Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, May 16, 2025

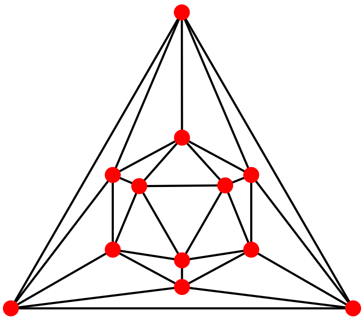
Prof. Dr. Alexander Wolff
Samuel Wolf

Exercise Sheet #3

Graph Visualization (SS 2025)

Exercise 1 – Canonical order and shift method for the icosahedron

Let G be the 1-skeleton of the icosahedron, i.e., the graph shown below.



a) Find a canonical order of G. **3 Points**

b) Draw G using the shift algorithm from the lecture. Show the intermediate drawings step by step. **5 Points**

Exercise 2 – Canonical orders for outerplanar graphs

A graph is *outerplanar* if it has a planar embedding such that all vertices are on the same face, usually the outer face. It is a *maximal outerplanar graph* if it is biconnected and internally triangulated.

Describe a special canonical order built precisely for maximal outerplanar graphs.

a) Reformulate the conditions (C1)–(C3) for maximal outerplanar graphs. That is, decide for each condition, whether it is necessary, can stay the same, or has to be adapted.

   Hint: Can we enforce a bound on the degree of v_{k+1} in G_{k+1}? **2 Points**

b) How can we adjust the algorithm CanonicalOrder for maximal planar graphs to obtain a canonical order for maximal outerplanar graphs? **4 Points**

Exercise 3 – An alternative shift algorithm

We want to examine an alternative algorithm for drawing a plane triangulation G:

- Let $(v_1, v_2, \dots, v_n)$ be a canonical order of the vertices of G.
- Draw v_1 at $(0,0)$, v_2 at $(2,0)$, and v_3 at $(1,1)$.
- Incrementally draw the graph $G_k = G[\{v_1, \dots, v_k\}]$ for $k \in \{4, 5, \dots, n\}$:

  Let $w_1, \dots, w_p, \dots, w_q, \dots, w_t$ be the vertices on the boundary of the outer face of G_{k-1} (in this order), where $w_1 = v_1$, $w_t = v_2$, and $w_p, \dots, w_q$ are the neighbors of v_k in G_{k-1}. As the x-coordinate of v_k, choose an integer value $x(v_k)$ with $x(w_p) < x(v_k) < x(w_q)$. If no such value exists, first shift the right part of the drawing to the right by 1; i.e., for $q \leq i \leq t$ move each $L(w_i)$ to the right by 1. Now choose the smallest positive integer y-coordinate for which the drawing stays planar and v_k lies on the outer face.

a) Argue why this algorithm always yields a planar drawing. In particular, in step 3, why does a suitable y-coordinate exist? **3 Points**

b) Find a sharp lower bound for the maximum area requirement of the resulting drawing: find an infinite family of graphs where making bad choices for the x-coordinate in step 3 gives exponentially large y-coordinates. **3 Points**

This assignment is due at the beginning of the next lecture, that is, on May 23 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on May 21 at 16:00 and the solutions will be discussed one week after that on May 28.