Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, May 23, 2025

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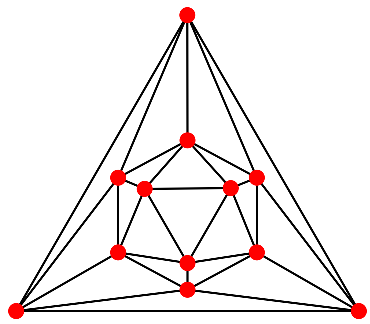
Exercise Sheet #4

Graph Visualization (SS 2025)

Exercise 1 – Schnyder realizer and Schnyder drawing for the icosahedron

Let G be the 1-skeleton of the icosahedron, i.e., the graph shown below.



a) Find a Schnyder labeling and the corresponding Schnyder wood of G. **3 Points**

b) Use the Schnyder wood from (a) to compute the Schnyder drawing of G on the $(2n-5) \times (2n-5)$ grid using the method of counting faces. **3 Points**

Exercise 2 – Fast construction of Schnyder realizer

In the lecture we have proven that every triangulated plane graph G has a Schnyder labeling and a Schnyder realizer. The proof yields a recursive algorithm: contract an edge $\{a, x\}$, find recursively a Schnyder forest in the resulting graph and then add x consistently back. A naive implementation of this algorithm yields a quadratic runtime, in particular, because we need to find the contracted edge. Explain how the algorithm can be improved to admit linear runtime.

Hint: Think about the candidate edges for contraction. How can you update them quickly? **5 Points**

Exercise 3 – Weak barycentric representations

Let G be a plane triangulated graph with a weak barycentric representation $v \in V(G) \mapsto (v_1, v_2, v_3) \in \mathbb{R}^3$. Let $A, B, C \in \mathbb{R}^2$ be points in general position.

Show that the function $f\colon v \in V(G) \mapsto v_1A + v_2B + v_3C$ yields a crossing-free drawing. **4 Points**

Exercise 4 – Fast calculation of barycentric coordinates

Let G be a triangulated plane graph with Schnyder realizer T_1, T_2, T_3. As in the lecture, for every inner vertex v of G and $i \in \{1, 2, 3\}$, let $v_i = |V(R_i(v))| - |V(P_{i-1}(v))|$, where $|V(R_i(v))|$ is the number of vertices in the region $R_i(v)$ (including the vertices on the boundary of $R_i(v)$ and v itself) and $|V(P_i(v))|$ is the number of vertices on the path from v to a_i in T_i.

Show that the values (v_1, v_2, v_3) can be calculated for every inner vertex v of G in linear total time.

Hint 1: Consider each $i \in \{1, 2, 3\}$ independently. (It suffices to consider v_1.)

Hint 2: Gather the necessary information by traversing T_1, T_2, and T_3. **5 Points**

This assignment is due at the beginning of the next lecture, that is, on May 30 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on May 28 at 16:00 and the solutions will be discussed one week after that on June 04.