3. Data Structures and Tolerant Retrieval

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After this lecture, you'll...

- Know what data structures are used for implementing inverted index
- Understand the pros and cons of hash tables and trees
- Know how to handle wildcard queries
- Be familiar with methods for handling spelling errors and typos in IR

Outline

- Recap of Lecture #2
- Data structures for inverted index
- Wild-card queries
- Spelling correction

Recap of the previous lecture

- Boolean retrieval
  - **Q:** How are queries represented in Boolean retrieval?
  - **Q:** How are documents represented for Boolean retrieval?
  - **Q:** How do we find relevant documents for a given query?
- Inverted index and finding relevant documents
  - **Q:** What is inverted index and what does it consist of?
  - **Q:** What are posting lists?
  - **Q:** How to merge posting lists?
  - **Q:** What is the computational complexity of the merge algorithm?
  - **Q:** What are skip pointers and what is their purpose?
- Phrase and proximity queries
  - **Q:** What is a biword index and what are its shortcomings?
  - **Q:** What is a positional index?
  - **Q:** How do we use positional index to answer phrase and proximity queries?

Recap of the previous lecture

- **Inverted index** is a data structure for computationally efficient retrieval
- Inverted index contains a list of references to documents for all index terms
  - For each term *t* we store the list of all documents that contain *t*
  - Documents are represented with their identifier numbers (ordinal, starting from 1)

„Frodo” -> [1, 2, 7, 19, 174, 210, 331, 2046]
„Sam” -> [2, 3, 4, 7, 11, 94, 210, 1137]
„blue” -> [2, 3, 24, 2001]

  - The list of documents that contains a term is called a **posting list** (or just **posting**)
  - **Q:** Postings are always sorted. Why?

Recap of the previous lecture

- So far, we learned how to handle
  - Regular Boolean queries
    - Standard merge algorithm over posting lists
    - Multi-term queries – optimizing according to lengths of posting lists
  - Phrase queries
    - Biword index
    - Positional index
  - Proximity queries
    - Positional index

- **Today we'll examine**
  - Data structures for implementing the inverted index
  - How to handle wild-card queries and spelling errors

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Data structures for inverted index

- Conceptually, an inverted index is a **dictionary**
  - Vocabulary terms (i.e., index terms) are keys
  - Posting lists are values

„Frodo” -> [1, 2, 7, 19, 174, 210, 331, 2046]
„Sam” -> [2, 3, 4, 7, 11, 94, 210, 1137]
„blue” -> [2, 3, 24, 2001]

- But the exact implementation is undefined
  - What data structures to use?
  - Where exactly to store different pieces of information – document frequencies, pointers to posting lists, skip pointers, token positions, ...?

Data structures for inverted index

- A naïve dictionary – an array/list of structures

Term	Doc. freq.	Pointer
a	656 265	→
aachen	65	→
blue	10 321	→
…	…	…
frodo	221	→

- Each element of the array is a structure consisting of:
  - The term itself
  - The number of documents in the collection in which the term appears
  - A pointer to the posting list of the term
- Structure size: char[N], int, pointer (int/long)

- **Q:** How to efficiently store the inverted index / dictionary in memory?
- **Q:** How to quickly look up elements at query time?

Data structures for inverted index

- Two main choices for implementing the inverted index dictionary
  - Hash tables
  - Trees

