# Seminar 'Optimization under Uncertainty'

1

robust, stochastic, and online optimization

Marie Schmidt

17.04.2024

# Agenda for today

- Course language?
- What's this course about?
- Administrative stuff

## Course language

- English or German for presentations and course communication?
- reports can be in English or German



















ightarrow variation in the driving times leads to different routes







## Seminar questions

How can we formulate optimization problems, when we do not have exact information on input parameters?

## Seminar questions

How can we formulate optimization problems, when we do not have exact information on input parameters?

What will be the impact on the (type of) solutions that we find?

## Seminar questions

How can we formulate optimization problems, when we do not have exact information on input parameters?

What will be the impact on the (type of) solutions that we find?

How can we find solutions to such optimization problems?



What is the best way from A to C?



What is the best way from A to C?



What is the best way from A to C - if there is a disruption on (B, C)?

9



What is the best way from A to C - if there is a disruption on (B, C)?



What should the traveler do?

What is the best way from A to C - if there is a disruption on (B, C)?



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc) We call the set of *scenarios* (possible travel times) *uncertainty set*  $\mathcal{U}$ .



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc) We call the set of *scenarios* (possible travel times) *uncertainty set* U. U could be finite,



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc) We call the set of *scenarios* (possible travel times) *uncertainty set* U. U could be finite, bounded,



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc) We call the set of *scenarios* (possible travel times) *uncertainty set* U. U could be finite, bounded, or unbounded.



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc) We call the set of *scenarios* (possible travel times) *uncertainty set* U. U could be finite, bounded, or unbounded.



What should the traveler do?

gain more information on the disruption (source, estimated length etc)
We call the set of *scenarios* (possible travel times) *uncertainty set U*.
*U* could be finite, bounded, or unbounded.
Possibly, we have a probability distribution on *U*.



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty* 



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty* 

• objective value in the worst case



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  $\mathcal{U}$ 

2.) define how to *evaluate* a solution *under uncertainty* 

• objective value in the worst case



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty* 

• objective value in the worst case


What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty* 

• objective value in the worst case

 $z_{wc}(P_1) = 1460, \ z_{wc}(P_2) = 120$ 



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

- objective value in the worst case
- expected objective value



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

- objective value in the worst case
- expected objective value

$$z_{\exp}(P_1) = 30$$
,  $z \exp(P_2) = 120$ 



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

- objective value in the worst case
- expected objective value
- worst-case ratio ('competitive ratio')



What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

 $z_{\rm cr}(P_2) = \frac{120}{20} = 6$ 

- objective value in the worst case
- expected objective value
- worst-case ratio ('competitive ratio')  $z_{cr}(P_1) = \frac{1460}{120} \approx 12.08$ ,



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

- objective value in the worst case
- expected objective value
- worst-case ratio ('competitive ratio')
- exp. ratio, worst-case/expected regret, probability to have the optimal solution, prob.to be 'on time', expected travel time in 90% of the cases, value at risk,...



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty*  $\rightarrow$  objective function



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

So far, we have only considered *paths* as possible solutions.



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

So far, we have only considered *paths* as possible solutions.

What if the traveler can wait at A to see how the situation evolves?

Strategy wait at A for at most 60 minutes to see if disruption has disappeared, if so, take  $P_1$ , otherwise, take  $P_2$  has competive ratio  $\frac{3}{2}$ 



What should the traveler do?

So far, we have only considered *paths* as possible solutions.

What if the traveler can wait at A to see how the situation evolves?

Strategy wait at A for at most 60 minutes to see if disruption has disappeared, if so, take  $P_1$ , otherwise, take  $P_2$  has competive ratio  $\frac{3}{2}$ 



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

So far, we have only considered *paths* as possible solutions.

What if the traveler can wait at A to see how the situation evolves?

Strategy wait at A for at most 60 minutes to see if disruption has disappeared, if so, take  $P_1$ , otherwise, take  $P_2$  has competive ratio  $\frac{3}{2}$ 



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty*  $\rightarrow$  objective function

3.) define the space of *feasible* solutions (in particular with respect to 'adjustability' of solution)



What is the best way from A to C - if there is a disruption on (B, C)?

What should the traveler do?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  ${\cal U}$ 

2.) define how to *evaluate* a solution *under uncertainty*  $\rightarrow$  objective function

3.) define the space of *feasible* solutions (in particular with respect to 'adjustability' of solution)

4.) solve the corresponding optimization problem



What is the best way from A to C - if there is a disruption on (B, C)?

What can people skilled in optimization do to help the poor traveler?

