

# Algorithmen, KI und Data Science 1 (AKIDS 1): **Graphs**

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# Breadth-First-Search



# Exercise 1.1

What is the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of input?

## Exercise 1.1 - Solution

- Check for each element  $v$  whether it is adjacent to  $u$
- Each iteration takes  $O(V)$
- We go through all vertices
- Runtime:  $O = (V^2)$

```
def bfs(G, s):
    for each vertex u in G.V-{s}:
        u.state = 0
        u.dist = inf # big int
        u.parent = null

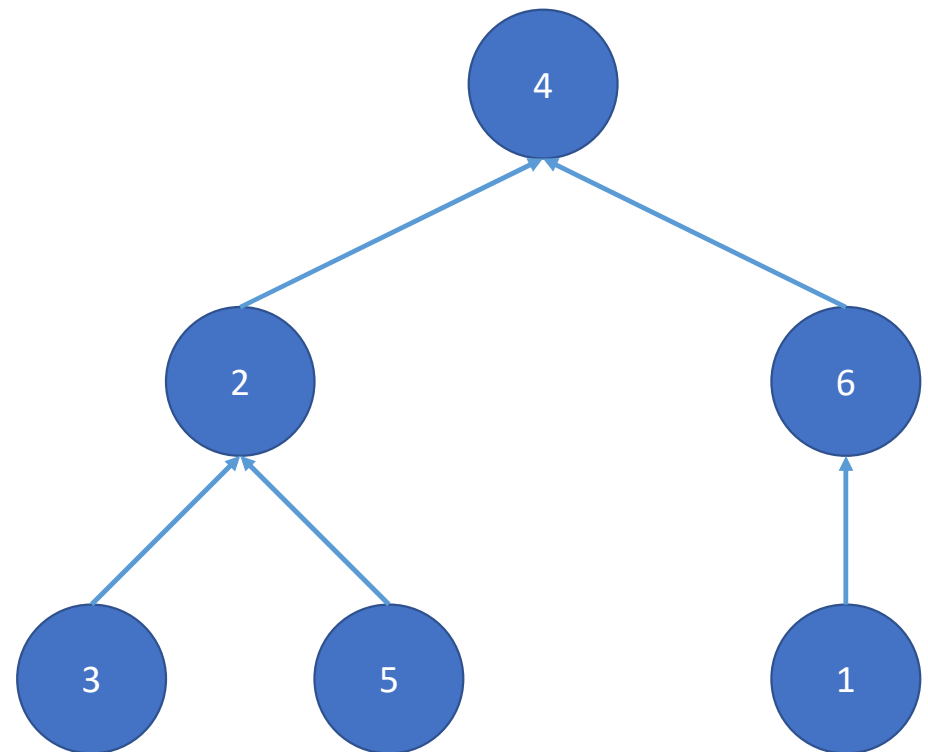
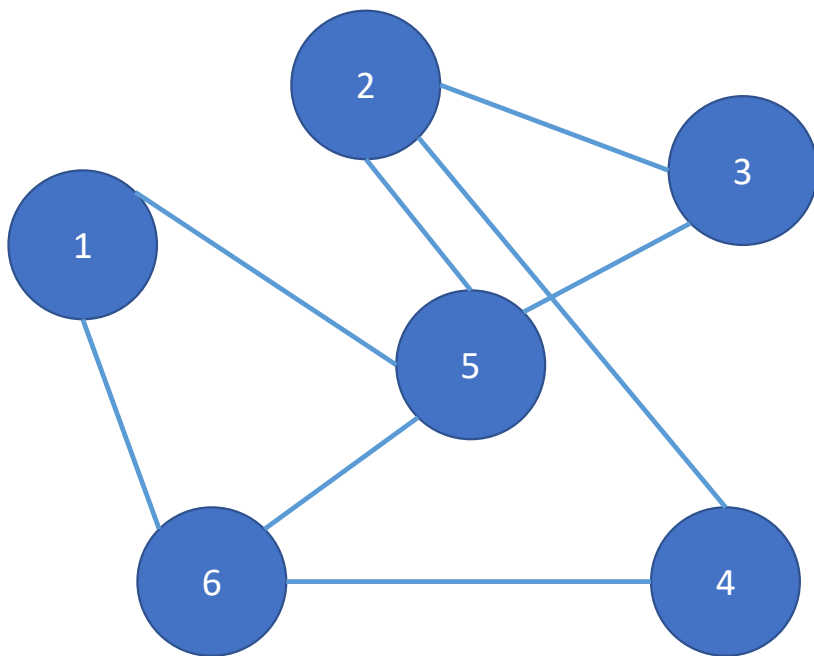
    qd = [] # empty queue
    s.state = 1 # discovered
    enqueue(qd, s)
    while not is_empty(qd):
        u = dequeue(qd)
        for vertex v in G.Adj[u]:
            if v.state == 0 # so far undiscovered:
                v.state == 1 # discovered
                v.dist == u.dist + 1
                v.parent = u
                enqueue(qd, v)
    u.state = 2
```



## Exercise 1.2

Does the BFS tree (see L10-slide 24) depend on the ordering of the adjacency lists?  
Explain by giving an example.

