Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #2

Graph Visualization (SS 2024)

Exercise 1 – Unit edge lengths

In a drawing of a graph G with *unit edge lengths* each edge is drawn as a line segment of length 1.

a) Prove or disprove that all trees admit a *crossing-free* drawing with unit edge lengths. **3 Points**

We now go one step further and consider drawings of G with unit edge lengths where the Euclidean distance between two vertices u and v is equal to the distance of u and v in G (i.e. equal to the number of edges of a shortest path from u to v).

b) Characterize the set of connected graphs that can be drawn in this way, i.e., show that *only* these graphs admit drawings with this property. **4 Points**

Exercise 2 – Adapting forces for positioning

In the force-directed approach, we may add additional forces to all or some vertices. Give functions with descriptions for forces that are suitable to

a) keep a vertex v close to a specified position, **1 Point**

b) position a vertex u close to the x-axis, **1 Point**

c) align an edge $\{a, b\}$ parallel to the y-axis (approximately), **1 Point**

d) draw directed edges upward. **1 Point**

Exercise 3 – Adapting forces for vertices with area > 0

The force-directed methods introduced in the lecture assume that all vertices are represented as points, i.e., disks with radius 0.

a) Which modifications are necessary to represent vertices as disks? **2 Points**

b) What about other convex shapes? **1 Point**

Exercise 4 – Tutte Drawings

Prove the following properties for Tutte drawings.

a) If G is connected, then a Tutte drawing can have vertex–vertex overlaps. **1 Point**

b) If G is 2-connected, then a Tutte drawing can have vertex–edge overlaps. **1 Point**

c) If G is 2-connected, then a Tutte drawing can have vertex–vertex overlaps. **1 Point**

d) In the literature, the Tutte forces are often described without dividing by the degree of the vertex:

$$f_{\text{attr}}(u, v) = \begin{cases} 0 & \text{, u fixed} \\ \|p_u - p_v\| \cdot \overrightarrow{p_u p_v} & \text{, else} \end{cases}$$

Find an example of a 3-connected graph where iteratively applying these forces does not find the equilibrium. Does that mean that no equilibrium exists?

Hint: Find a situation where a vertex "shoots" too far over the optimum position. **3 Points**

This assignment is due at the beginning of the next lecture, that is, on May 3 at 10:15 am. Please submit your solutions via WueCampus. The solutions will be discussed in the tutorial session on May 8.