Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, June 13, 2025

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Exercise Sheet #7

Graph Visualization (SS 2025)

Exercise 1 – Contact representations by line segments

Show that there exists a planar graph that does not admit a contact representation by line segments, even when the line segments can have any slopes and lengths. (In a contact representation of line segments, we insist that (i) no two line segments may overlap or intersect and (ii) no point in the plane is contained in more than two line segments.) **6 Points**

Exercise 2 – Contact representations by equilateral triangles

Show that there is a planar graph that does not have a contact representation by homothetic equilateral triangles, that is, equilateral triangles that only differ in scale and translation, but not in rotation. (E.g., all triangles have a horizontal edge and the third corner above this horizontal edge.) Note that a contact point is allowed to be contained in more than two triangles. **6 Points**

Exercise 3 – Square contact representations of maximal outerplanar graphs

Recall that a planar graph is *outerplanar* if it has a planar embedding such that all vertices are on the outer face. An outerplanar graph G is *maximal* outerplanar if, for every pair of distinct non-adjacent vertices u and v, $G + (u, v)$ is not outerplanar.

Let $(v_1, v_2, \ldots, v_n)$ be a canonical order of a maximal outerplanar graph G. That is, $V(G) = \{v_1, v_2, \ldots, v_n\}$, the graph G_i induced by $\{v_1, v_2, \ldots, v_i\}$ is a maximal outerplanar graph, and the following conditions hold:

- the edge (v_1, v_2) lies on the outer face; and
- for each $i \in \{3, 4, \ldots, n\}$, v_i has exactly two neighbors on the outer face of G_{i-1}.

Show that every maximal outerplanar graph has a contact representation by squares and that, using the canonical order, it can be computed in linear time. **8 Points**

This assignment is due at the beginning of the next lecture, that is, on June 20 at 10:15. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 11 at 16:00 and the solutions will be discussed one week after that on June 18.