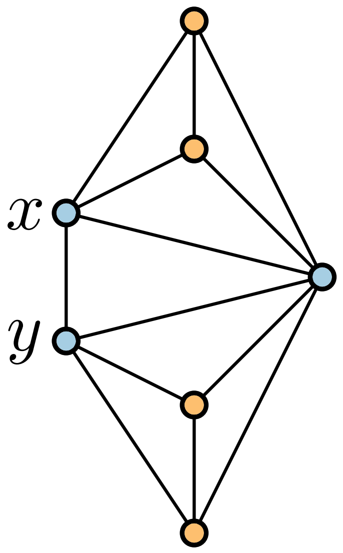

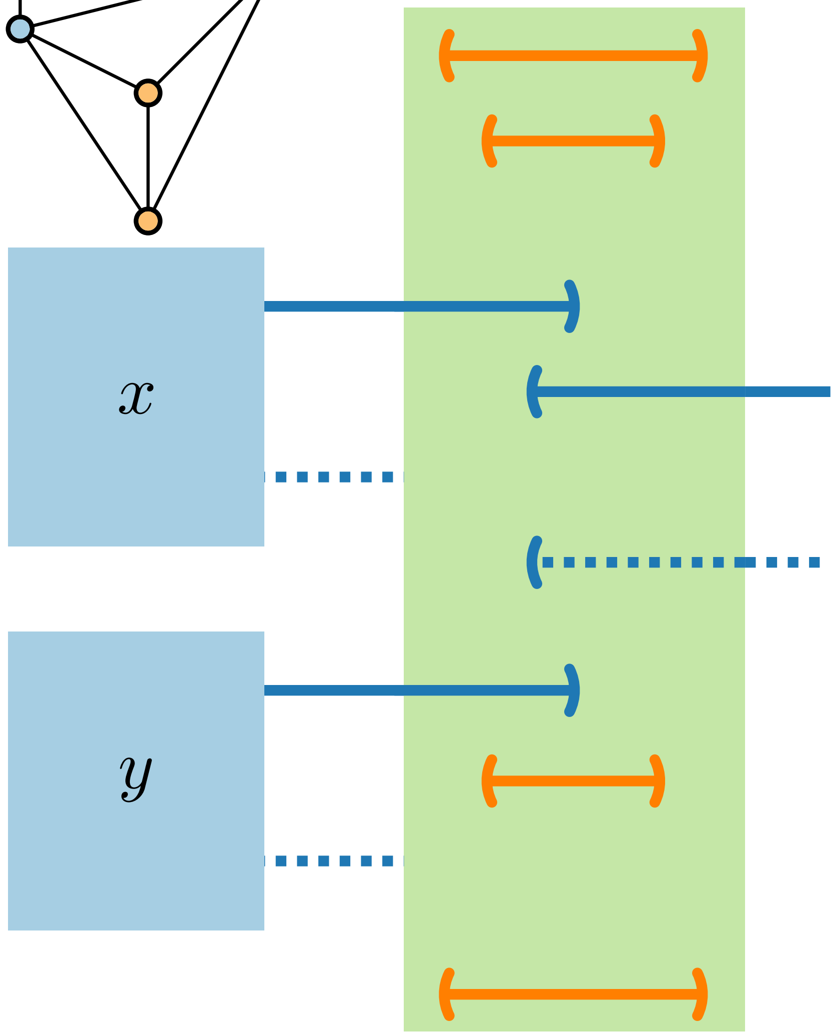

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