Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #10

Graph Visualization (SS 2025)

Exercise 1 – Visibility representations

Let $x > 0$, let $y > 0$, and let $R = [0, x] \times [0, y] \subset \mathbb{R}^2$ be an axis-parallel rectangle in the plane that contains a set S of pairwise disjoint horizontal line segments. Let the line segments $[0, x] \times \{0\}$ and $[0, x] \times \{y\}$ be contained in S.

Let G be the directed graph with vertex set S and arc set

$E = \{(u, v) \mid v \text{ is above } u \text{ and there is a vertical line of sight of width} > 0 \text{ between them}\}$.

a) Show that G is an st-graph. **2 Points**

b) Show that G is upward planar. **2 Points**

Exercise 2 – Computing coordinates for a visibility representation

We want to compute an ε-bar visibility representation ψ of an st-graph G for a given fixed $\varepsilon > 0$. (Recall that an st-graph is an embedded planar graph.) In addition to G, we are also given a minimum vertical distance between each pair of bars that correspond to adjacent vertices and a minimum width for each bar. More precisely, we are given a function $h\colon E(G) \to \mathbb{R}^+$ that returns, for an edge (u, v) of G, a lower bound for the vertical distance of the bars $\psi(u)$ and $\psi(v)$. We are also given a function $w\colon V \to \mathbb{R}^+$ that returns, for a vertex v of G, a lower bound for the width of the bar $\psi(v)$.

a) Give a linear-time algorithm in pseudocode that computes the y-coordinates of the bars in a visibility representation of minimum height. **6 Points**

b) To obtain a bar visibility representation, it remains to determine the x-coordinates. Describe a linear-time algorithm that computes the x-coordinates of the bars in a visibility representation of minimum width. **6 Points**

Exercise 3 – Simplifying the NP-hardness construction

Analyze the NP-hardness reduction that we discussed in the lecture. Is it possible to remove the topmost orange bar in the OR'-gadget? **4 Points**

This assignment is due at the beginning of the next lecture, that is, on July 18 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on July 16 at 16:00 and the solutions will be discussed one week after that on July 23.