Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #5

Graph Visualization (SS 2025)

Exercise 1 – Simple upward planar graphs

Prove or disprove that the following graph classes are upward planar:

a) directed acyclic graphs whose underlying undirected graph is a simple cycle; **2 Points**

b) directed acyclic graphs whose underlying undirected graph is a tree. **2 Points**

Exercise 2 – False friends

In the lecture we introduced three necessary conditions for a digraph G to be upward planar (planar, acyclic, bimodal). Show that these conditions are not sufficient. In other words, find a directed graph with an embedding that adheres to all three conditions and prove that the graph (not only this specific embedding) is *not* upward planar.
4 Points

Exercise 3 – Refinement of the outer face

Let G be a directed acyclic graph that has a set of global sources S and a set of global sinks T. Let G be given in a planar embedding with a set of faces F, and an outer face f_0. Let $\Phi\colon S \cup T \to F$ be a consistent assignment of the large angles of the sinks and sources to the incident faces. We consider the situation in which the inner faces have been refined already, i.e., there exist no large angles in the inner faces any more.

Show how you can complete the transformation into a planar st-graph by refining f_0.
4 Points

Exercise 4 – Upward planar drawings of series-parallel graphs

In contrast to general acyclic graphs, all series-parallel graphs have an upward planar drawing.

Describe an algorithm that generates for a series-parallel graph with a given embedding an upward planar drawing such that the embedding is preserved. In the output drawing, edges are allowed to have up to two bends, but all bends and vertices must have integer coordinates on a grid of polynomial size.

You can assume that the series-parallel graph is given with its decomposition tree.

Hint: Try to draw each edge such that it consists of three line segments where the middle segment is vertical.

a) Describe your algorithm. **4 Points**

b) Argue why your algorithm generates an upward planar drawing. **2 Points**

c) Give tight upper bounds on the width and the height of the drawings generated by your algorithm (with respect to m, the number of edges of the given graph). **2 Points**

This assignment is due at the beginning of the next lecture, that is, on June 6 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 04 at 16:00 and the solutions will be discussed one week after that on June 11.