Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, January 30, 2026

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Homework Assignment #12
Approximation Algorithms (Winter Semester 2025/26)

Exercise 1 – Heuristics for STEINER FOREST

In the lecture, we considered the following two heuristics for STEINER FOREST. Show that both of them can return arbitrarily bad solutions (in terms of approximation factor). Let G be the given graph, let $c\colon E(G) \to \mathbb{N}$ be the given edge costs, and let R be the given set of terminal pairs.

a) For each pair of terminals in R, compute a shortest path that connects the two terminals. Return the union of the paths. **[3 points]**

b) Compute a Steiner tree for the terminal set $\bigcup R$.

   Full points are given only if you show that the approximation guarantee is arbitrary bad even if G is a complete metric graph. **[4 points]**

Exercise 2 – Naive Primal–Dual Approach

Consider the first primal-dual algorithm PRIMALDUALSTEINERFORESTNAIVE from the lecture. Construct an instance for which PRIMALDUALSTEINERFORESTNAIVE returns a solution with approximation factor greater than 2. **[5 points]**

Exercise 3 – Special cases of STEINER FOREST

Given a graph G, edge costs $c\colon E(G) \to \mathbb{N}$, and a set R of terminal pairs, show that the algorithm PRIMALDUALSTEINERFOREST(G, c, R) returns an optimal solution if

a) $R = V \times V$, **[4 points]**

b) $R = \{s, t\}$ for some $s, t \in V$. **[4 points]**

Please hand in your solutions via WueCampus until 10:00 on Friday, February 6.