1.) gain more information on the disruption (source, estimated length etc)  $\rightarrow$  uncertainty set  $\mathcal{U}$ 

2.) define how to *evaluate* a solution *under uncertainty*  $\rightarrow$  objective function

3.) define the space of *feasible* solutions (in particular with respect to 'adjustability' of solution)

4.) solve the corresponding optimization problem

# Modeling cycle for optimization problems



## Problems in the focus of this seminar



• shortest path problem

## Problems in the focus of this seminar



shortest path problem

knapsack problem



## Presentation schedule

- Introduction to the shortest path problem ( $\approx$  8.5.)
- Shortest path under uncertainty 1
- Shortest path under uncertainty 2

• Introduction to the knapsack problem ( $\approx$  5.6.)

- Knapsack under uncertainty 1
- Knapsack under uncertainty 2

.....

## Presentation schedule

presentation dates will be assigned after this session

- Introduction to the shortest path problem (pprox 8.5.)
- Shortest path under uncertainty 1
- Shortest path under uncertainty 2

• Introduction to the knapsack problem ( pprox 5.6.)

- Knapsack under uncertainty 1
- Knapsack under uncertainty 2

- (Other optimization problem under uncertainty 1) ( $\approx$  10.7.)
- (Other optimization problem under uncertainty 2)

## Presentation schedule

presentation dates will be assigned after this session

- Introduction to the shortest path problem (pprox 8.5.)
- Shortest path under uncertainty 1 Shortest path under uncertainty 2. • Introduction to the knapsack problem ( pproxindividual, based on a Knapsack under uncertainty 1 paper,(normally) 1 per Knapsack under uncertainty 2 session, approx. 45 minutes
  - (Other optimization problem under uncertainty 1) ( $\approx$  10.7.)
  - (Other optimization problem under uncertainty 2)

#### 12 presentation dates will be Presentation schedule assigned after this session • Introduction to the shortest path problem ( $\approx$ 8.5.) group, 'the basics', short (at most 30 minutes), Shortest path under uncertainty 1 same day as first Shortest path under uncertainty 2 individual presentation of the block • Introduction to the knapsack problem ( $\approx$ 5.6.) individual, based on a Knapsack under uncertainty 1 paper,(normally) 1 per • Knapsack under uncertainty 2 session, approx. 45 minutes

- (Other optimization problem under uncertainty 1) ( $\approx$  10.7.)
- (Other optimization problem under uncertainty 2)

## Paper selection for the individual presentations

After this session:

- consult list of papers (WueCampus)
- indicate preferences and preknowledge on WueCampus until Friday at noon
- paper assignment and schedule (including dates) will be published on WueCampus on Friday afternoon

← read/scan paper before selection!

All students presenting a 'shortest path' paper prepare a joint group presentation, containing

 basic information on the *deterministic* problem: definition, complexity, (illustration of) solution methods

All students presenting a 'shortest path' paper prepare a joint group presentation, containing

 basic information on the *deterministic* problem: definition, complexity, (illustration of) solution methods

All students presenting a 'shortest path' paper prepare a joint group presentation, containing

- basic information on the *deterministic* problem: definition, complexity, (illustration of) solution methods
   deterministic means without uncertainty
- one (or several) *uncertain* problem instances that will be used in the individual presentations to illustrate concepts & methods

Tip: use (a deterministic version of) the same instance to illustrate the methods for the deterministic problem

All students presenting a 'shortest path' paper prepare a joint group presentation, containing

- basic information on the *deterministic* problem: definition, complexity, (illustration of) solution methods
   deterministic means without uncertainty
- one (or several) *uncertain* problem instances that will be used in the individual presentations to illustrate concepts & methods

Tip: use (a deterministic version of) the same instance to illustrate the methods for the deterministic problem

The same holds for the students with a 'knapsack' paper.

All students presenting a 'shortest path' paper prepare a joint group presentation, containing

- basic information on the *deterministic* problem: definition, complexity, (illustration of) solution methods
   deterministic means without uncertainty
- one (or several) *uncertain* problem instances that will be used in the individual presentations to illustrate concepts & methods

Tip: use (a deterministic version of) the same instance to illustrate the methods for the deterministic problem

The same holds for the students with a 'knapsack' paper.

Students who present on a *different* problem include this part in their individual presentation.

## Individual presentations: Structure

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?

2. What could go wrong if we just took the deterministic solution?

3. Which solutions are considered 'feasible'?

4. How are solutions *evaluated under uncertainty*? Tip: illustrate this on the deterministic solution

5. Solution method from paper, illustrated on example(s) from introductory group presentation

(6. Own ideas - see next slide)

## Individual presentations: Structure

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

- 1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?
- 2. What could go wrong if we just took the deterministic solution?
- 3. Which solutions are considered 'feasible'?
- 4. How are solutions *evaluated under uncertainty*? Tip: illustrate this on the deterministic solution
- 5. Solution method from paper, illustrated on example(s) from introductory group presentation
- (6. Own ideas see next slide)

If you present on a problem that is not shortest path or knapsack, this includes problem definition & methods for the deterministic case.

