Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

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Homework Assignment #6
Approximation Algorithms (Winter Semester 2024/25)

Exercise 1 – METRIC-k-CLUSTER

Let G be a complete graph with edge weights $c\colon E(G) \to \mathbb{Q}_{\geq 0}$ that satisfy the triangle inequality. Let k be a positive integer. We want to find a partition of $V(G)$ into k sets of vertices $V_1, \dots, V_k$, called *clusters*, such that the weight of the most expensive intra-cluster edge is minimized. In other words, we have to minimize

$$\max_{1 \leq i \leq k,\ u,v \in V_i} c(u,v).$$

a) Devise a factor-2 approximation algorithm for this problem. **[7 points]**

b) Show that under the assumption $P \neq NP$, there exists no factor-$(2-\varepsilon)$ approximation algorithm for this problem, where $\varepsilon > 0$.

Hint: Use the hardness of the graph coloring problem: Given a graph G and a parameter k, can the vertices of G be colored with at most k colors such that no two adjacent vertices share a color? **[6 points]**

Exercise 2 – Greedy for METRIC-k-CENTER

Consider the following greedy algorithm for METRIC-k-CENTER.

Algorithm 1: Greedy-Metric-k-Center(graph G, cost function c, int $k > 0$)

```
Pick an arbitrary vertex v ∈ V(G)
S ← {v}
while |S| < k do
    u ← vertex with c(u, S) = max_{v∈V(G)} c(v, S)
    S ← S ∪ {u}
```

Show that this algorithm is a factor-2 approximation for METRIC-k-CENTER. **[7 points]**

Please hand in your solutions via WueCampus until 10:00 on Friday, December 6.