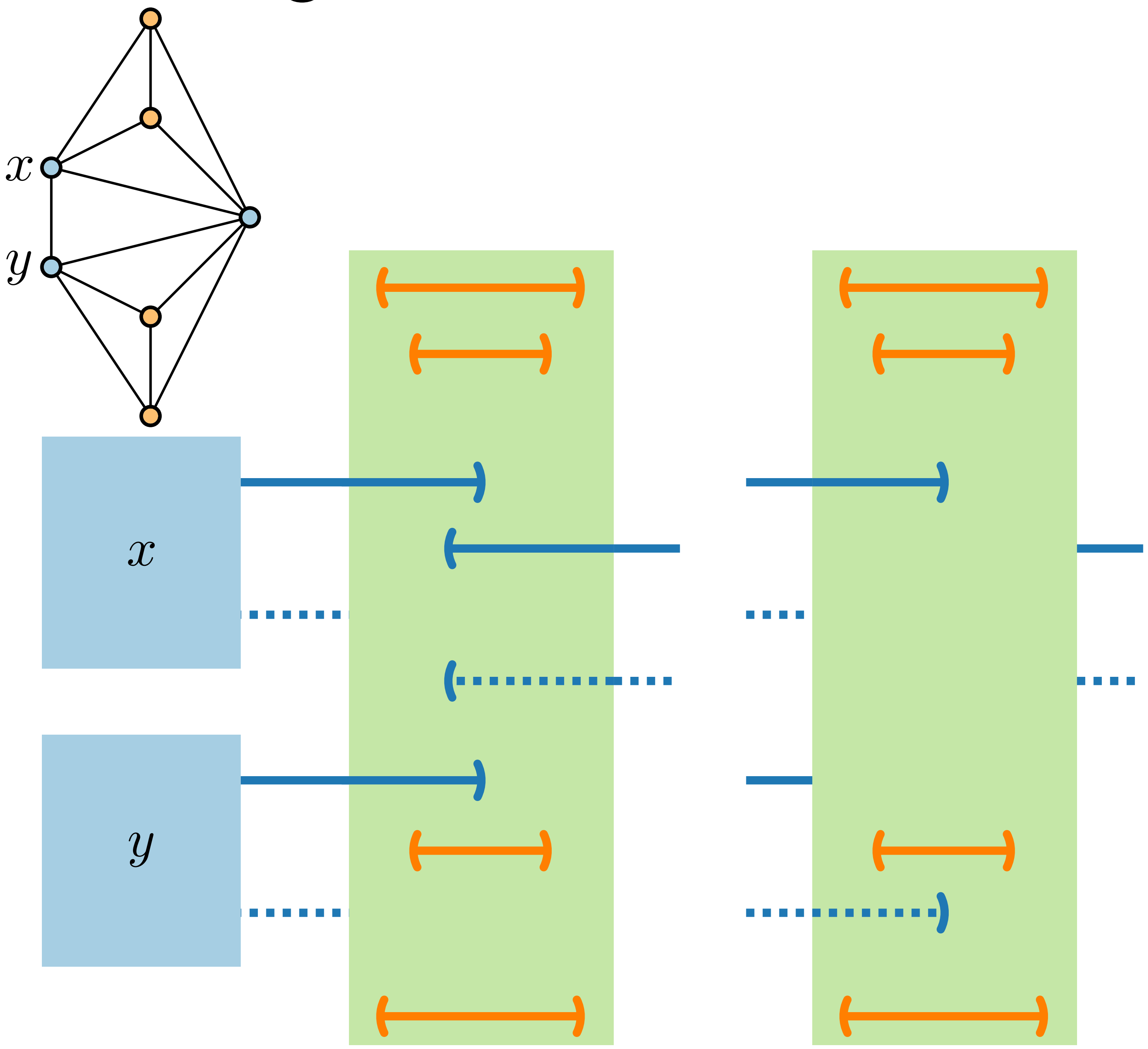

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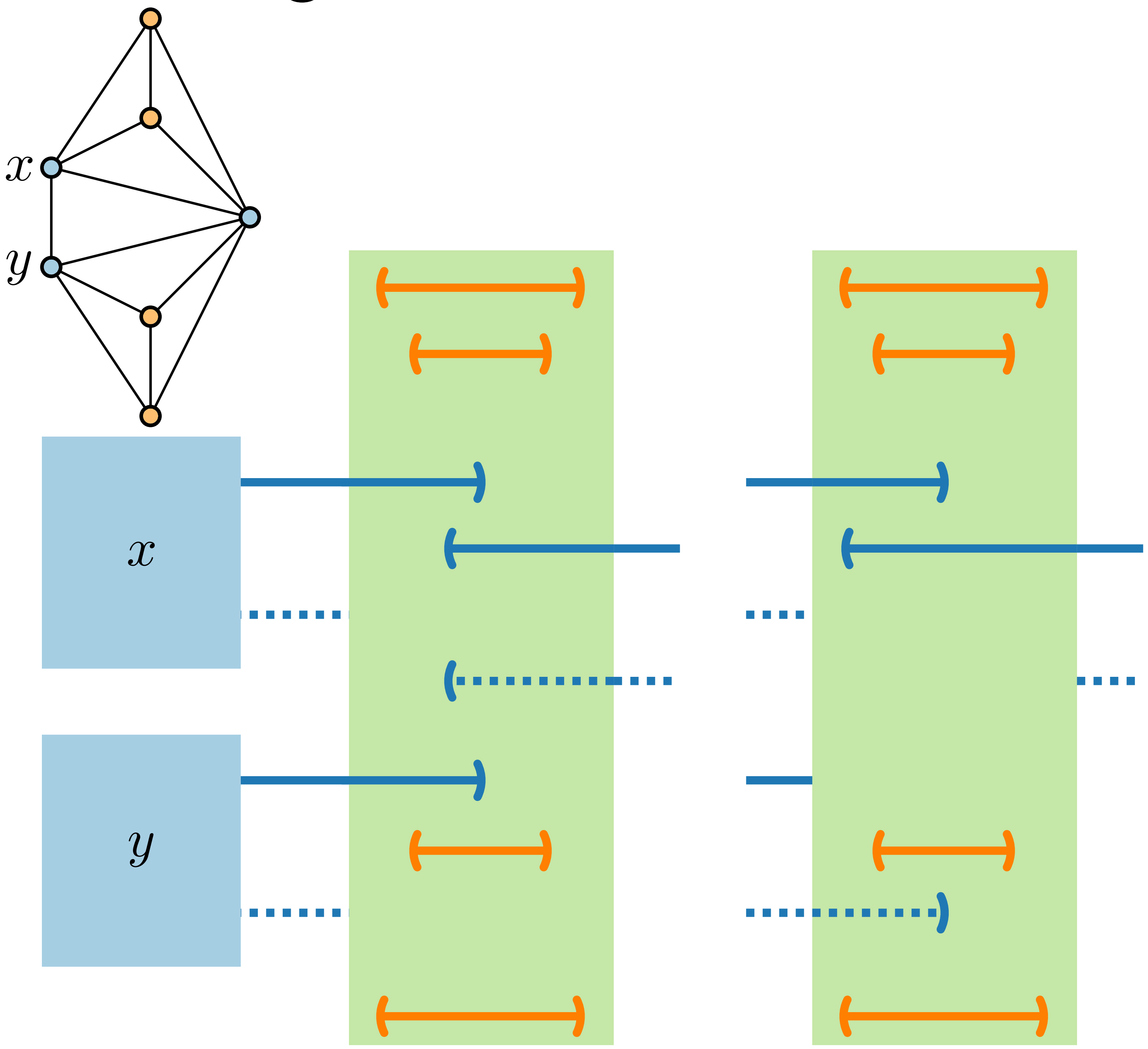