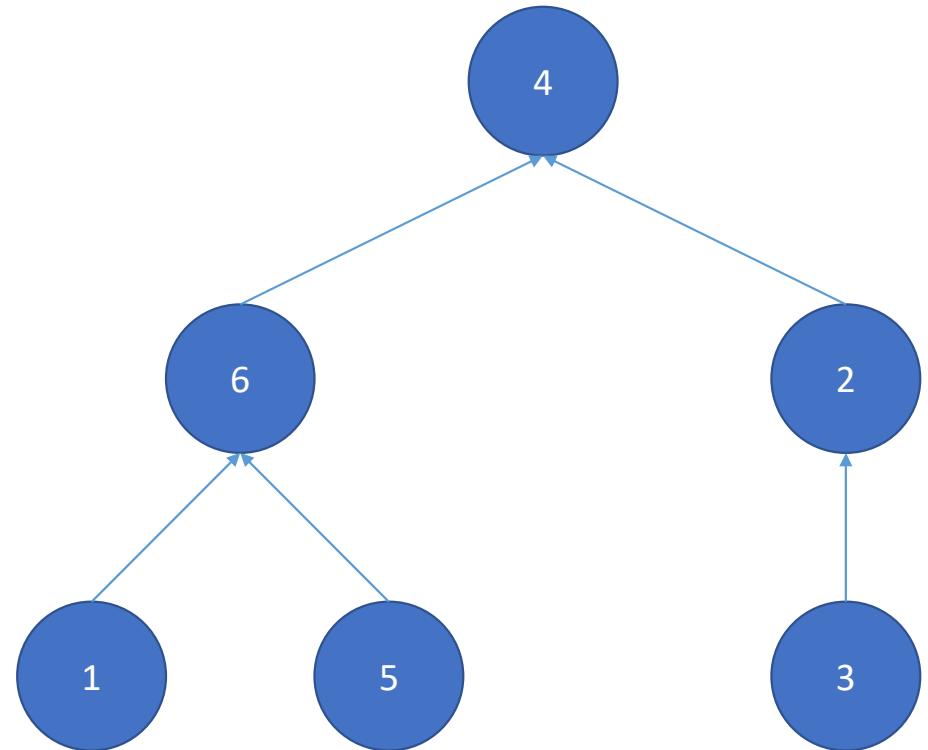
## Exercise 1.2 – From lecture slide





## Exercise 1.2 - Solution

- Change order of 2 and 6 in adjacency list of 4





# Depth-First-Search



# Exercise 2.1

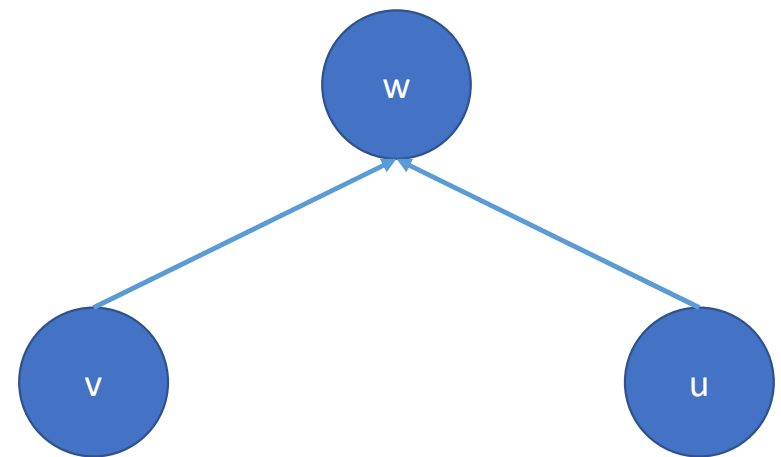
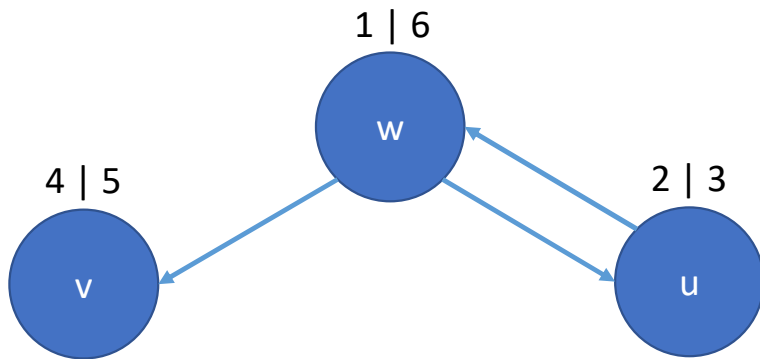
## Exercise 2.1 – Recap DFS

```
dfs(G)
  for each vertex u in G.V
    u.state = 0
  time = 0
  for each vertex u in G.V
    if u.state == 0
      dfs_visit(G, u)
```

```
dfs_visit(G, u)
  time = time + 1
  u.state = 1 # visited
  u.vt = time
  for each vertex v in G.Adj[u]
    if v.state == 0 # if v unvisited
      dfs_visit(G, v)
  u.state = 2 # finished
  time = time + 1
  u.ft = time
```

Give a counterexample to the conjecture that if a directed graph  $G$  contains a path from  $u$  to  $v$ , and if  $u.vt < v.vt$  in a depth-first search of  $G$ , then  $v$  is a descendant of  $u$  in the depth-first tree/forest produced.

## Exercise 2.1 - Solution



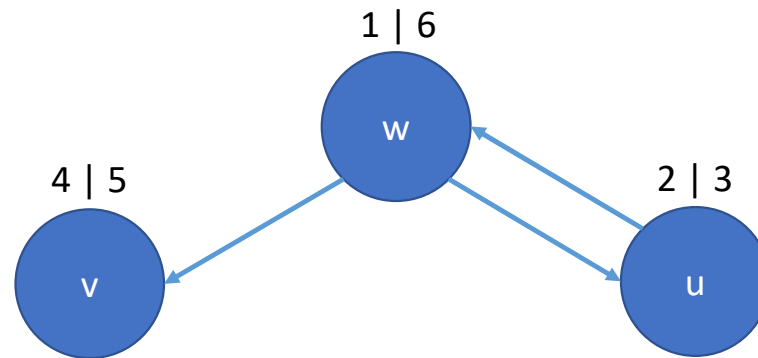


## Exercise 2.2

Give a counterexample to the conjecture that if a directed graph  $G$  contains a path from  $u$  to  $v$ , then any depth-first search must result in  $v.vt \leq u.ft$ .



## Exercise 2.2 - Solution





# Topological Sort



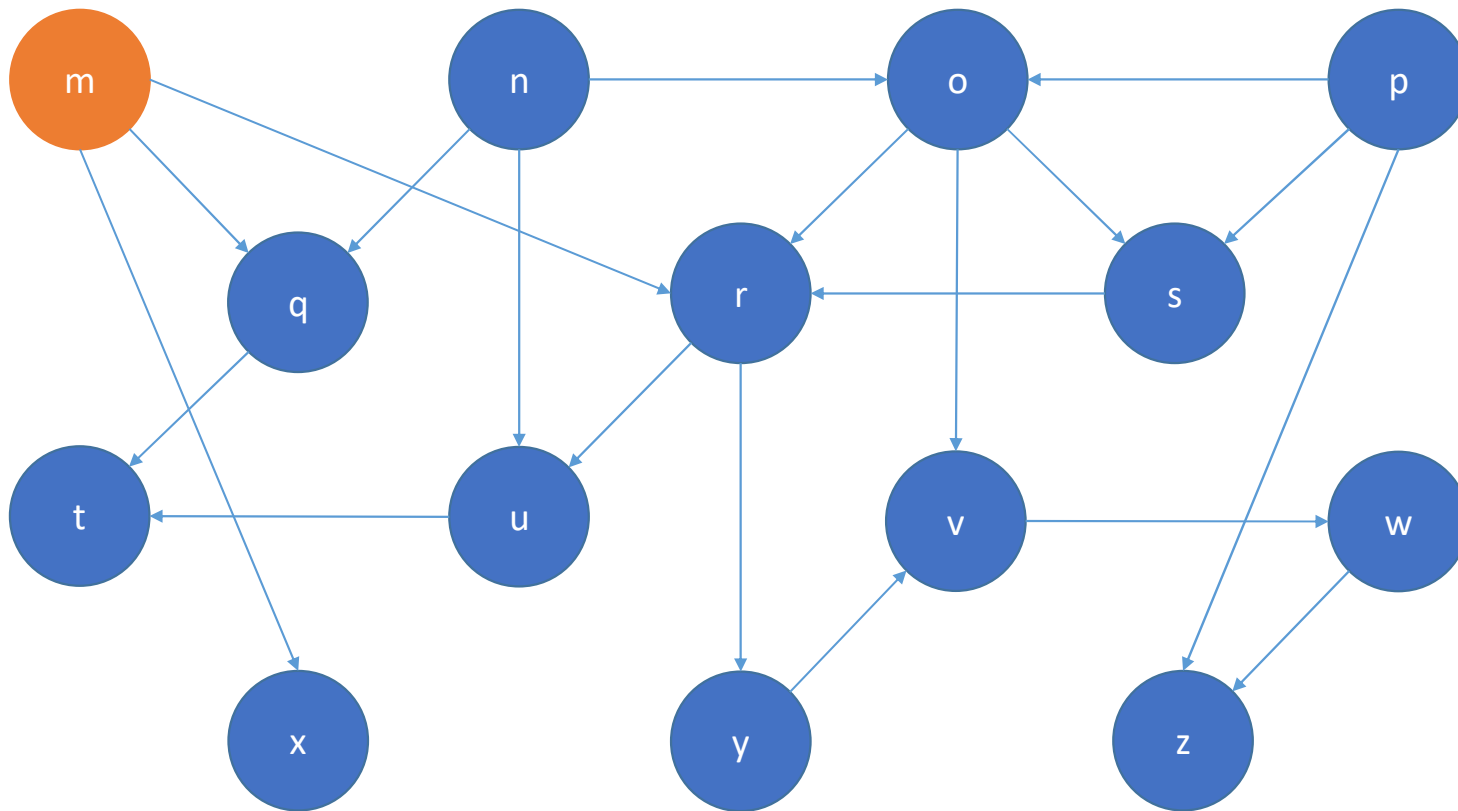
# Exercise 3.1

Show the ordering of vertices produced by topological-sort when it is run on the below graph. Include start and finish times for each vertex. Assume that the DFS procedure considers the vertices in alphabetical order and assume that each adjacency list is ordered alphabetically.

