Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, June 20, 2025

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Exercise Sheet #8

Graph Visualization (SS 2025)

Exercise 1 – MINIMUM FEEDBACK (ARC) SET

Let G be a directed graph. For a set $E' \subseteq E(G)$, let $E'_{\mathrm{rev}} := \{vu \mid uv \in E'\}$ be the set of reversed edges. A minimum-cardinality set $E^\star \subseteq E(G)$ is called

- a MINIMUM FEEDBACK ARC SET if $G_{\mathsf{FAS}} = (V(G), E(G) \setminus E^\star)$ is acyclic;
- a MINIMUM FEEDBACK SET if $G_{\mathsf{FS}} = (V(G), (E(G) \setminus E^\star) \cup E^\star_{\mathrm{rev}})$ is acyclic.

Show that for any set $E^\star \subseteq E(G)$ it holds that $E^\star$ is a MINIMUM FEEDBACK SET if and only if $E^\star$ is a MINIMUM FEEDBACK ARC SET. **6 Points**

Exercise 2 – Optimal one-sided crossing minimization

We consider the problem of one-sided crossing minimization, i.e., we are given a bipartite graph G, where $V(G) = L_1 \cup L_2$, with a permutation π_1 of L_1, and we search for a permutation π_2 of L_2 that minimizes the number of crossings.

Suppose that, for π_1, there exists a permutation $\pi_2^\star$ of L_2 such that no two edges cross.

a) Show that in this case the *barycenter heuristic* also yields a permutation π'_2 that results in no crossings. **3 Points**

b) Show that in this case the *median heuristic* also yields a permutation π''_2 that results in no crossings. **3 Points**

Exercise 3 – Planar drawings

Let G be an upward-planar graph. Does the Sugiyama framework always yield an upward-planar drawing of G if we use, for the layering, the recursive linear-time algorithm to minimize the number of layers, and then the median heuristic for crossing minimization, which finds a non-crossing solution if such a solution exists?

Justify your answer. **4 Points**

Exercise 4 – Precedence-Constrained Multi-Processor Scheduling

Give an infinite class of instances for which the $(2 - 1/W)$-approximation algorithm for the scheduling problem PRECEDENCE-CONSTRAINED MULTI-PROCESSOR SCHEDULING from the lecture yields schedules of length $(2-1/W)$ OPT. It suffices to consider a fixed value for W; for example, $W = 2$. **4 Points**

This assignment is due at the beginning of the next lecture, that is, on July 4 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 25 at 16:00 and the solutions will be discussed on July 9.