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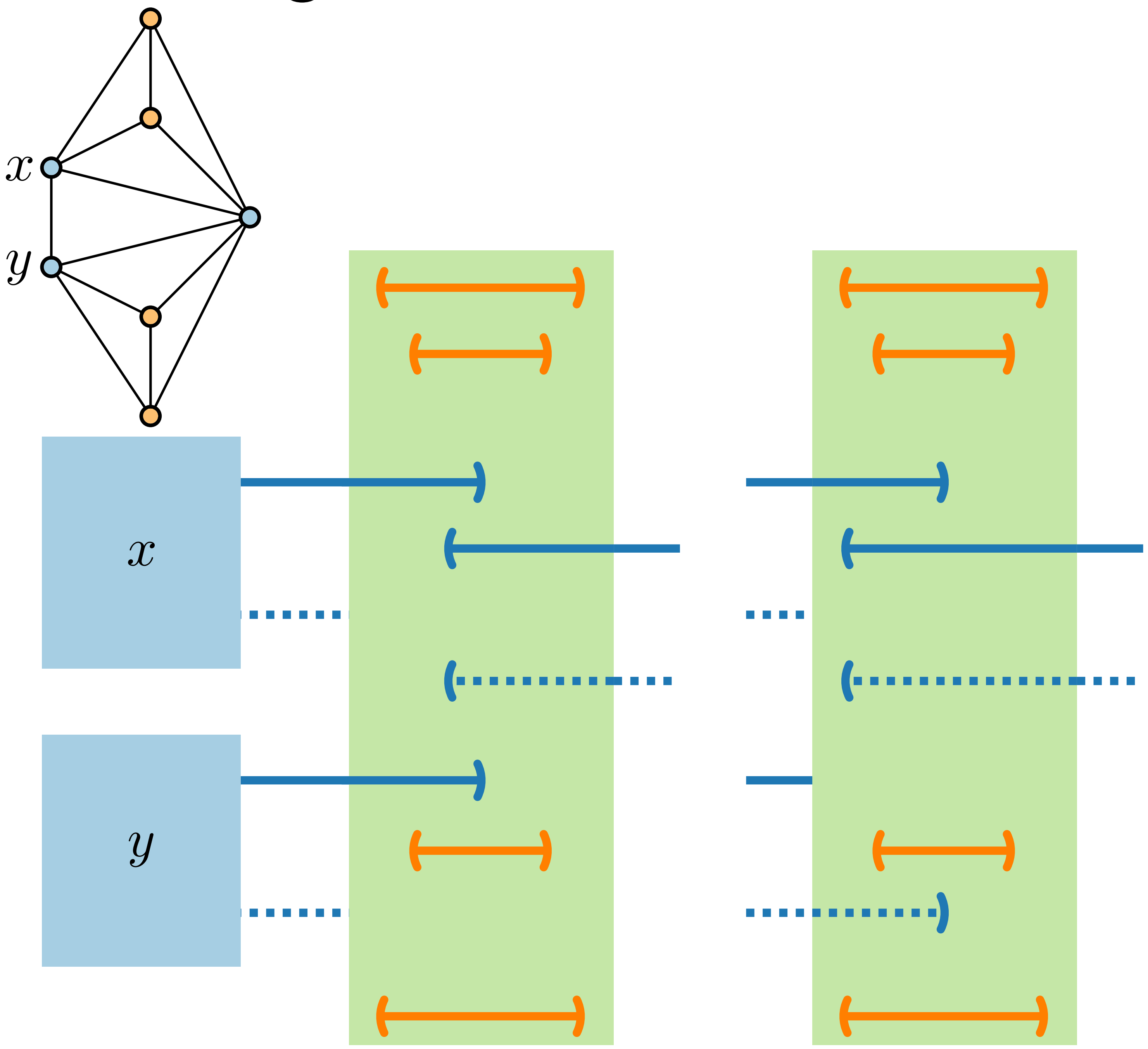