## Exercise 3.1 – Recap topological sort

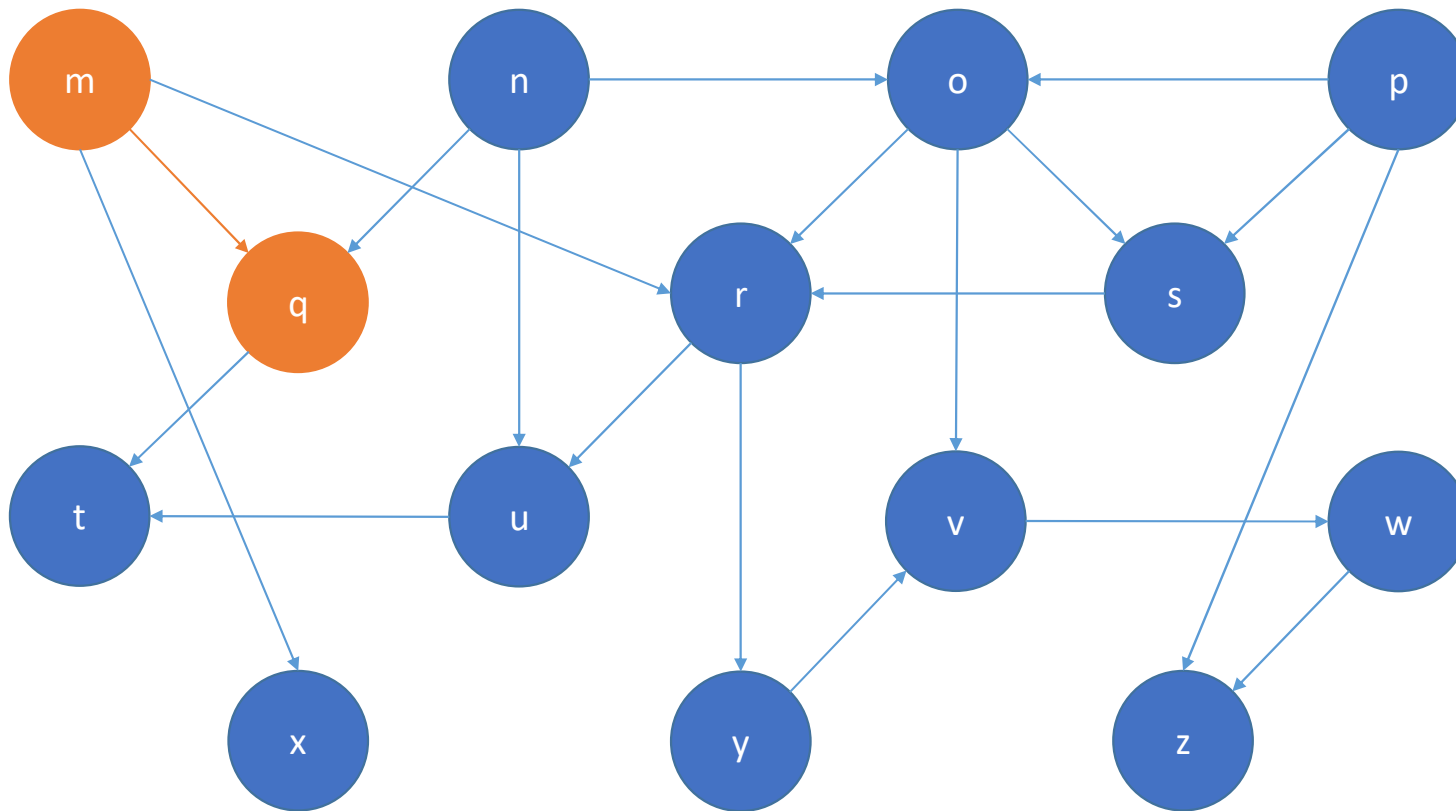
- Topological sort states:
  - Given a graph  $G$  (directed and acyclical) and edge  $(u, v)$ ,  $u$  appears before  $v$  in the topological sorting
- Algorithm:
  - Run DFS
  - Output the vertices in descending ft order

# Exercise 3.1



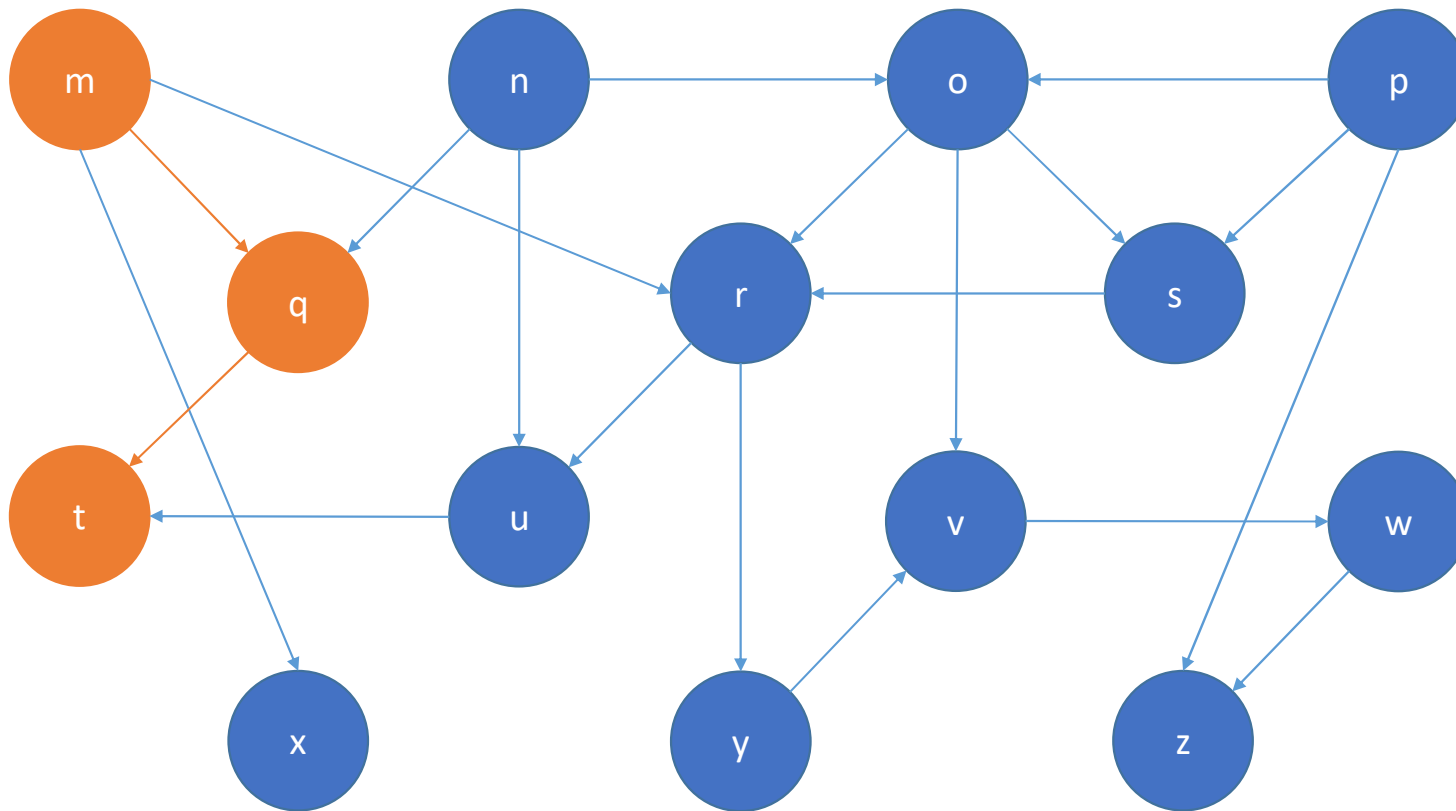
	vt	ft
m	1	
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x		
y		
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## Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	
r		
s		
t		
u		
v		
w		
x		
y		
z		

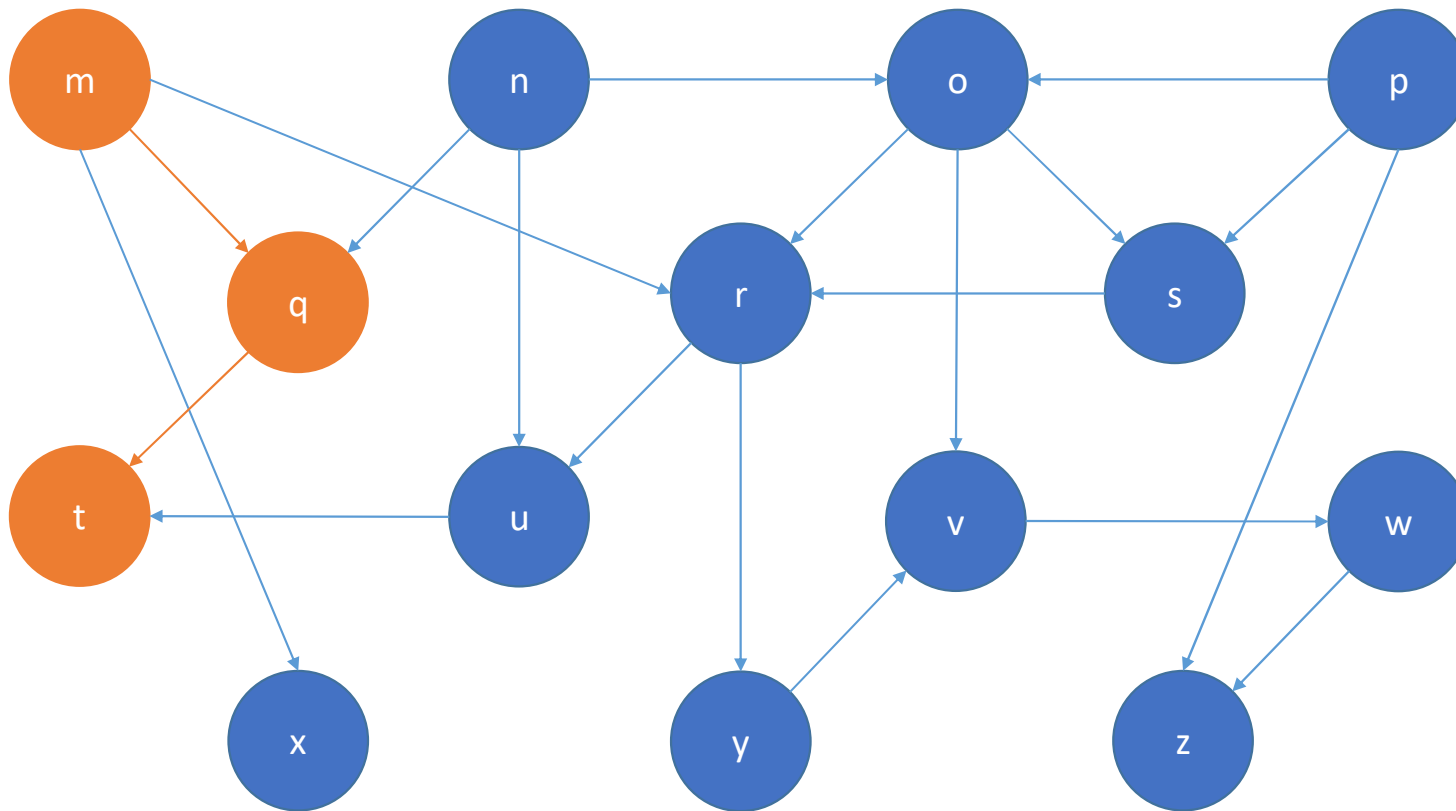
## Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	
r		
s		
t	3	
u		
v		
w		
x		
y		
z		

