Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, December 12, 2025

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Homework Assignment #8
Approximation Algorithms (Winter Semester 2025/26)

Exercise 1 – Greedy for the KNAPSACK Problem

Consider the following greedy approach for the KNAPSACK problem: Iterate over the objects in descending order according to the ratio between profit and size, i. e., $\frac{\mathrm{profit}(a_1)}{\mathrm{size}(a_1)} \geq \ldots \geq \frac{\mathrm{profit}(a_n)}{\mathrm{size}(a_n)}$. Add the current object to the knapsack if there is enough space left.

Show that this approach does not lead to a constant approximation factor.

[5 points]

Exercise 2 – 1/2-Approximation for the KNAPSACK Problem

Consider the following algorithm for the KNAPSACK problem: Let $a_1, \ldots, a_n$ be the objects in descending order according to the ratio between profit and size. Assume that each individual object fits in the knapsack. Now determine the smallest index k such that the total size of the first k objects exceeds the capacity of the knapsack. Finally, return the better of the two solutions $\{a_1, \ldots, a_{k-1}\}$ and $\{a_k\}$.

Show that this algorithm is a $1/2$-approximation.

[7 points]

Exercise 3 – MINIMUM MAKESPAN SCHEDULING on Two Machines

Consider the following problem: Given n jobs with runtimes $t_1, \ldots, t_n \in \mathbb{N}$, distribute them over two machines such that the total makespan is minimum. More formally, we search for a partitioning of $T := \{t_1, \ldots, t_n\}$ into two sets M_1 and M_2 such that $t(M_1, M_2) := \max\{\sum M_1, \sum M_2\}$ is minimized.

Develop an FPTAS for this problem.

[8 points]

Suggestion: First, develop an exact algorithm with pseudo-polynomial runtime. Then, scale and round accordingly.

Please hand in your solutions via WueCampus until 10:00 on Friday, December 19.