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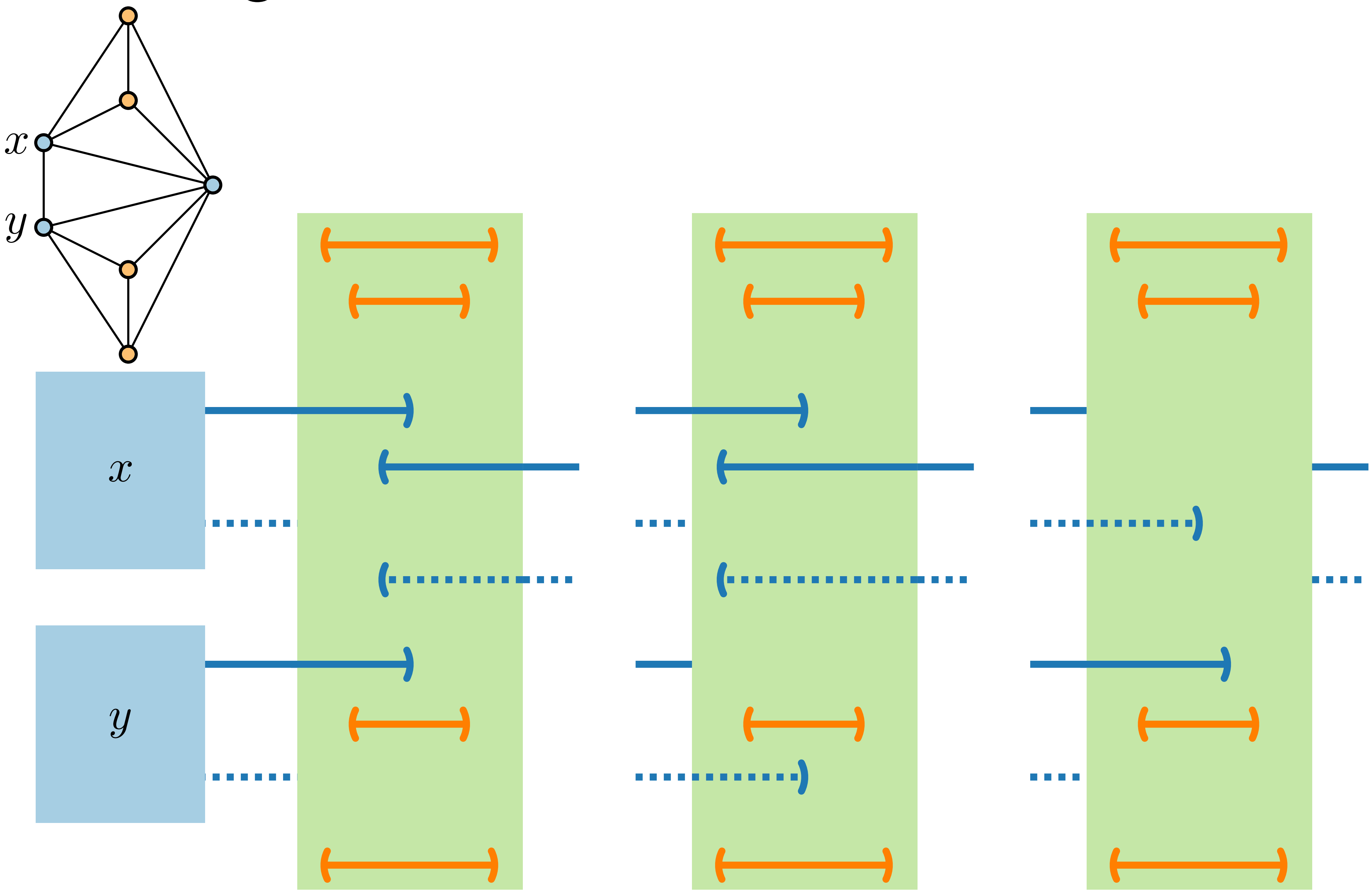