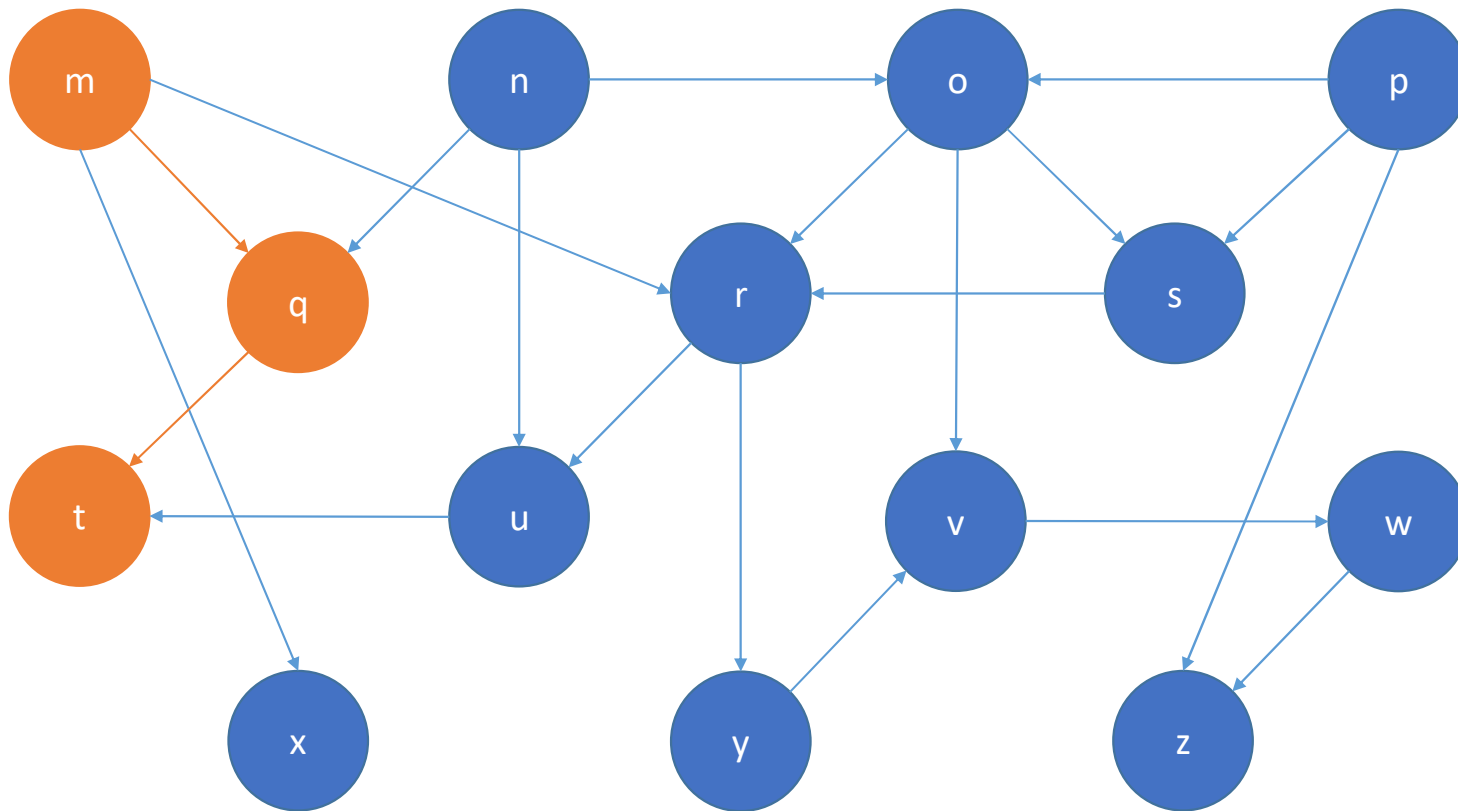


## Exercise 3.1



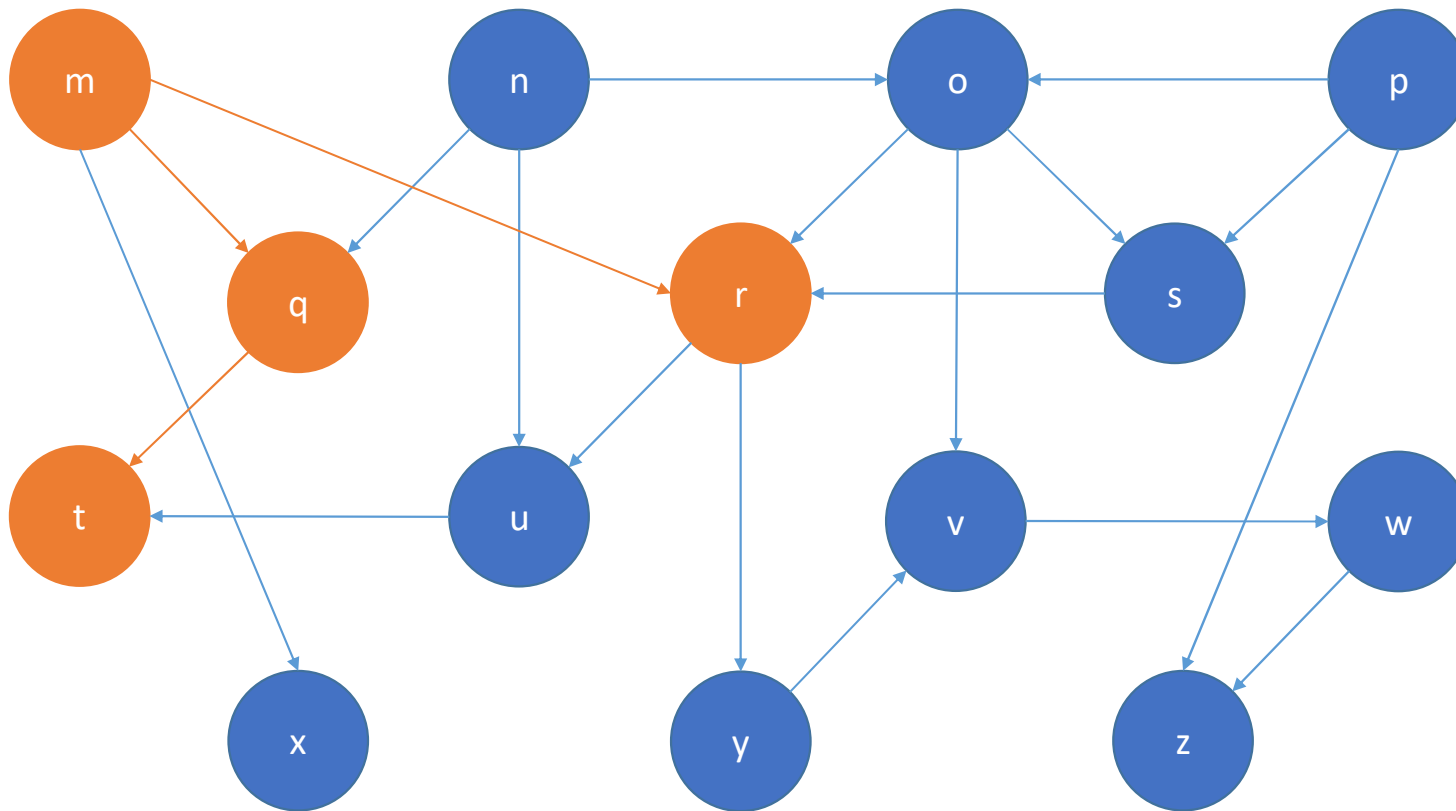
	vt	ft
m	1	
n		
o		
p		
q	2	
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s		
t	3	4
u		
v		
w		
x		
y		
z		

