Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, November 5, 2024

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Homework Assignment #3
Approximation Algorithms (Winter Semester 2024/25)

Exercise 1 – FEEDBACK VERTEX SET on tournaments

A *tournament* is a directed graph G that contains exactly one of the edges (u, v) and (v, u) for each pair of vertices $u \neq v$. The FEEDBACK VERTEX SET problem asks for a smallest set of vertices whose removal from G results in an acyclic graph.

Show that there exists a factor-3 approximation algorithm for FEEDBACK VERTEX SET on tournaments. **[5 points]**

Hint: Show that it suffices to destroy all cycles of length 3. Then, develop an approximation-preserving reduction to SET COVER with frequency $h = 3$ (see Exercise 2 in Homework Assignment #2).

Exercise 2 – SENDER–RECEIVER

Let G be a graph with non-negative edge costs $c: E(G) \to \mathbb{Q}_{\geq 0}$. Let S and R be two disjoint sets of vertices, which we call *sender* and *receiver*, respectively. The *Sender–Receiver* problem asks for a minimum-cost subgraph of G where every receiver is connected to some sender by a path.

Hint: Introduce an additional vertex and connect it to G appropriately. Using an approximation-preserving reduction can simplify the solutions for questions a) and c) considerably.

a) Show that an *exact* solution can be found in polynomial time if $S \cup R = V(G)$ holds. **[4 points]**

b) *Bonus:* Show that the general version of the problem (where $S \cup R$ and $V(G)$ don't necessarily coincide) is NP-hard. **[3 extrapoints]**

c) Give a factor-2 approximation algorithm for the general version of the problem. **[3 points]**

Exercise 3 – Greedy for MULTIWAY CUT

A natural greedy algorithm for computing a multiway cut in a graph G with given terminals $s_1, \ldots, s_k$ is the following: Based on G, compute a cheapest s_i–s_j cut for each pair (s_i, s_j) that is still connected, and remove the edges in the cheapest of these cuts from G. Repeat until all terminal pairs are separated.

Show that this algorithm has approximation ratio $2 - 2/k$. **[8 points]**

Hint: Consider the partitioning $V_1, \ldots, V_k$ of $V(G)$ (with $s_i \in V_i$ for each $i \in \{1, \ldots, k\}$) induced by an optimal sequence of cuts. Assume that the terminals are indexed $s_1, \ldots, s_k$ such that $c(V_1, \bar{V}_1) \leq \cdots \leq c(V_k, \bar{V}_k)$. Charge each cut chosen by the algorithm by one of the cuts of type $c(V_j, \bar{V}_j)$. Make sure that each cut $c(V_j, \bar{V}_j)$ is charged at most once.

Please hand in your solutions via WueCampus by 10:00 on Friday, November 15.