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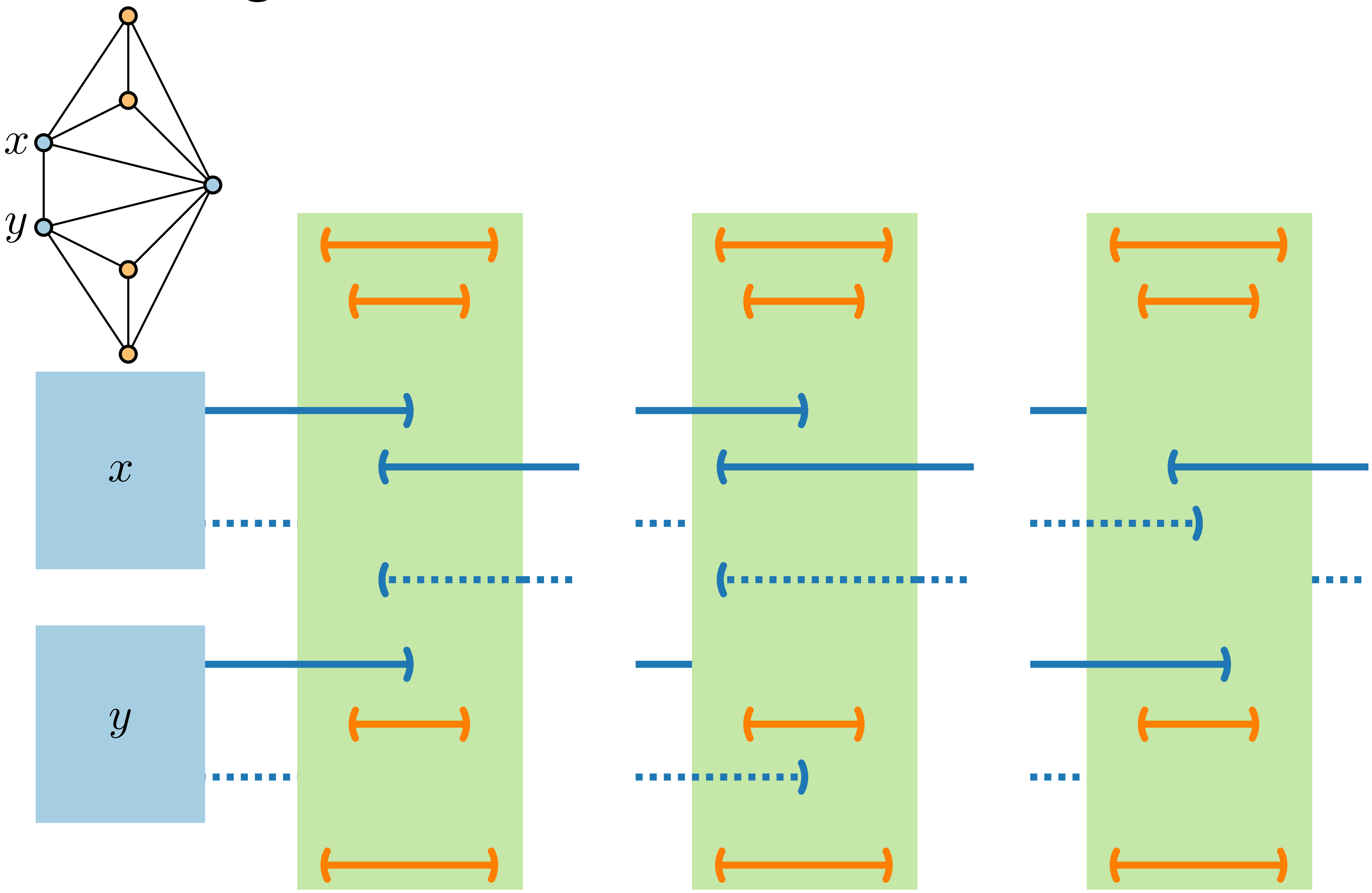

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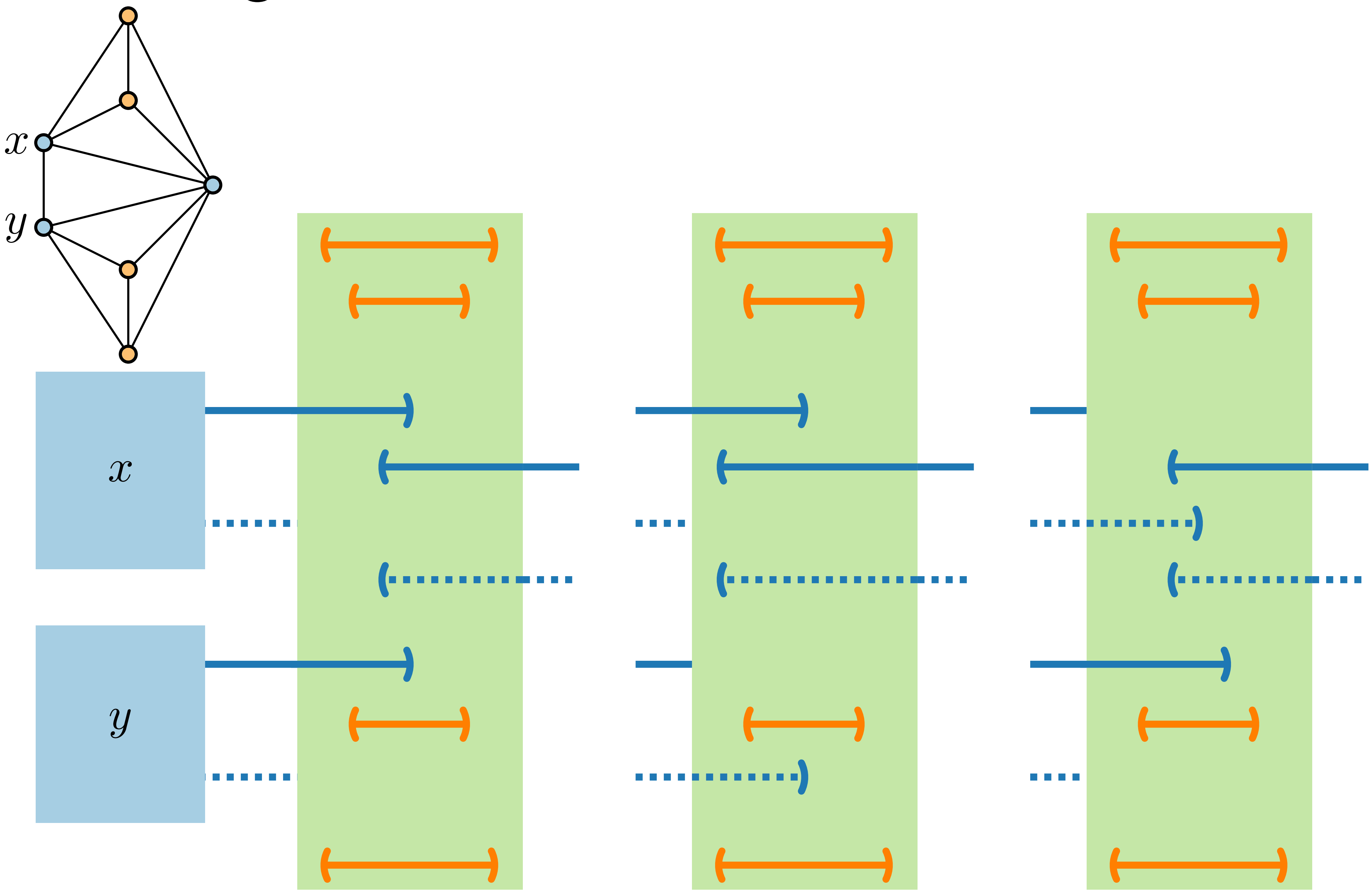