# Exercise 3.1



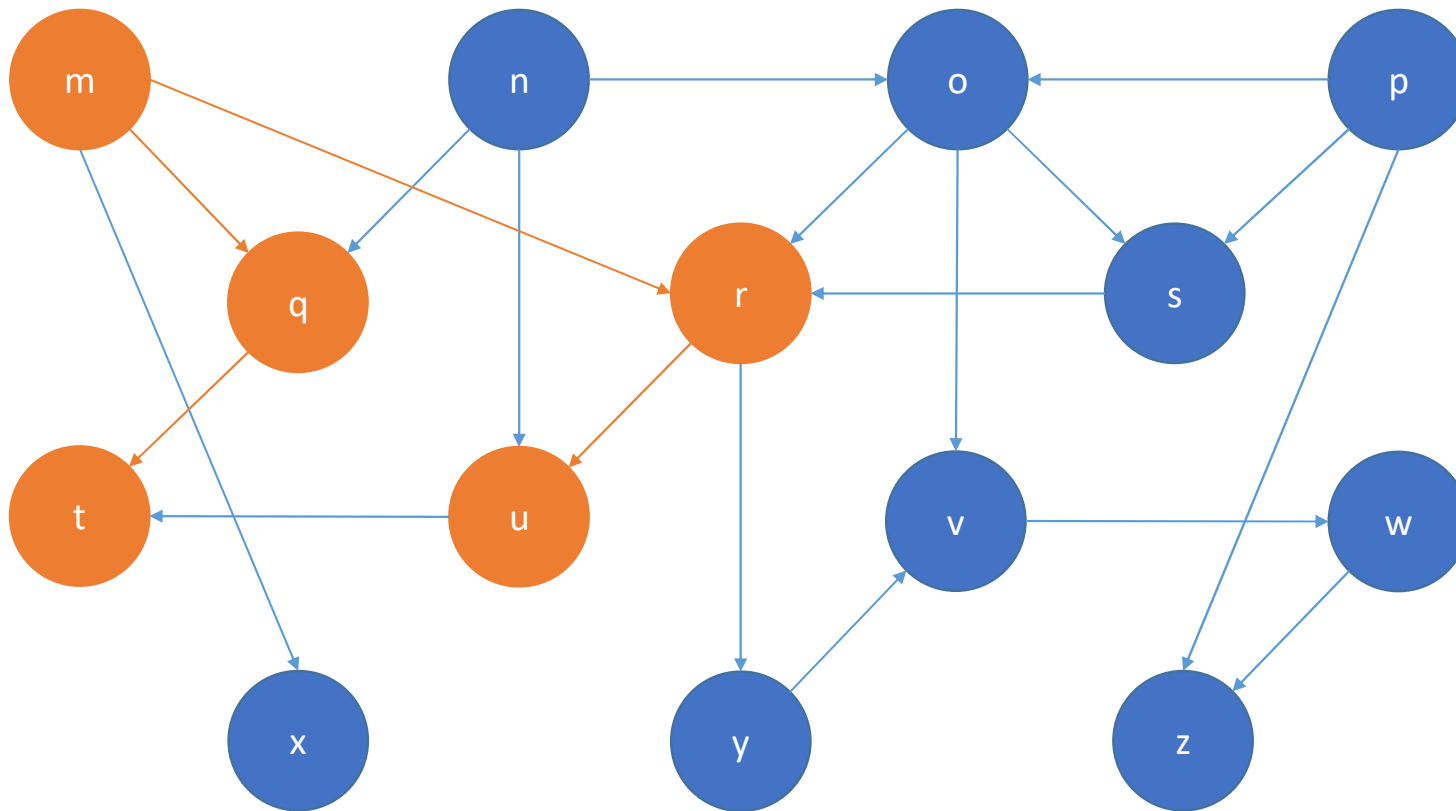
	vt	ft
m	1	
n		
o		
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q	2	5
r		
s		
t	3	4
u		
v		
w		
x		
y		
z		

## Exercise 3.1



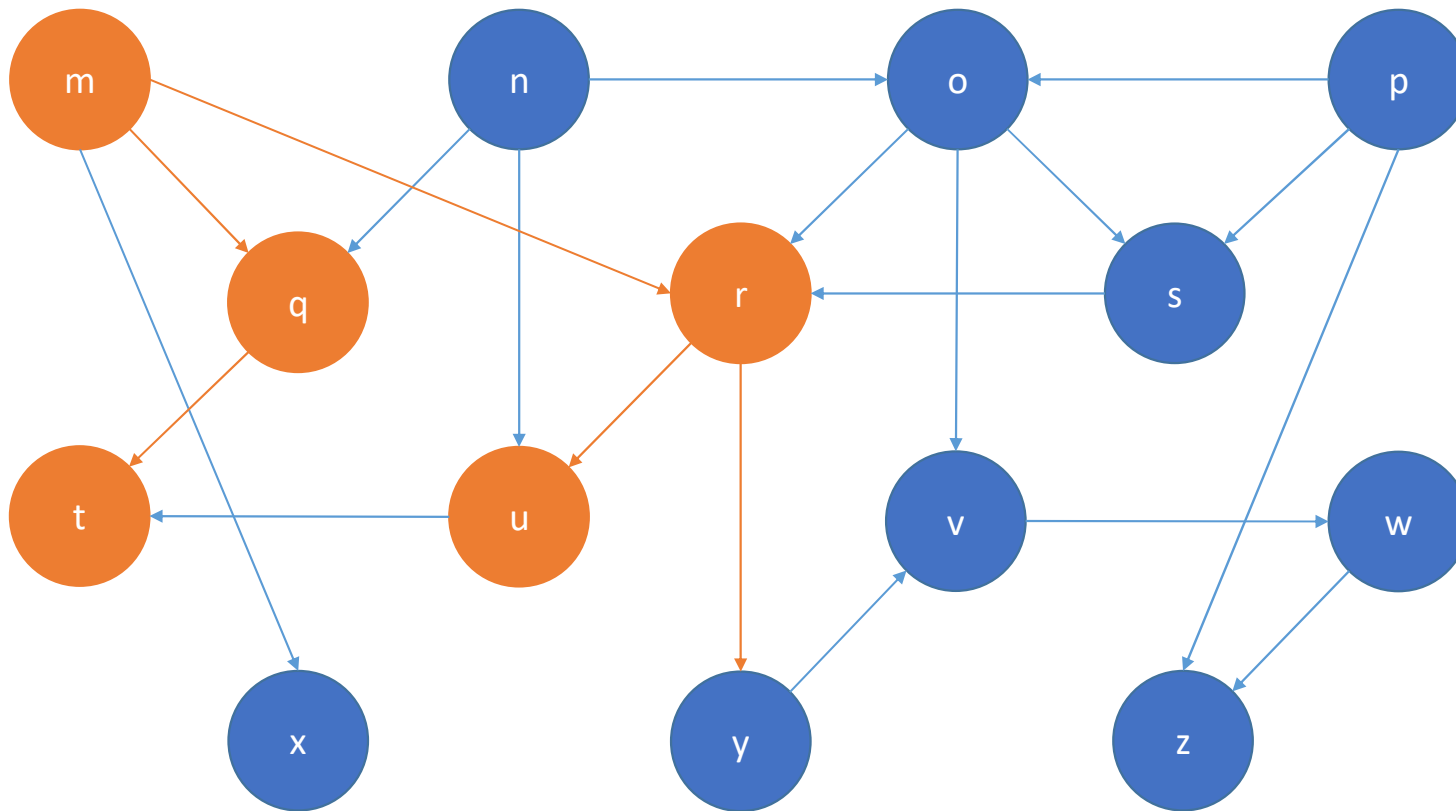
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q	2	5
r	6	
s		
t	3	4
u		
v		
w		
x		
y		
z		

