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9. Exercise for "Algorithmen, KI & Data Science 1"

1 Metaheuristic Search

1. How do state space search problems differ from discrete constraint optimization problems?

    - Starting point: set of initial states (small subset of state space) vs whole state space
    - Finding next state: compute successor states based on allowed transitions vs compute neighborhood states (usually based on distance)
    - Ending point: goal state based on goal test (small subset of state space) vs. no explicit ending state, try to find state with maximum/minimum value of objective function f
    - Problem to solve: How to get to the goal state with minimal cost/maximal gain vs. find the state with minimal/maximal value
    - Guidance: Specific heuristics (estimate distance to goal state) to trim the number of paths to explore vs. problem agnostic metaheuristics to find next best solution in the neighborhood

2. Define the 0-1 knapsack problem as discrete constrained optimization problem. The 0-1 knapsack problem is defined as follows: given a set of N items, each with a weight w and a value v, determine the subset of items to choose such that the total weight is less than or equal to a given limit W and the total value is as large as possible.

Example 0-1 knapsack problem:
You are packing your bag pack for a trip. You have the following items at home:

- knife, weight 2, value 6
- water, weight 7, value 9
- sleeping bag, weight 5, value 7
- mobile phone, weight 1, value 2
- jacket, weight 2, value 3

With $W = 7$, you could pack {knife, sleeping bag} with weight 7 and value 13 or {mobile phone, jacket} with weight 3 and value 10 or ..., but you would not be allowed to pack {knife, water} with weight 9.

- $X = (x_1, x_2, ..., x_N)$, where N is the total number of items
- $D_1 = D_2 = ... = D_N \in \{0, 1\}$ because an item is either included in the knapsack or not
- $f(X) = \sum_{i=1}^{N} v_i x_i$, where v_i refers to the value of item x_i
- Constraint: $\sum_{i=1}^{N} w_i x_i \leq W$, where w_i refers to the weight of item x_i

3. How does simulated annealing escape local optima?

   Simulated annealing allows non-improving steps with a decreasing probability through the search.

4. How does genetic algorithm escape local optima?

   Genetic algorithm uses mutation (random change of values in a chromosome) to escape local optima (e.g., element change or element swap)

2 Constrained Satisfaction Problems

1. How do state space search problems differ from constrained satisfaction problems?

   a) Type of problems: Optimal path problems vs finding and any solution (all equally good)

b) Type of states: Complex, non-factorable states vs factorable states

c) Type of transitions: Explicit state transitions defined by the nature of the problem vs transitions from k to k+1 assigned variables

2. How many solutions does the map coloring problem from lecture 18 have? How many if two colors are allowed?

   Starting form SA, there are three possible colors to choose from. Next going to WA which has two colors left. All other regions in the mainland have only one color left. Therefore, there are $2 \times 3 = 6$ choices for the mainland. As T is not connected to the mainland, we can choose any of the three colors for T. This results in $3 \times 6 = 18$ possible solutions. If only two colors are allowed no solution exists.

3. Explain why choosing the variable that is most constrained, but the value that is least constrained is considered a good heuristic?

   Choosing the variable that is most constrained allows the algorithm to fail early. Therefore, large parts of the search tree are pruned. Choosing the value that is least constrained gives higher chances on avoiding conflicts in upcoming steps. Hence, it increases the chances of finding and actual solution.

4. Consider the problem of placing k knights on an $n \times n$ chessboard such that no two knights are attacking each other, where k is given and $k \leq n^2$. Choose a CSP formulation giving your (a) variables, (b) the domain of each variable, and (c) the constraints.

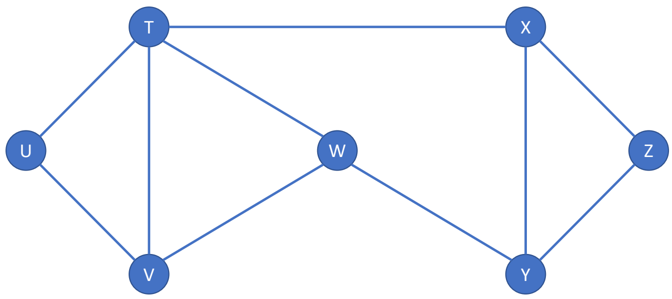
   (a) Variables: $X = (x_1, x_2, ..., x_k)$, where k is the number of knights placed on the chess board and x_i represents the position of each knight.

   (b) Domain: $x_i \in \{(a, 1), (a, 2), ..., (h, 8)\}$, where each tuple is a position on the chess board

   (c) Constraints: Two knights cannot share the same position AND a move from any knight does not result in two knights sharing the same position

5. Consider the following graph that represents a coloring problem. Each vertex of the graph should be colored with either blue (B), green (G) or red (R), such that adjacent nodes do not have the same color (similar to the coloring problem shown

in lecture 18).



Fill out the table below by applying the backtracking algorithm with inference, degree heuristic and least constraining value heuristic to the coloring problem. In case the heuristics do not produce a single next best move, process the variables and values in lexicographical order. The assignment of a value to a variable is depicted with "(a)".

	T	U	V	W	X	Y	Z
Init	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R
1. Iter	B (a)	G,R	G,R	G,R	G,R	B,G,R	B,G,R
2. Iter							
3. Iter							
4. Iter							
5. Iter							
6. Iter							
7. Iter							
8. Iter							

	T	U	V	W	X	Y	Z
Init	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R	B,G,R
1. Iter	B (a)	G,R	G,R	G,R	G,R	B,G,R	B,G,R
2. Iter		G,R	G,R	G,R	G,R	B (a)	G,R
3. Iter		R	G (a)	R	G,R		G,R
4. Iter		R		R	G (a)		R
5. Iter		R (a)		R			R
6. Iter				R (a)			R
7. Iter							R (a)

6. Implement the class $ColoringCSP$ and the additional methods in the provided $.ipynb$ to solve coloring problems similar to the one presented in lecture 18. You should select variables and colors in lexicographical order. In the provided $.ipynb$, you can find a cell that provides examples on how to use the class' attributes. Implement the following methods:

   a) $complete(s, csp)$: The method takes the current assignment s and the csp as input. It returns $True$ if the assignment is complete and $False$ otherwise.

b) $select\_unassigned\_var(csp)$: The method takes the csp as input. It returns an unassigned variable in lexicographical order.

c) $order\_values(csp)$: The method takes the csp as input. It returns the values in lexicographical order.

d) $violates(s, v, val)$: The method is a class method of $ColoringCSP$. It takes the current assignment s, and the new variable v together with its new value val as input. It returns $True$ if the new assignment violates the constraints and $False$ otherwise.

e) $backtrack(s, csp)$: Implement the naive backtracking as shown in lecture 18.