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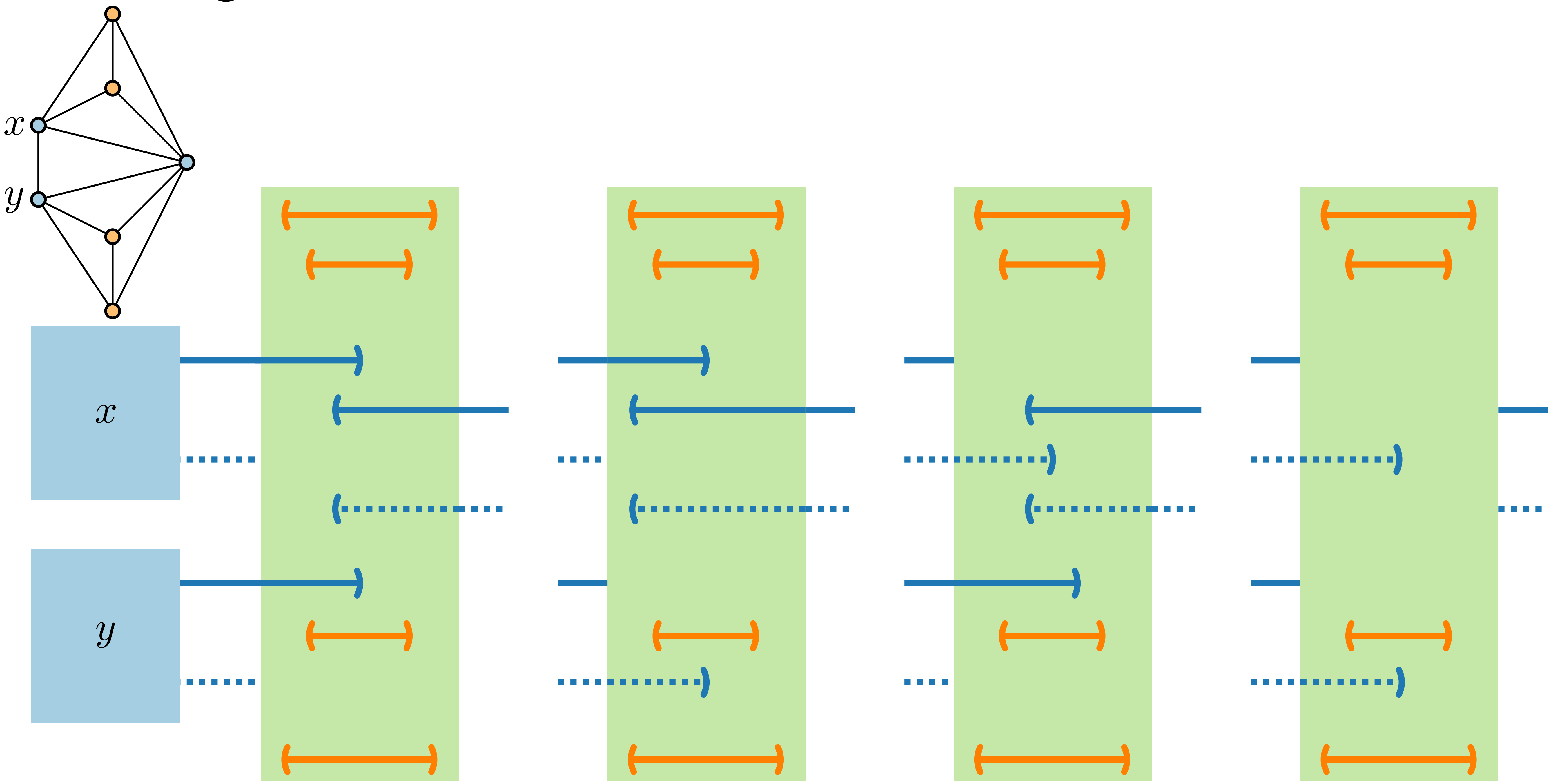

