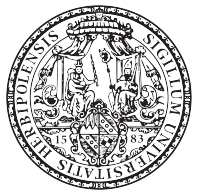

Introduction to Informatics
WiSe 2024/2025

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Exercise Sheet 04

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Please upload your solutions to WueCampus as a scanned document (image format or pdf), a typesetted PDF document, and/or as a `jupyter` notebook.

1. **Pencil-and-Paper Algorithm**

   (a) The pencil-and-paper algorithm for addition, as developed in the lecture for the decimal system (base 10), should now be adapted to work for numeral systems with base k, where k is a positive integer greater than 1 and does not exceed the number of distinct digits and letters available in the alphabet (e.g., $k \leq 36$, using digits 0–9 and letters A–Z). For example, adapt the algorithm to binary ($k = 2$) or hexadecimal ($k = 16$). Use the algorithm from the group exercise in the lecture and generalize it to handle addition for numbers represented as sequences of digits in such k-nary numeral systems.

2. **Sorting Algorithms**

   (a) BucketSort is a sorting algorithm that distributes elements into several "buckets" based on their values. Each bucket is then sorted individually, often using another sorting algorithm or recursively applying Bucket Sort. Finally, the contents of the buckets are combined to produce the sorted output.

   Look up the Bucket sort algorithm for integers in a fixed range and implement it in a Jupyter notebook.

   (b) Use matplotlib to plot the runtime of the Bucket sort algorithm as a function of the length of the input array. To get a meaningful result, for each length of the input array, run the algorithm several times for different random inputs and calculate the average runtime.

   Hint: Use the time package from python to measure the runtime of the algorithm. (`https://docs.python.org/3/library/time.html`)

   (c) Compare the runtime of the BucketSort algorithm to that of BubbleSort and MergeSort introduced in the lecture. Therefore plot the runtime of the three algorithms as a function of the length of the input array.

   (d) Explain whhy the BucketSort algorithm for integers in a fixed range takes less than $n \log_2 n$ steps on average.