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?

2. What could go wrong if we just took the deterministic solution?

3. Which solutions are considered 'feasible'?

4. How are solutions *evaluated under uncertainty*? Tip: illustrate this on the deterministic solution

5. Solution method from paper, illustrated on example(s) from introductory group presentation

(6. Own ideas - see next slide)

Activate your audience:

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?

2. What could go wrong if we just took the deterministic solution?

3. Which solutions are considered 'feasible'?

4. How are solutions *evaluated under uncertainty*? Tip: illustrate this on the deterministic solution

5. Solution method from paper, illustrated on example(s) from introductory group presentation

(6. Own ideas - see next slide)

#### Activate your audience:

You present uncertainty set, optimality concept, and (idea of) the method.

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?

2. What could go wrong if we just took the deterministic solution?

3. Which solutions are considered 'feasible'?

4. How are solutions *evaluated under uncertainty*? Tip: illustrate this on the deterministic solution

5. Solution method from paper, illustrated on example(s) from introductory group presentation

(6. Own ideas - see next slide)

#### Activate your audience:

You present uncertainty set, optimality concept, and (idea of) the method. And let the other students do/help with the rest.

(0. Preliminaries: any concepts that are relevant for your paper, specifically, and that other may not know)

1. What is uncertain (and why)? How is the uncertainty set defined? Do we have a probability distribution?

2. What could go wrong if we just took the deterministic solution?

3. Which solutions are considered 'feasible'?

4. How are solutions *evaluated under uncertainty*? Tip illustrate this on the deterministic solution

5. Solution method from paper,

illustrated on example(s) from introductory group presentation

(6. Own ideas - see next slide)

often most suitable for activating the audience

#### Activate your audience:

You present uncertainty set, optimality concept, and (idea of) the method. And let the other students do/help with the rest.

#### Report

At the end of the term, each student hands in a report consisting of the following parts:

- 1. basic information on the *deterministic* problem
- 2. example instances
- 3. uncertainty & uncertainty set
- 4. solution space (feasibility) and evaluation under uncertainty
- 5. solution method
- 6. own ideas

#### Report

At the end of the term, each student hands in a report consisting of the following parts:

- 1. basic information on the *deterministic* problem
- 2. example instances

group work, all 'shortest path students'submit the same (and all 'knapsack students' submit the same), individual for other problems

- 3. uncertainty & uncertainty set
- 4. solution space (feasibility) and evaluation under uncertainty
- 5. solution method

individual work

6. own ideas

individual or group work

### Report

At the end of the term, each student hands in a report consisting of the following parts:

1. basic information on the *deterministic* problem

pprox 2 pages

- 2. example instances group work, all 'shortest path students'submit the same (and all 'knapsack students' submit the same), individual for other problems
- 3. uncertainty & uncertainty set
- 4. solution space (feasibility) and evaluation under uncertainty
- 5. solution method

individual work

pprox 2 pages

 $\approx 10 \text{ pages}$ 

6. own ideas

individual or group work
#### Report

At the end of the term, each student hands in a report consisting of the following parts:

1. basic information on the *deterministic* problem

pprox 2 pages

- group work, all 'shortest path students'submit the same (and all 'knapsack students' submit the same), individual for other problems
- 3. uncertainty & uncertainty set
- 4. solution space (feasibility) and evaluation under uncertainty
- 5. solution method

2. example instances

pprox 10 pages



# Grading criteria

- clear structure
- concepts and methods are clearly presented and well understood
- appropriate level of abstraction
- questions well answered
- appropriate length
- on group work component: all group members contribute
- specific to presentation: well-designed interactions, audience succesfully activated

## WueCampus and WueStudy

WueCampus: Course materials (including lecture slides) and course information, forum for questions and discussions, course communication, paper list, link for paper choice/assignment

Please sign up.

# WueCampus and WueStudy

WueCampus: Course materials (including lecture slides) and course information, forum for questions and discussions, course communication, paper list, link for paper choice/assignment

Please sign up.

**WueStudy:** here you register for the examination. Important: register until **31.05.**2024 or we cannot book your grade!

# Questions and support

Do you have any questions now?

presentation preparation:

- slides to be sent 10 days before presentation (Sunday evening) to marie.schmidt@uni-wuerzburg.de
- slide feedback meeting 7 days in advance (after seminar)

further question? meeitng timeslots bookable on doodle