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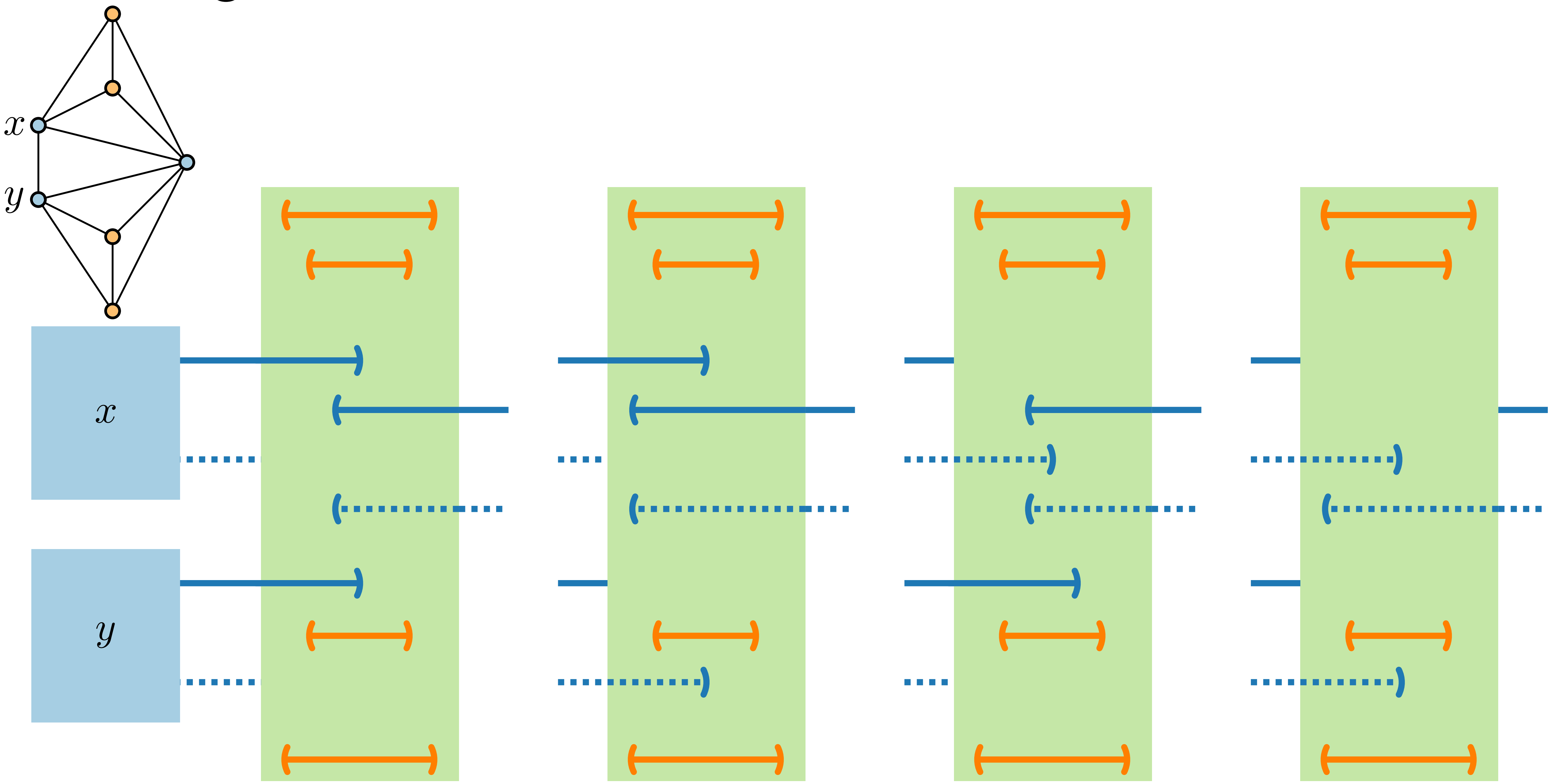