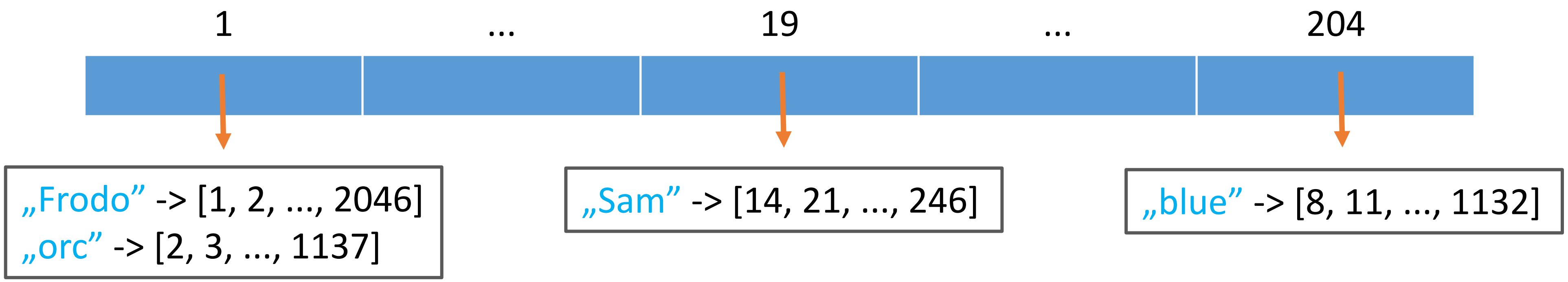
- Both are regularly used in IR systems
- Both have advantages and shortcomings

Inverted index dictionary as a hash table

- **Hash table** is a common data structure for implementing dictionaries, i.e., a structure that maps keys to values as associative arrays
- Hash tables rely on **hash functions** – functions that for a given input value (i.e., key) computes the index in the array where the value is stored
  - Ideally, each key would be assigned a different index
  - Most hash functions are imperfect – they may compute the same value for several different keys – this is called a collision
    - **Q:** how to account for collisions?
- Each vocabulary term is „hashed" into an integer value
  - *hf(„Frodo")* = 1, *hf(„Sam")* = 19, *hf(„blue") = 204*

Inverted index dictionary as a hash table

- *hf(„Frodo”)* = 1, *hf(„Sam”)* = 19, *hf(„blue”)* = 204, *hf(„orc”)* = 1
- **Associative array**



- If the hash function maps the key to the bucket with more than one entry, then the linear search through the bucket is performed

Inverted index dictionary as a hash table

- The main advantage of hash table is **fast lookup**
  - **Q:** What is the complexity of the lookup?
  - **A:** O(1)

