Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, January 14, 2025

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Homework Assignment #10
Approximation Algorithms (Winter Semester 2024/25)

Exercise 1

Show that assuming $P \neq NP$, there exists no approximation algorithm for MINIMUMDEGREESPANNINGTREE with ratio $\alpha < 3/2$.

[3 points]

Exercise 2

Let G be a graph with an (unknown) Hamiltonian path and let $n = |V(G)|$. Give a polynomial-time algorithm that finds a simple path of length $\Omega\left(\frac{\log n}{\log \log n}\right)$.

[5 points]

Exercise 3

In the lecture, we learned about a local search algorithm that finds a spanning tree with a provable upper bound on the maximum degree. To prove the efficiency of the algorithm, we used the potential function

$$\Phi(T) = \sum_{v \in V(G)} 3^{\deg_T(v)},$$

where T is the current spanning tree.

Prove that the potential function decreases with every edge flip by at least factor $\frac{2}{27n^3}$, that is, $\Phi(T') \leq (1 - \frac{2}{27n^3})\Phi(T)$, where T' is the tree after the edge flip. **[5 points]**

Exercise 4

Two students, Peter and Susi, study the approximation algorithm for MINIMUMDEGREESPANNINGTREE from the lecture. This algorithm finds a spanning tree with maximum degree at most $2 \cdot \mathrm{OPT} + \ell$, where $\ell := \lceil \log_2 n \rceil$. Peter and Susi want to improve the quality of this result and argue as follows:

Peter: „If we want to find better results, then we have to choose a smaller ℓ! By this formula, this guarantees a smaller maximum degree of the spanning tree!“

Susi: „But shouldn't we choose a larger ℓ? That allows more flips, and shouldn't that give us a better result?!“

a) Settle the dispute between Peter and Susi. Generalize the result from the lecture by allowing values $\ell := \lceil \log_b n \rceil$ for arbitrary $b > 1$. For $b = 2$, you should obtain the result from the lecture as a special case. **[4 points]**

Peter: „Then we agree! But what does that mean for the running time?“

b) How does the choice of ℓ (or b) affect the running time of the algorithm? **[3 points]**

Please hand in your solutions via WueCampus until 10:00 on Friday, January 24.