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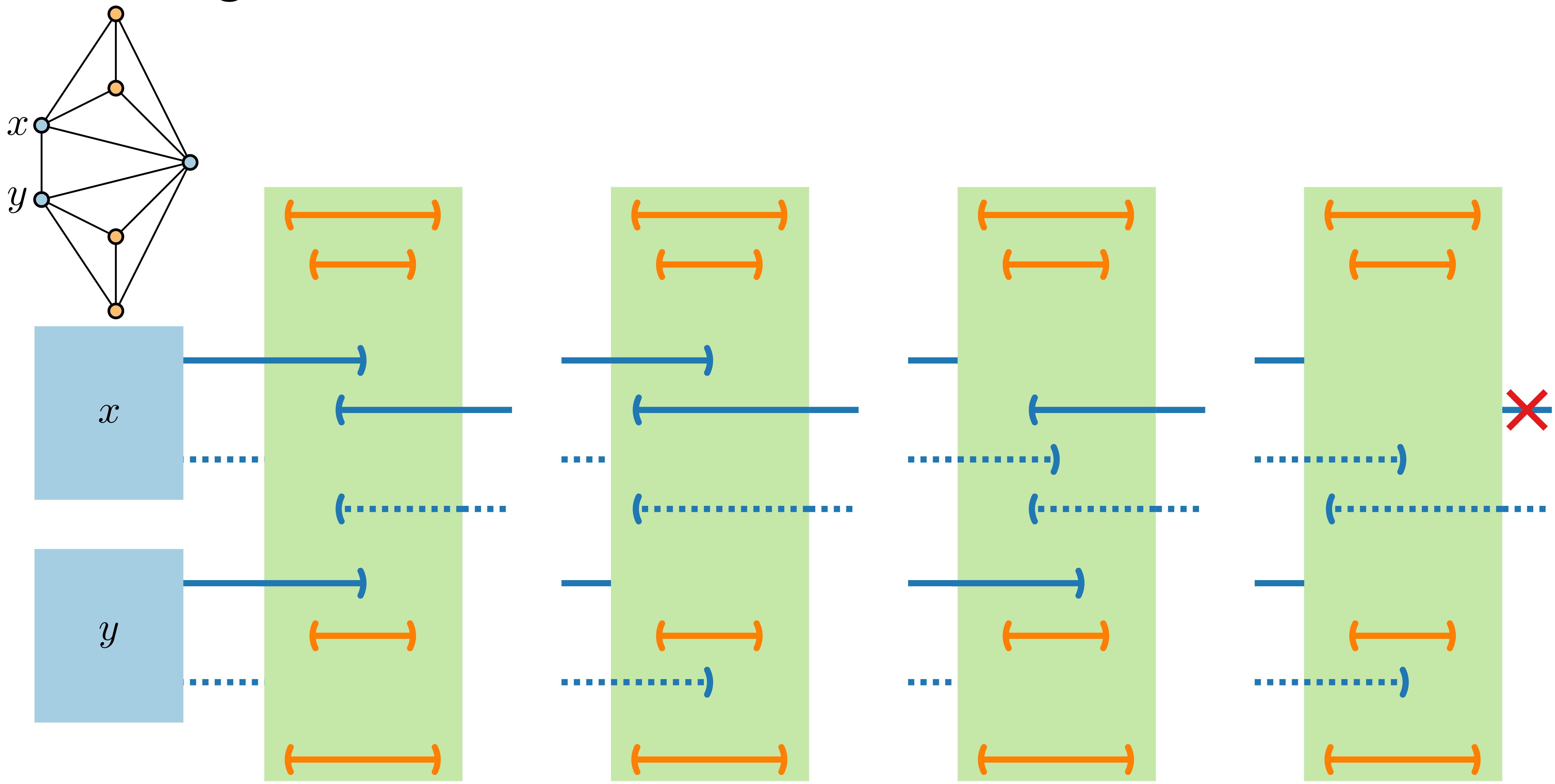

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Open Problems:

- Can ~~*rectangular*~~ ε-bar visibility representation extension be solved in polynomial time for st-graphs? For DAGs?

Discussion

- *Rectangular* ε-bar visibility representation extension can be solved in $O(n \log^2 n)$ time *for st-graphs*.
- ε-bar visibility representation extension is NP-complete.
- ε-bar visibility representation extension is NP-complete for (series-parallel) st-graphs when restricted to the *integer grid* (or if any fixed $\varepsilon > 0$ is specified).

Open Problems:

- Can ~~*rectangular*~~ ε-bar visibility representation extension be solved in polynomial time for st-graphs? For DAGs?

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Open Problems:

- Can ~~*rectangular*~~ ε-bar visibility representation extension be solved in polynomial time for st-graphs? For DAGs?
- Can *strong* bar visibility recognition / representation extension be solved in polynomial time for st-graphs?

Literature

Main source:

- [Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18]
  The Partial Visibility Representation Extension Problem

Referenced papers:

- [Tamassia, Tollis '86] Algorithms for visibility representations of planar graphs
- [Wismath '85] Characterizing bar line-of-sight graphs
- [Chaplick, Dorbec, Kratochvíl, Montassier, Stacho '14]
  Contact representations of planar graphs: Extending a partial representation is hard
- [Andreae '92] Some results on visibility graphs
- [Garg, Tamassia '01]
  On the Computational Complexity of Upward and Rectilinear Planarity Testing
- [Gutwenger, Mutzel '01] A Linear Time Implementation of SPQR-Trees
- [de Berg, Khosravi '10] Optimal Binary Space Partitions in the Plane