## Exercise 3.1



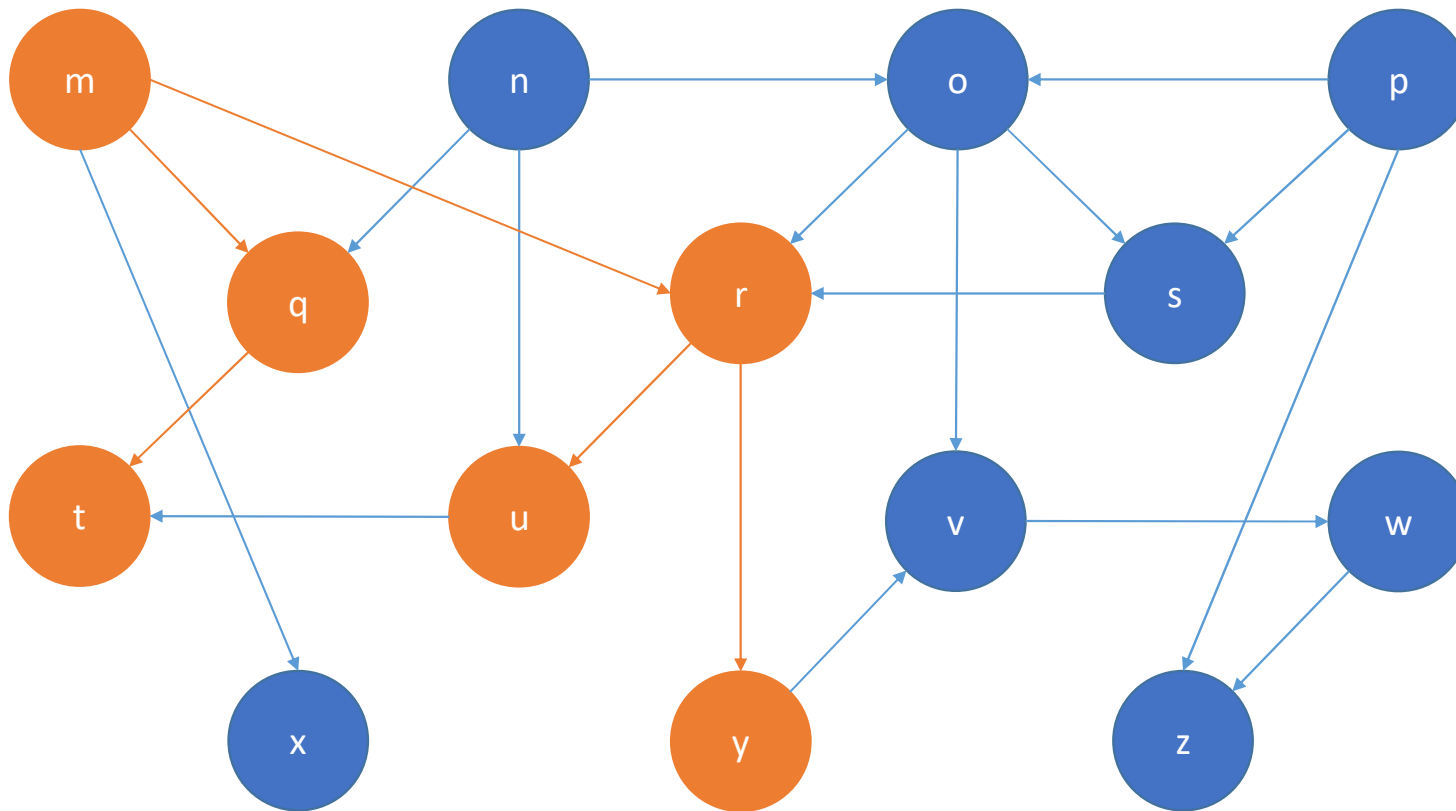
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m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	
v		
w		
x		
y		
z		

## Exercise 3.1



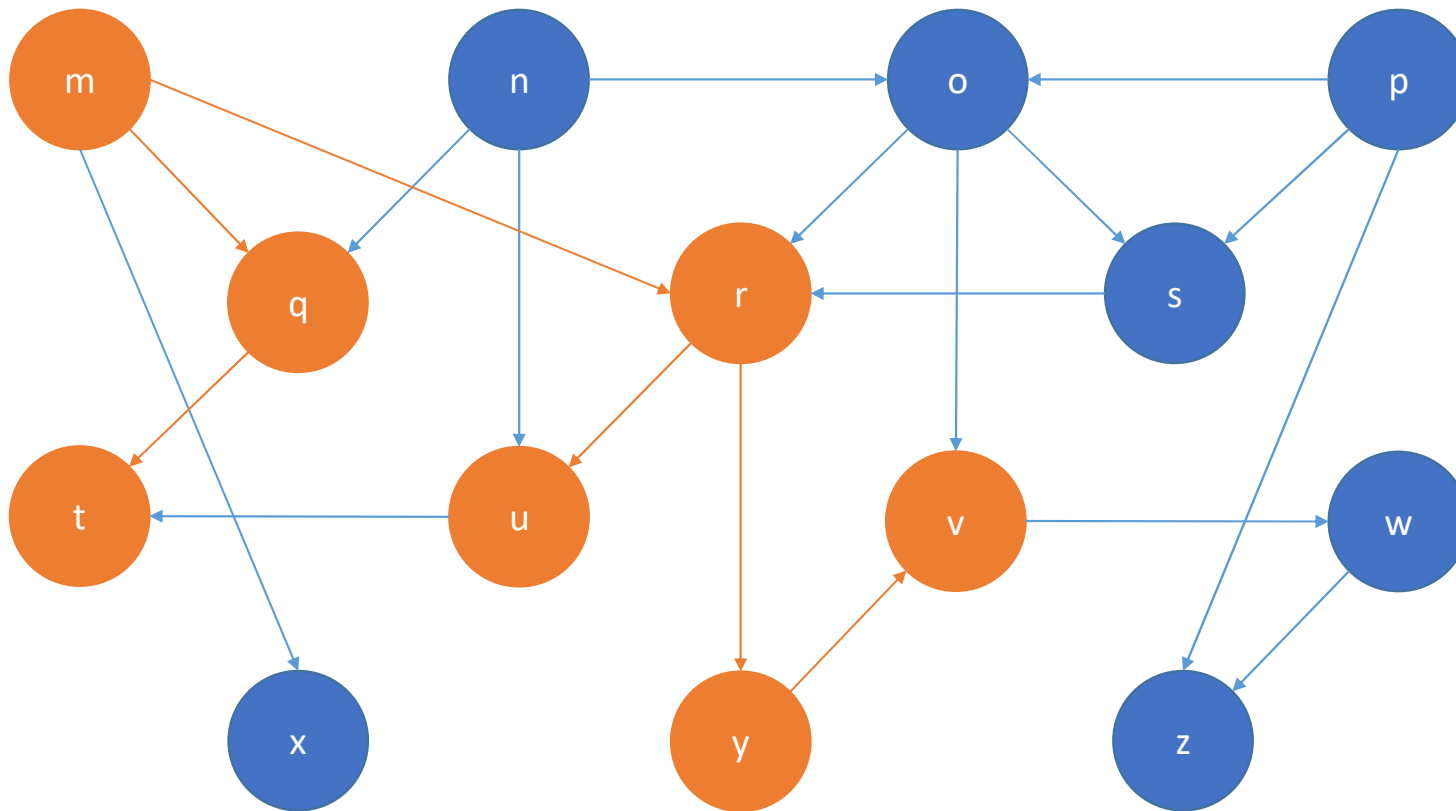
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v		
w		
x		
y		
z		

