Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, Dezember 19, 2025

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Homework Assignment #9
Approximation Algorithms (Winter Semester 2025/26)

Exercise 1 – Euclidean TSP on an Integer Grid

In the lecture, the following simplifying assumptions for Euclidean TSP were made: All n input points lie within a square of side length $L = 4n^2$. Further, all points have integer coordinates.

Show that these assumptions are justifiable, or more precisely: Show that if a polynomial approximation scheme (PTAS) for the restricted version of Euclidean TSP above exists, then there exists also a PTAS for the unrestricted version.

a) First, explain how your algorithm processes an instance I_u of the unrestricted version. Your algorithm can make use of the approximation scheme of the restricted version for that.

[5 points]

b) Next, show that your algorithm yields a tour of length at most $(1 + \varepsilon)\,\mathrm{OPT}_u$ for any $\varepsilon > 0$, where OPT_u is the length of a shortest tour for I_u.

[7 points]

Hint: It suffices to prove the statement for instances with many points, i.e., for all $n \geq f(\varepsilon)$ where f is an arbitrary function. Further, it suffices to show that your algorithm yields a tour of length at most $(1 + 2\varepsilon)\mathrm{OPT}_u$ for any $\varepsilon > 0$.

Exercise 2 – Well-Behaved Tours

Let I be an instance of the Euclidean TSP that satisfies the assumptions from Exercise 1. We consider the basic dissection for this instance. Show that there exists a shortest well-behaved tour that visits each portal of the basic dissection at most twice.

[8 points]

Hint: Consider a portal p on a grid line l that is visited more than twice by a shortest well-behaved tour. This portal splits the tour into *ears*, which are the maximum partial tours between two visits of p. Then apply shortcuts.

Please hand in your solutions via WueCampus until 10:00 on Friday, January 16.