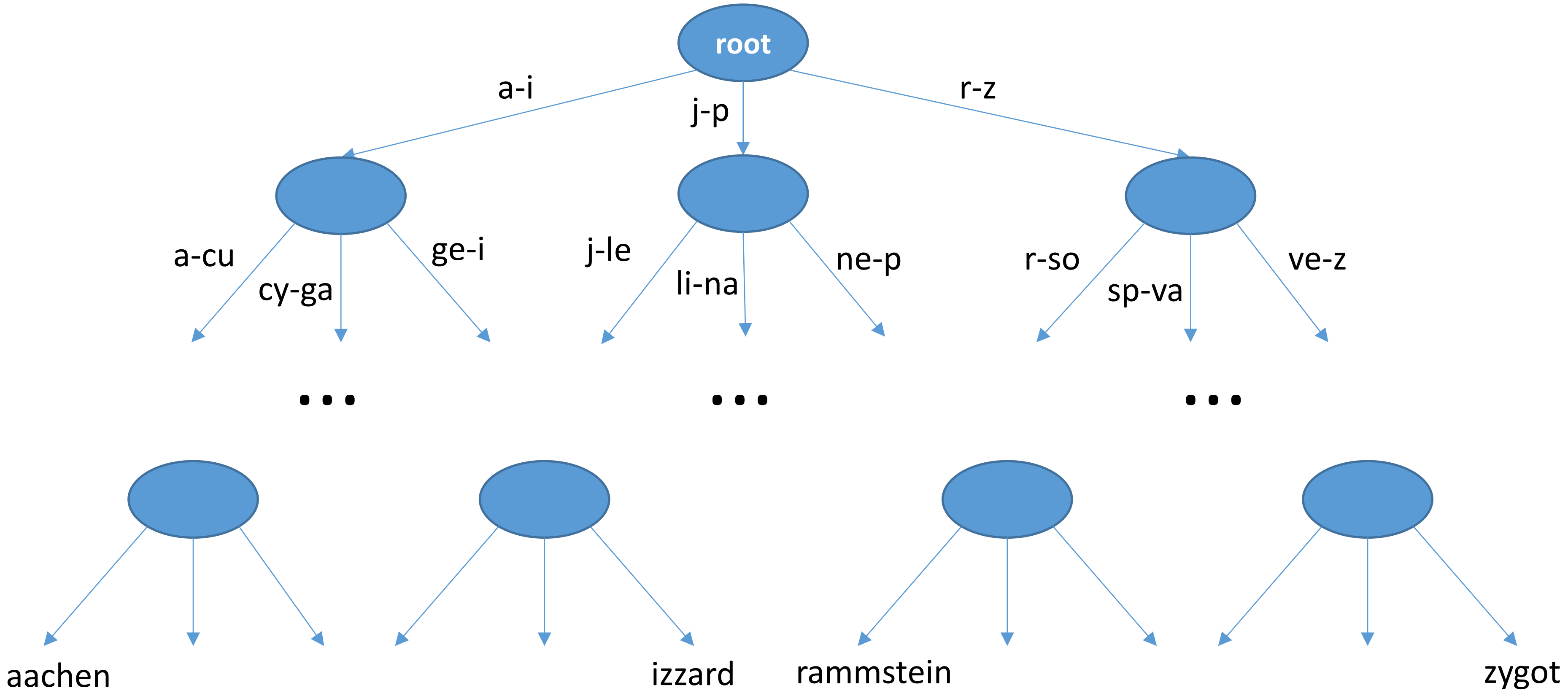
- Shortcomings:
  - Hash functions are sensitive to minor differences in strings
    - Close strings not assigned same or close buckets
      - E.g., *hf(„judgment”)* = 12, *hf(„judgement”)* = 354)
  - As such, they do not support prefix search
    - Important for tolerant retrieval
  - Constant vocabulary growth means occasional rehashing for all terms
    - **Q:** Why do we need to rehash if the vocabulary grows?

Inverted index dictionary as a tree

- Trees divide the vocabulary in the hierarchical form
  - Each node in the tree captures the subset of the vocabulary
  - Nodes closer to the root represent larger vocabulary subsets
  - Nodes closer to leaves encompass narrower subsets
  - Actual **vocabulary terms** are found in leaf nodes of the tree
  - The division of vocabulary is usually alphabetical

- Trees should be created in a balanced fashion
  - Each node in the tree should have approximately the same number of children
  - Subtrees of nodes at the same depth should have approx. the same number of leaves

Inverted index dictionary as a tree



Inverted index dictionary as a tree

- **Q:** What is the lookup complexity for a balanced tree with a node degree N which stores vocabulary containing |V|?
  - **A:** Lookup complexity is equal to the depth of the tree, so the complexity is $O(\log_N |V|)$
- The central design decision is the degree of the nodes in an index tree, i.e., the number of child nodes a parent node should have
  - Large node degree N
    - Shallow trees, but a large number of children to go through linearly
  - Small node degree N
    - Small number of children to linearly search, but deep trees
- Advantage
  - Can handle prefix search
- Shortcoming
  - Lookup complexity ($O(\log_N |V|)$) bigger than for hash tables ($O(\mathbf{1})$)

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Wild-card queries

- **Wild-card queries** are queries in which an asterisk sign stands for any sequence of characters
  - Wild-card term (with an asterisk) represents a group of terms and not a single term
- Trailing wild-card queries (aka prefix queries, * at the end)
  - E.g., „mon*” is looking for all documents containing any word beginning with „mon”
  - Easy to handle with **B-tree index**: retrieve all words w in range $mon \leq w < moo$
- Leading wild-card queries (* at the beginning)
  - E.g., „*mon” is looking for all documents containing any word ending with „mon”
  - Can be handled with an additional B-tree that indexes vocabulary terms backwards
    - Retrieve all words w in range: $nom \leq w < non$
- **Q:** How to handle queries with the wild-card in the middle?
  - Retrieve documents containing any word satisfying the wild-card query „co*tion”?

