Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, July 18, 2025

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Exercise Sheet #11

Graph Visualization (SS 2025)

Exercise 1 – Crossing ratio of k-planar multigraphs

In the lecture, we have shown that the crossing ratio of 1-planar graphs is $\rho_{1-pl}(n) = (n-2)/2$ by constructing a family of graphs with crossing number 2 and 1-planar crossing number $n-2$.

Find a family of k-planar multigraphs (i.e., there can be multiple copies of every edge) such that the crossing ratio is $k(n-2)/2$. **4 Points**

Exercise 2 – 1-planar orthogonal drawings

Biedl & Kant[1] gave a linear-time algorithm that constructs for every plane maxdeg-4 graph (except the octahedron) an orthogonal drawing with at most 2 bends per edge.

a) Devise a linear-time algorithm to create, for a given embedded maxdeg-4 1-planar graph, an orthogonal 1-planar drawing. How many bends does the drawing at most require with respect to n? You can ignore the octahedron for now.
5 Points

b) Show that the octahedron admits an orthogonal 1-planar drawing that does not exceed this number of bends. **3 Points**

Exercise 3 – Straight-line drawings of optimal 1-planar graphs with one bend

In the lecture, we have seen that a 1-planar graph G that admits a 1-planar straight-line drawing can have at mots $4n-9$ edges. Hence, an optimal 1-planar graph cannot have a 1-planar straight-line drawing. However, if we allow a single bend (in total, not per edge), we can draw every optimal 1-planar graph.

a) Does this imply that every 1-planar graph can be drawn with a single bend?
2 Points

b) Construct a straight-line drawing with a single bend of the optimal 1-planar graph on 8 vertices. **4 Points**

[1] Therese C. Biedl, Goos Kant: **A better heuristic for orthogonal graph drawings.** Comput. Geom. 9(3): 159–180 (1998)

Exercise 4 – Decomposition of IC-planar graphs

Prove that every IC-planar graph can be decomposed into a planar graph and a matching, that is, there is a 2-coloring of the edges where one color induces a planar graph and the other color induces a matching. **2 Points**

This assignment is due at the beginning of the next lecture, that is, on July 25 at 10:15. Please submit your solutions via WueCampus. Questions can be asked in the tutorial session on July 23 at 16:00. The solutions will not be discussed.