Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, June 6, 2025

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Exercise Sheet #6

Graph Visualization (SS 2025)

Exercise 1 – Higher-degree vertices in orthogonal layouts

Let G be an arbitrary graph with an embedding $\mathcal{E}$. Our goal is to draw G orthogonally, while preserving the embedding, such that all vertices of degree greater than 4 are represented by rectangles instead of points. To achieve this, we replace every vertex v having $\deg(v) > 4$ by a ring of vertices $v_1, \ldots, v_{\deg(v)}$ such that the edges incident to v are distributed among the vertices $v_1, \ldots, v_{\deg v}$ (see figure below). The embedding $\mathcal{E}$ is modified accordingly during this step. Let G' with embedding $\mathcal{E}'$ be the result of this replacement step.

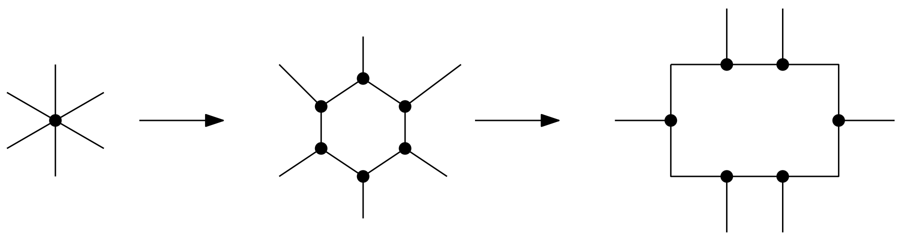

FIGURE 1: Replacement of a vertex of higher degree by a ring, which will be represented by a rectangle in the orthogonal drawing.

Modify the flow network by Tamassia such that it provides a bend-minimal orthogonal description of $(G', \mathcal{E}')$ in which every ring representing a vertex of higher degree is a rectangle with the property that (i) no vertices are placed in any of its four corners and (ii) every side of the rectangle contains at least one vertex. **6 Points**

Exercise 2 – From flow to orthogonal representation

Let G be a plane graph of vertex degree at most 4 with the embedding given by the set of faces F and the outer face f_0. Let X be a flow of cost k in the corresponding flow network $N(G)$. We consider the orthogonal description $H(G)$ belonging to X (as in the lecture).

Show that $H(G)$ fulfills property (H3) on the angle sum of the faces for orthogonal descriptions, that is, argue that

$$\sum_{r \in H(f)} C(r) = \begin{cases} -4 & \text{if } f = f_0 \\ +4 & \text{otherwise.} \end{cases}$$

holds for every face f. **4 Points**

Exercise 3 – Edge bending left and right in orthogonal representation

Let G be a plane graph with embedding $\mathcal{E}$, and let $H(G)$ be a bend-minimal orthogonal description of $(G, \mathcal{E})$. Is it possible that there exists an edge such that, in $H(G)$, it bends to the right as well as to the left?

Prove this claim (by giving an example) or disprove it (by showing that such an edge cannot exist). **3 Points**

Exercise 4 – Algorithm for the refinement step

The flow networks for compaction in the lecture only work for orthogonal representations where every face is rectangular. In the lecture, we learned how to refine the faces such that every face becomes rectangular.

Formulate the algorithm to do the refinement of an inner face. Write a method (ideally in commented pseudocode) that takes as input the face representation of a face f and returns an orthogonal representation of the refinement of the face. **7 Points**

This assignment is due at the beginning of the next lecture, that is, on June 13 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 11 at 16:00 and the solutions will be discussed one week after that on June 18.