Wild-card queries

- Example query
  - „co*tion” (we want: *coordination*, *comotion*, *cohabitation*, *connotation*, ...)

Idea:

1. Lookup „co*” in the forward B-tree of the vocabulary
2. Lookup „*tion” in the backward B-tree of the vocabulary
3. Intersect the two obtained term sets

- Unfortunately, this is too expensive (too slow) for most real-time IR settings
  - We need to fetch the relevant documents with a single lookup into index
  - We need to enrich the index somehow
    - This will increase the index size, but memory is usually less of an issue

Wild-card queries and permuterm index

- Idea: index all character-level permutations of terms

- **Permuterm index** additionally stores permutations of vocabulary terms
- We add a special „end-of-term" character ($) and store all permutations:
  - E.g., „comotion" -> „$comotion", „n$comotio", „on$comoti", „ion$comot", „tion$como", „otion$com", „motion$co", „omotion$c"

- **Q:** How to use **permuterm index** for **middle-wild-card queries**?
  - **A:** Permute the wild-card query until you obtain a trailing query (asterisk at the end)
  - E.g., „co*tion" -> „$co*tion" -> „tion$co*"
  - We know how to handle trailing wild-card queries – „tion$co*" can now be handled by a single permutex index tree

Wild-card queries and permuterm index

- Queries supported by permuterm index
  - Exact queries: for „X” we look up „$X”
  - Trailing wild-card query: for „X*” we look up „$X*”
  - Leading wild-card query: for „*X” we look up „X$*”
  - General wild-card query: for „X*Y” we look up „Y$X*”

- **Q:** How would you handle the query „X*Y*Z” with the permuterm index?
  - **A:** Here we have no option but to fire two lookups into the index
    1. Retrieve the postings for „X*Z” (by looking up „Z$X*”)
    2. Retrieve the posting list for the query „*Y*” (by looking for „$Y*”)
    3. Intersect the two retrieved lists of terms

Character indexes

- Next idea:
  - How about we index all character n-grams (sequences of n characters) instead of whole terms?
  - We surround all terms with term-boundary symbols ($) and create lists of all sequences of *n* consecutive character within terms
  - Example: „Frodo and Sam fought the orcs” (stopwords removed; lemmatized)
    - Terms: $frodo$, sam, $fight$, orc
    - Char. 3-grams: fr, fro, rod, odo, do; sa, sam, am; fi, fig, igh, ght, ht; or, orc, rc

- We need to keep the second inverted index
  - For each character n-gram maintain the list of vocabulary terms that contain it
  - E.g., „$fr” -> [„freak”, „freedom”, ..., „frodo”, „frozen”]
    „sam” -> [„asamoah”, „balsam”, „disambiguate”, ..., „sam”, „subsample”]

Wild-card queries and character indexes

- Query for character n-grams and merge results (AND operator)
- Example: query „mon*” and 3-gram character indexing
  - Query is transformed into: „$m” AND „mo” AND „on”
  - **Q:** What might be the issue with this transformation?
  - **A:** Conjunction of character 2-grams might yield false positives
    - For example: moon, motivation, moderation, etc.
    - Compare this issue with the false positives of biword index from Lecture #2
  - Retrieved terms must be post-filtered against the query to eliminate false positives
    - Term contains „mon”?

