Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, January 21, 2025

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Homework Assignment #11
Approximation Algorithms (Winter Semester 2024/25)

Exercise 1 – MAX-SAT

a) Show that the following algorithm is a 1/2-approximation for MAX-SAT. Let τ be an arbitrary truth assignment, and let τ' be its complement. In other words, a variable is `true` in τ if and only if it is `false` in τ'. Compute the weight of the clauses satisfied by τ and by τ', then output the better assignment. **[4 points]**

In the following, let a k-CNF-SAT instance denote a formula in conjunctive normal form where each clause contains exactly k literals and every variable occurs at most once per clause.

b) Show that every 3-CNF-SAT instance with at most seven clauses is satisfiable. **[2 points]**

c) Recall the algorithm that outputs a truth assignment uniformly at random. It runs in polynomial time, and we proved a lower bound on the expected weight of the satisfied clauses. We then derandomised this algorithm. This makes the algorithm slower and more complicated. Consider instead the following algorithm for MAX-3-CNF-SAT: keep guessing truth assignments until you find one that satisfies 7/8 of the clauses. Show that this algorithm runs in expected polynomial time. Compare the advantages and disadvantages of these three algorithms for MAX-3-CNF-SAT. **[8 points]**

d) Recall the MAXCUT problem: Given a graph G, find a set S with $\emptyset \subsetneq S \subsetneq V(G)$ such that the number of edges between S and its complement is as large as possible.

Solve MAXCUT by using a hypothetical algorithm for MAX-2-CNF-SAT. **[6 points]**

Please hand in your solutions via WueCampus until 10:00 on Friday, January 31.