

Behauptung: $U_1 \leq G_1, U_2 \leq G_2 \rightarrow U_1 \times U_2 \leq G_1 \times G_2$

Beweis: $U_1 \times U_2 \neq \emptyset$, denn $(e_1, e_2) \in U_1 \times U_2$

$a, b \in U_1 \times U_2, a = (a_1, a_2), b = (b_1, b_2)$

$$\begin{aligned} ab^{-1} &= (a_1, a_2) \cdot (b_1, b_2)^{-1} \\ &= (a_1, a_2) \cdot (b_1^{-1}, b_2^{-1}) \\ &= \left(\underbrace{a_1 b_1^{-1}}_{\in U_1}, \underbrace{a_2 b_2^{-1}}_{\in U_2} \right) \\ &\in U_1 \times U_2 \quad (\text{da } U_1 \leq G_1, U_2 \leq G_2) \end{aligned}$$