- Resulting terms are then looked up in the term-document inverted index

Character indexes

- Comparison with permuterm index
  - Advantage: space efficient (less space needed than for permuterm index)
  - Shortcoming: slower than using permuterm index
    - A Boolean query (and term-level merges) needs to be performed for every query term

- Wild-card queries in general
  - Often not supported by Web search engines (not at the character level anyways)
  - Found in some desktop or library search systems
  - Wild-cards are conceptually troubling as well
    - User must know what they don't know (i.e., where to put the asterisk)
      - Used a lot in domain-specialized early Boolean retrieval systems – in part as a term normalization technique (before stemming or lemmatization were widespread)
    - If we have several options in mind, we can just run several concrete queries

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Spelling correction

- Primary use-cases for spelling correction
  1. Correcting documents during indexing
  2. Correcting user queries on-the-fly

- Two flavors of spelling correction
  1. Isolated words
     - Check each word on its own for errors in spelling
     - Will not catch typos that result in another valid word
       - E.g., „from" → „form"
  2. Context-sensitive spelling correction
     - Correctness evaluated by looking at surrounding words as well
     - E.g., „Frodo went *form* Gondor to Mordor"

Document correction

- Correction should occur prior to indexing
  - Aiming to have only valid terms in the vocabulary
  - Smaller vocabulary, i.e., the term dictionary contains fewer entries

- We do not change the original documents
  - Just perform correction when normalizing terms before indexing

- Common types of errors for certain types of documents
  1. OCR-ed documents – „rn” vs. „m”, „O” vs. „D”
  2. Digitally-born documents often have QWERTY keyboard typos – errors from close keys – „O” vs. „I”, „A” vs. „S”, etc.

Query correction

- Primary focus is on correcting errors from queries
  - **Q:** Failing to fix errors in queries has more serious consequences than omitting to fix errors in documents. Why?

- With respect to user interface, we have two options
  1. Silently retrieving documents according to the corrected query
  2. Return several suggested „corrected" query alternatives to the user
     - „Did you mean?" option

Isolated word correction

- **The idea:** using reference lexicon of correct spellings (i.e., lexicon of valid terms)
- Two approaches for obtaining a reference lexicon
  1. Existing lexicons like
     - Standard wide-coverage lexicon of a language (e.g., Webster's English dictionary)
     - Domain-specific lexicons (e.g., lexicon of legal terms)
  2. Lexicon built from large corpora
     - E.g., all the words on the web or in Wikipedia
     - **Q:** Do we want to keep absolutely all terms from large corpora?

Isolated word correction

- Given a reference lexicon and the query term (a character sequence from the query), we do the following:
  1. Check if the query term *Q* is in the reference lexicon
  2. If the term *Q* is not in the reference lexicon, find the entry *Q'* from the lexicon that is „closest” to the query term *Q*

- How do we define „closest”?
  - We need some similarity/distance measure
  - We will examine several options
    1. Edit distance (also known as Levenshtein distance)
    2. Weighted edit distance
    3. Character n-gram overlap

Spelling correction – edit distance

- **Edit distance** between two strings *S* and *S'* is the minimal number of **operations** required to transform one string into the other
  - What are the „operations"?
- We typically consider operations at the character level
  - Character insertion („frod" → „frodo")
  - Character deletion („frpodo" → „frodo")
  - Character replacement („frido" → „frodo")
  - Less often: transposition of adjacent characters („fordo" → „frodo")
    - Transposition equals „deletion" + „insertion"?
    - **Q:** Why introducing it as a separate operation?
- Levenshtein distance: counts insertions, deletions and replacements
- Damerau-Levenshtein distance: additionally counts transpositions as a single operation
- Algorithm based on dynamic programming

Dynamic programming

- For detailed explanation of dynamic programming see
  Cormen, Leiserson, Rivest, and Stein. „Introduction to Algorithms”
- **Optimal substructure**: the optimal solution of the problem contains within itself the subsolutions, i.e., the optimal solutions to subproblems
- **Overlapping subsolutions**: we can recycle subsolutions – i.e., avoiding repeating the computation for the same subproblems over and over again
- **Q:** What would be a „subproblem” for the edit distance computation?
  - **A:** the edit distance between two prefixes of input strings
- **Q:** Do we have many subproblem repetition for edit distance?
  - **A:** most distances between same pair of prefixes are needed 3 times (as a subproblem of computing distance for insertion, deletion, and substitution)

