Gruppe 1 - Aufgabe 2

Behauptung:

$$Z(G_1 \times G_2) = Z(G_1) \times Z(G_2)$$

Beweis:

"$\supseteq$ "

Sei $(x_1, x_2) \in Z(G_1) \times Z(G_2)$

$\Rightarrow \forall y_1 \in G_1; \forall y_2 \in G_2 : x_1 y_1 = y_1 x_1 \wedge x_2 y_2 = y_2 x_2$

$$\begin{aligned} (x_1, x_2)(y_1, y_2) &= (x_1 y_1, x_2 y_2) \\ &= (y_1 x_1, y_2 x_2) \\ &= (y_1, y_2)(x_1, x_2) \end{aligned}$$

$\Rightarrow (x_1, x_2) \in Z(G_1 \times G_2)$

"$\subseteq$"

Annahme: $\exists (x_1, x_2) \in Z(G_1 \times G_2) : (x_1, x_2) \notin Z(G_1) \times Z(G_2)$

$\Rightarrow \exists \hat{y_1} \in G_1 : x_1 \hat{y_1} \neq \hat{y_1} x_1 \vee \exists \hat{y_2} \in G_2 : x_2 \hat{y_2} \neq \hat{y_2} x_2$

$$\begin{aligned} (x_1, x_2)(\hat{y_1}, \hat{y_2}) &= (x_1 \hat{y_1}, x_2 \hat{y_2}) \\ &\neq (\hat{y_1} x_1, \hat{y_2} x_2) \\ &= (\hat{y_1}, \hat{y_2})(x_1, x_2) \end{aligned}$$

$\Rightarrow (x_1, x_2) \notin Z(G_1 \times G_2)$