Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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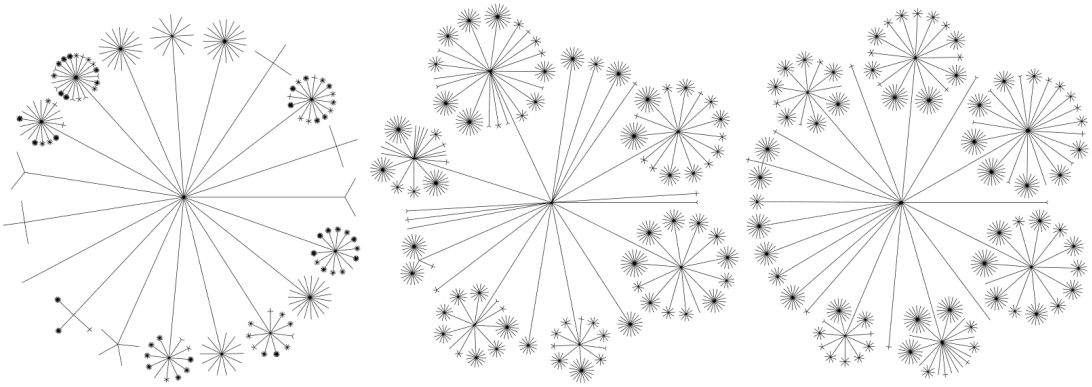
Prof. Dr. Alexander Wolff
Samuel Wolf

Exercise Sheet #1

Graph Visualization (SS 2025)

Exercise 1 – Drawing conventions & aesthetics of balloon layouts

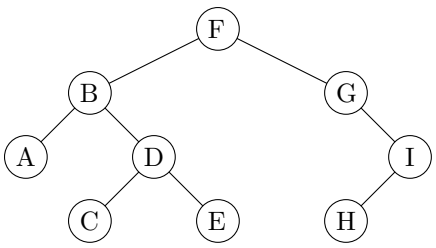
The three drawings of the same tree below are drawn with a *balloon layout*. Try to find at least two common drawing conventions and two possible drawing aesthetics to optimize for this layout style. **2 Points**

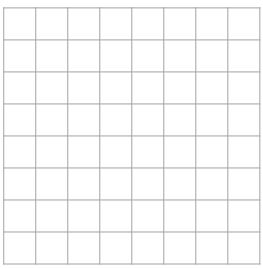


Exercise 2 – Binary trees with pre- and postorder coordinates

Let T be a binary tree with root r. For each $v \in V(T)$, let $x(v) := \text{preorder}(v)$ and $y(v) := \text{postorder}(v)$. Recall that $T(v)$ denotes the subtree rooted at v.

You may use the graph and grid below to try an example.





a) Prove that this coordinate assignment yields a planar drawing of T. **4 Points**

b) Give tight bounds on the area requirement of the generated drawing. **2 Points**

c) Prove that if you direct all edges of T such that they "point away" from r – that is, all vertices can be reached from r – then all arcs in the drawing point downwards.
2 Points

Exercise 3 – Lower bound on the area of right-heavy HV-drawings

Prove that there are trees for which the right-heavy HV-layout algorithm from the lecture produces drawings with area $\Omega(n \log n)$, where n is the number of vertices in the tree.
4 Points

Exercise 4 – Space-saving HV-drawings of complete binary trees

Let T be a *complete binary tree* of height h, that is, a binary tree where all vertices of depth $0, \ldots, h-1$ have exactly 2 children and all vertices of depth h are leaves. Consider the following HV-drawing algorithm.

Algorithm 1: BalancedHVDraw(node v, height h)

if $v ==$ nil **then return** $\emptyset$
$v_l \leftarrow$ left child of v
$v_r \leftarrow$ right child of v
$\Gamma_1 \leftarrow$ BalancedHVDraw(v_l, $h-1$)
$\Gamma_2 \leftarrow$ BalancedHVDraw(v_r, $h-1$)
if h odd **then return** horizontal combination of Γ_1 and Γ_2
if h even **then return** vertical combination of Γ_1 and Γ_2

a) Prove that BalancedHVDraw produces a drawing of T with area $O(n)$.

Hint: use induction on the height of the tree with the following hypothesis. The area (width $\times$ height) of the drawing for odd height is $(2\sqrt{n+1}-3) \times (\frac{3}{2}\sqrt{n+1}-2)$ *and for even height is* $(\sqrt{2(n+1)}-2) \times (\frac{3}{2}\sqrt{2(n+1)}-3)$.
4 Points

b) Give tight constant upper and lower bounds on the aspect ratio (i.e., the ratio between the width and the height) of the drawing generated with input T.
2 Points

This assignment is due at the beginning of the next lecture, that is, on May 2 at 10:15 am. Please submit your solutions via WueCampus. Questions can be asked in the tutorial session on April 30 at 16:00 and the solutions will be discussed on May 7.