Levenshtein distance

- Let *a* and *b* be two strings between which we measure edit distance (with |a| and |b| being their respective lengths):
- Mathematically, the Levenshtein distance $lev_{a,b}(|a|, |b|)$ is computed as follows:

$$\operatorname{lev}_{a,b}(i,j) = \begin{cases} \max(i,j) & \text{if} \min(i,j) = 0, \\ \min \begin{cases} \operatorname{lev}_{a,b}(i-1,j) + 1 \\ \operatorname{lev}_{a,b}(i,j-1) + 1 \\ \operatorname{lev}_{a,b}(i-1,j-1) + 1_{(a_i \neq b_j)} \end{cases} & \text{otherwise.} \end{cases}$$

- Where $1(a_i \neq b_j)$ is the indicator function equal to **0** if $a_i = b_j$ and 1 otherwise
- Once we compute $lev_{a,b}(i, j)$ for some pair (i, j) we store it in memory so we don't compute it again when needed in another recursive thread
- Directly implementing this formula requires **recursion**

Example – Levenshtein recursively

- For the example, we will follow only one thread of recursion (first subproblem)
- „sany” vs. „sam”
  - min(lev(„san”, „sam”) + 1, lev(„sany”, „sa”) + 1, lev(„san”, „sa”) + 1)
- „san” vs. „sam”
  - min(lev(„sa”, „sam”) + 1, lev(„san”, „sa”) + 1, lev(„sa”, „sa”) + 1)
- „sa” vs. „sam”
  - min(lev(„s”, „sam”) + 1, lev(„sa”, „sa”) + 1, lev(„s”, „sa”) + 1)
- „s” vs. „sam”
  - min(lev(„”, „sam”) + 1, lev(„s”, „sa”) + 1, lev(„”, „sa”) + 1)
- „” vs. „sam”
  - return 3

Levenshtein distance – non-recursive version

- We can avoid the recursion if we start from the recursive algorithm's end condition – return max(i, j) if min(i, j) = 0
- Then compute the edit distances of larger prefixes from smaller prefixes

```
LevenshteinDistance(s1, s2)
for i ← 0 to |s1|
do m[i, 0] = i
for j ← 0 to |s2|
do m[0, j] = j
for i ← 1 to |s1|
do for j ← 1 to |s2|
    do if s1[i] = s2[j]
        then m[i, j] = min{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]}
        else  m[i, j] = min{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]+1}
return m[|s1|, |s2|]
```

Example – Levenshtein non-recursively

	_	s	a	m
_	0	1	2	3
s	1	0	1	2
a	2	1	0	1
n	3	2	1	1
y	4	3	2	**2**

Damerau-Levenshtein distance

- Standard edit distance counts transposition of adjacent characters as two edits
  - E.g., „frodo” vs. „fordo”
  - two character replacements: „r” -> „o” in position 2 and „o” -> „r” in position 3

- However, transposing adjacent characters is usually a single typing error
  - **Damerau-Levenshtein distance** introduces **transposition** as the fourth atomic distance operation
  - **Q:** How would you integrate transposition as a single distance operation into the edit distance algorithm?
  - **A:** $d(i,j)$ additionally needs to consider $d(i-2, j-2) + 1(a_{i-1} = b_j \;\&\; a_i = b_{j-1})$ when looking the edit distances of prefixes

Weighted edit distance

- Sometimes we want to assign smaller distance to common errors
  - The weight of an operation (deletion, insertion, replacement, transposition) depends on the caharcter(s) involved
- Motivation: better capture common OCR or typing errors
  - E.g., On a QWERTY keyboard, letter „m” is much more likely to be mis-typed as „n” than as „q”
  - Thus, the replacement operation „m” -> „n” should be assigned smaller edit distance than „m” -> „q”
- Additional input required
  - Data structure (e.g., weight matrix) containing operation weights for (combinations of) characters
- **Q:** How to integrate weighting into the edit distance algorithm based on dynamic programming?