## Exercise 3.1



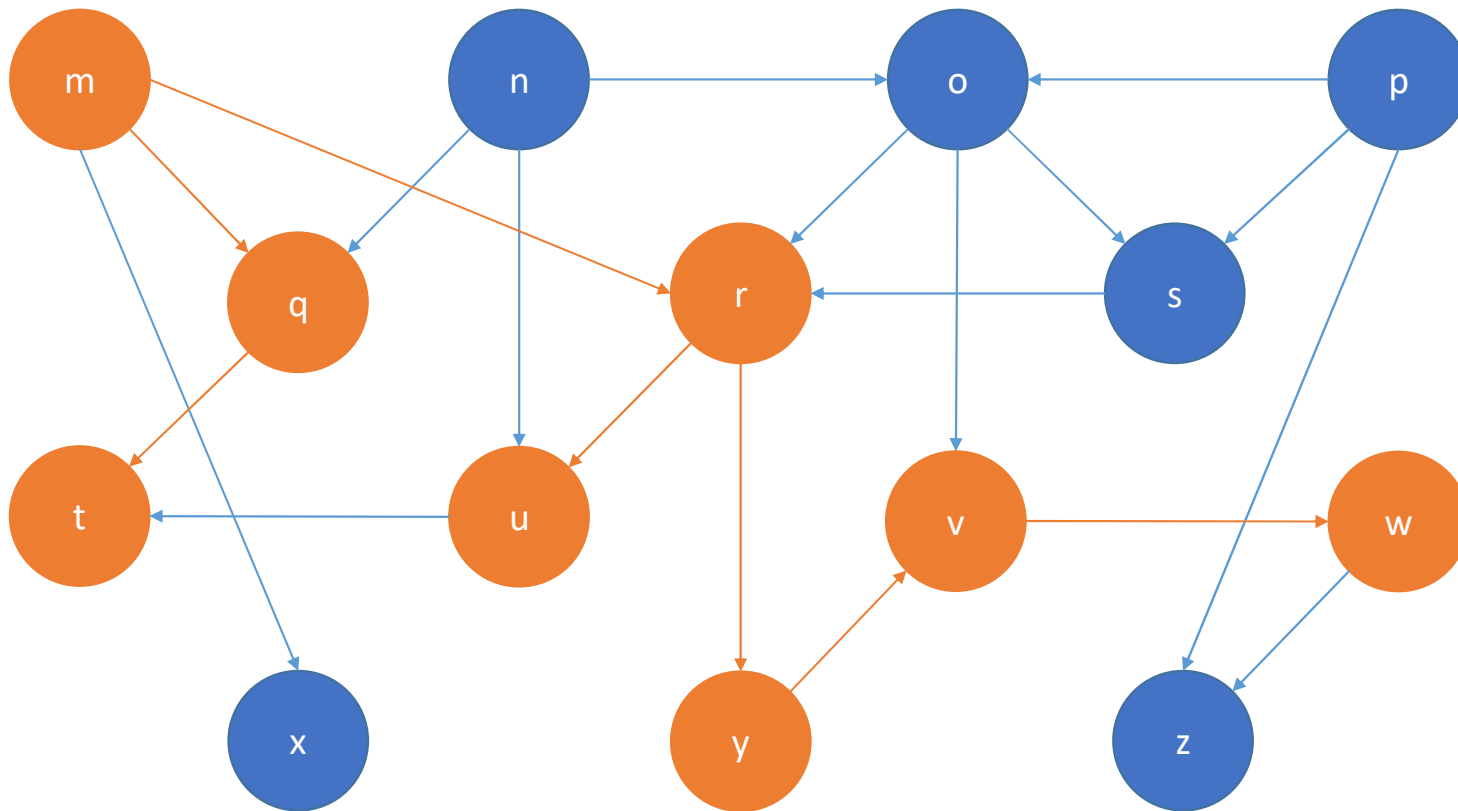
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n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v		
w		
x		
y	9	
z		

## Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	
w		
x		
y	9	
z		

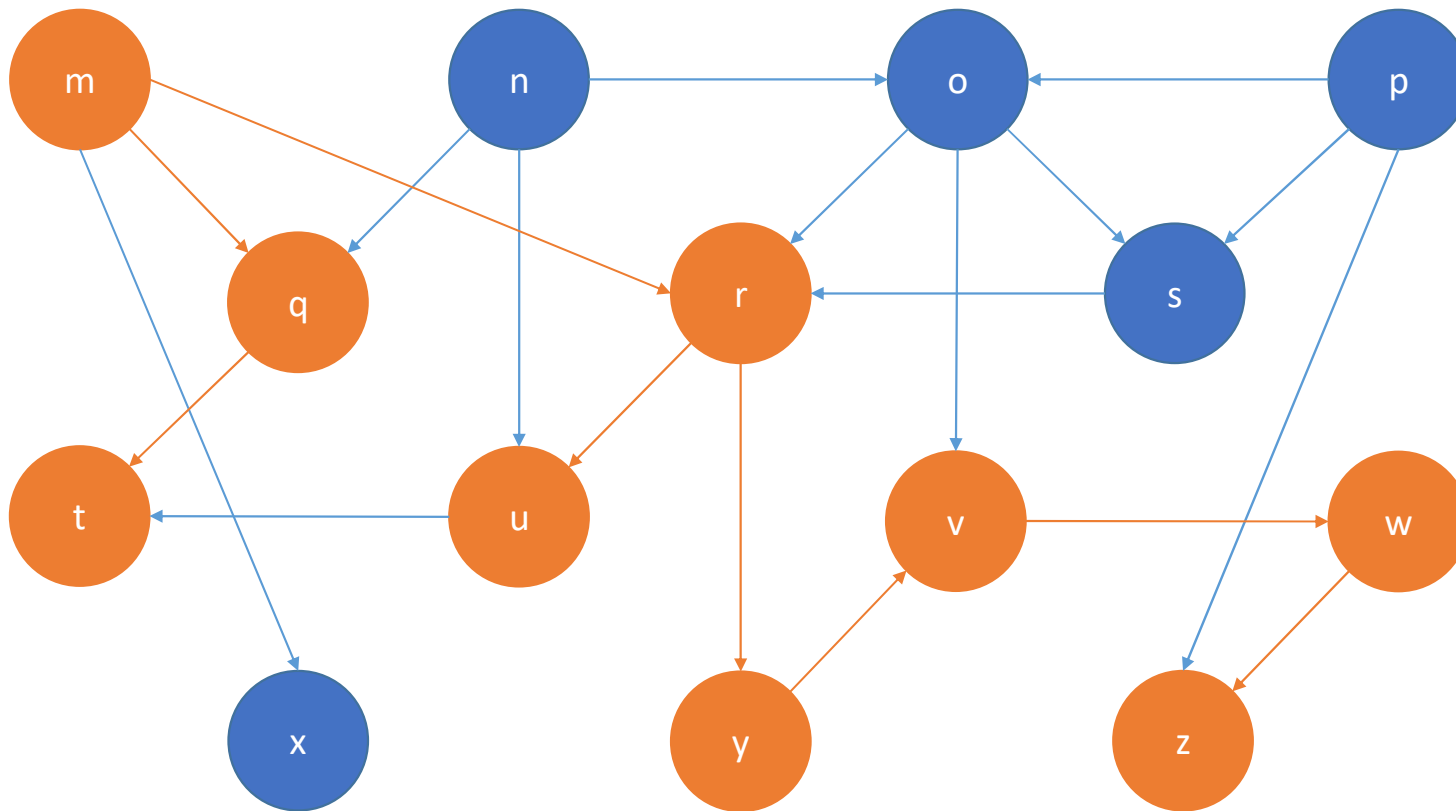
# Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	
w	11	
x		
y	9	
z		