Using edit distances

- Given a (misspelled) query we need to find the closest dictionary term

- **Q:** How do we know (or assume) that the query is misspelled in the first place?
  - **A:** We don’t find the query term in the vocabulary dictionary
  - With this strategy, we cannot capture typos like „from” -> „form”

- Finding closest dictionary term
  - Compute edit distance between the query term and each of the dictionary terms?
    - Too slow (the dictionaries are usually rather large)
  - We need to somehow pre-filter the „more promising” dictionary entries

N-gram index for spelling correction

- **Idea:** use the character n-gram index to pre-filter dictionary candidates

1. Enumerate all character n-grams in the query string
   - E.g., 3-grams in „frodso” -> „fro”, „rod”, „ods”, „dso”
2. Retrieve all vocabulary terms containing any of the obtained character n-grams
   - Using the inverted index of character n-grams
3. Treshold the obtained list of candidates on the number or percentage of matching character n-grams
4. Compute the edit distances between the query term and the remaining dictionary candidates
5. Select the candidate with the smallest edit distance as the correction

Character n-gram overlap

- Can be used as
  - A measure for pre-filtering candidates in order to reduce the number of edit distance computation
  - As a self-standing distance measure, alternative to Levenshtein distance
- Example
  - Suppose the query is „fpodo bigginss” and the text is „frodo baggins” and we are computing the overlap in character 3-grams
  - {„fpo”, „pod”, „odo”, „big”, „igg”, „ggi”, „ins”, „nss”} vs. {„fro”, „rod”, „odo”, „bag”, „agg”, „ggi”, „ins”}
  - We have 3 matching 3-grams: „odo”, „ggi”, and „ins”
    - That’s 3 out of 8 for the query and 3 out of 7 for the text
- **Q:** What should we take as measure of proximity/distance?
  - Is raw count of matching n-grams good choice?

Character n-gram overlap

- Raw count of matching character n-grams is not a good choice
  - Does not account for the length of terms in comparison
  - Two distinct but long terms may have a large raw count of matching n-grams
    - E.g., „collation” and „collaboration” have 5 matching 3-grams
  - We need to normalize the score with the length of terms
- **Jaccard coefficient** – a commonly used measure of set overlap

$$|X \cap Y| / |X \cup Y|$$

- Simple alternative: averaged length-normalized overlap

$$0.5 \cdot \left(|X \cap Y| / |X| + |X \cap Y| / |Y| \right)$$

Context-sensitive spelling correction

- Example:
  - Suppose the text is „Frodo fled from Mordor back to Gondor”
  - Suppose the query is „fled form Gondor”
- To identify the misspelling „form” -> „from” we need to take into account the context, i.e., surrounding words
- Context-sensitive error correction steps
  1. For each term of the query, retrieve dictionary terms that are sufficiently close
     - „fled” -> {„fled”, „flew”, „flea”}; „form” -> {„form”, „from”}; „gondor” -> {„gondor”}
  2. Combine all possibilities (i.e., all combinations of candidates for each term)
     - „fled form gondor”, „fled from gondor”, „flew form gondor”, „flew from gondor”, „flea form gondor”, „flea from gondor”,
  3. Rank the possibilities according to some criteria

Context-sensitive spelling correction

- **Hit-based** spelling correction
  - Rank the candidate combinations according to the number of hits (no. documents that contain those combinations)
  - Return the candidate with the largest number of hits
- **Log-based** spelling correction
  - Rank the candidates according to the number of appearances in the query logs (i.e., the number of times the same query was posed before)
  - Useful only if you have <u>a lot of users who fire a lot of queries</u>
- **Probabilistic** spelling correction (e.g., based on language modeling)
  - Ranking according to probabilities of term sequences
  - E.g., P(„fled form gondor”) = P(„fled”) * P(„form” | „fled”) * P(„gondor” | „form”)

Now you...

- Know what data structures you can use for implementing inverted index
- Understand the pros and cons of hashtables and trees
- Know how to handle wildcard queries
- Are familiar with methods for handling spelling errors and typos in IR