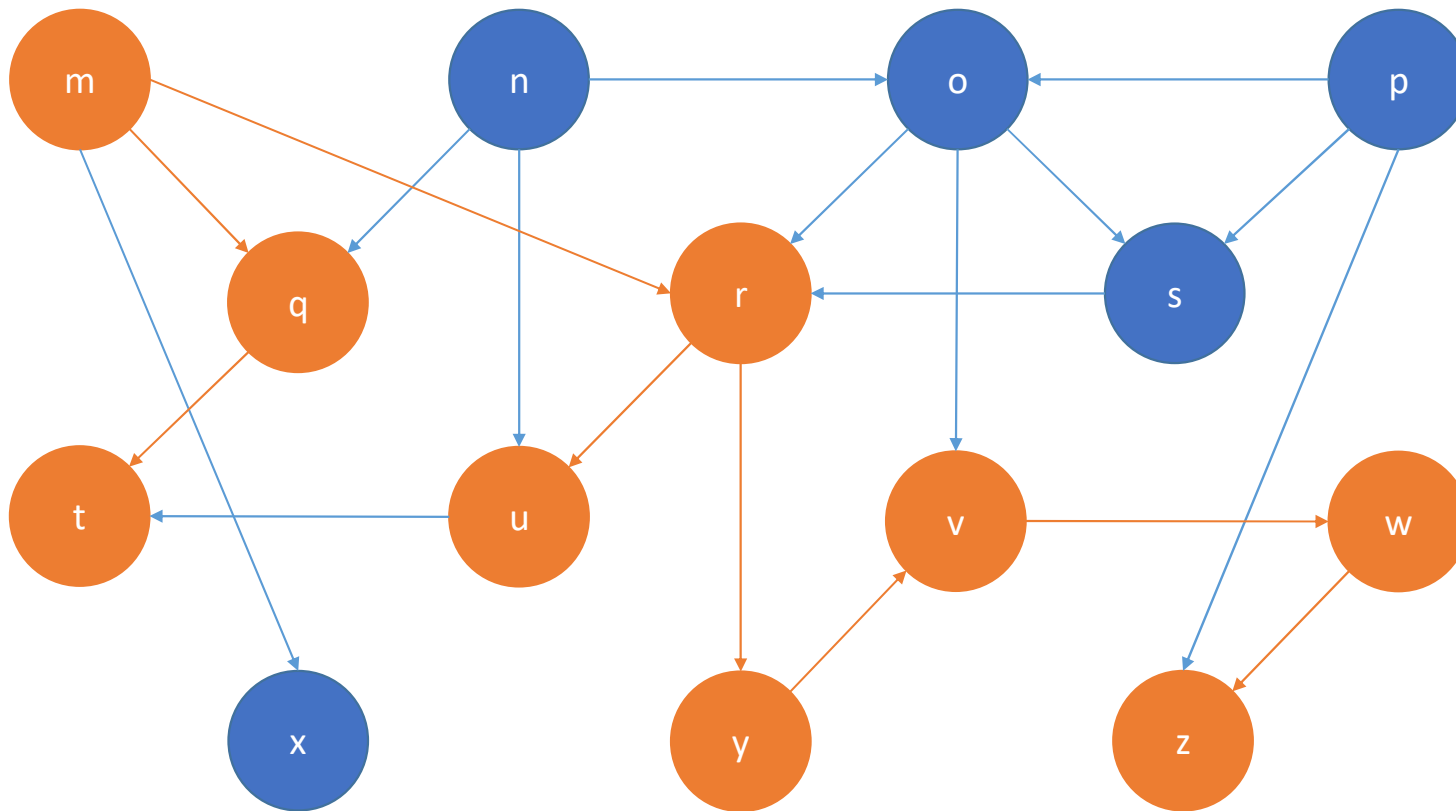


## Exercise 3.1



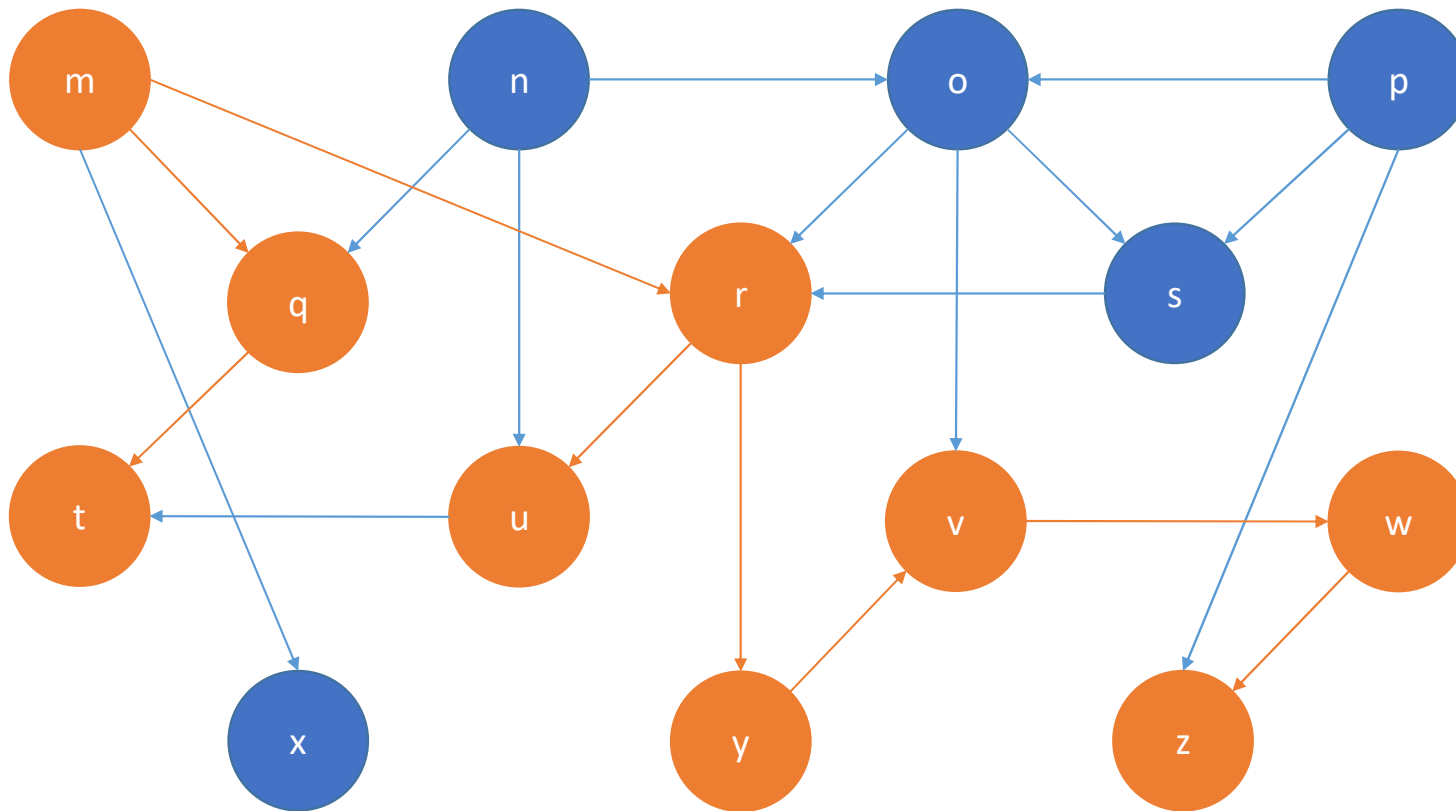
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	
w	11	
x		
y	9	
z	12	

## Exercise 3.1



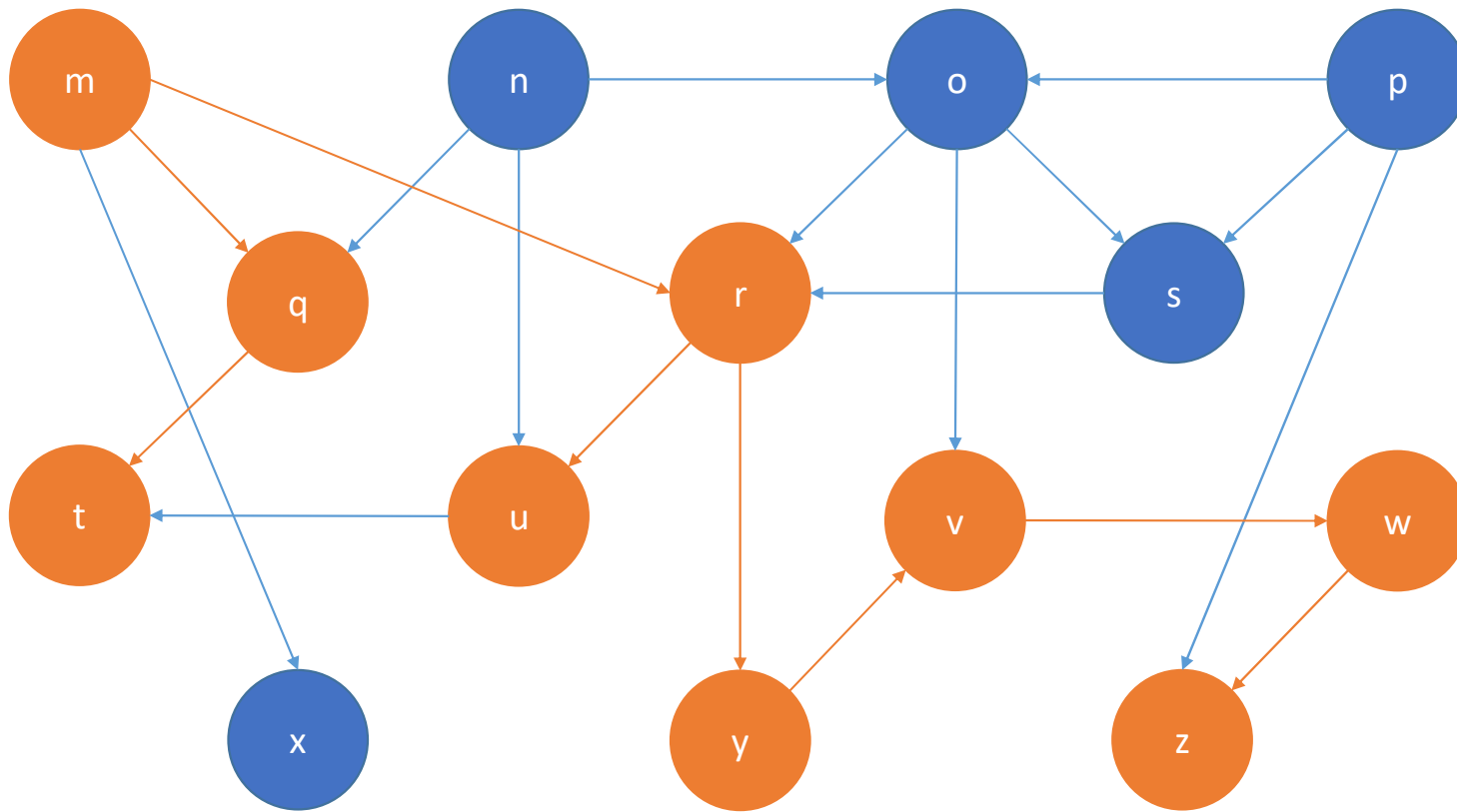
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	
w	11	
x		
y	9	
z	12	13

## Exercise 3.1



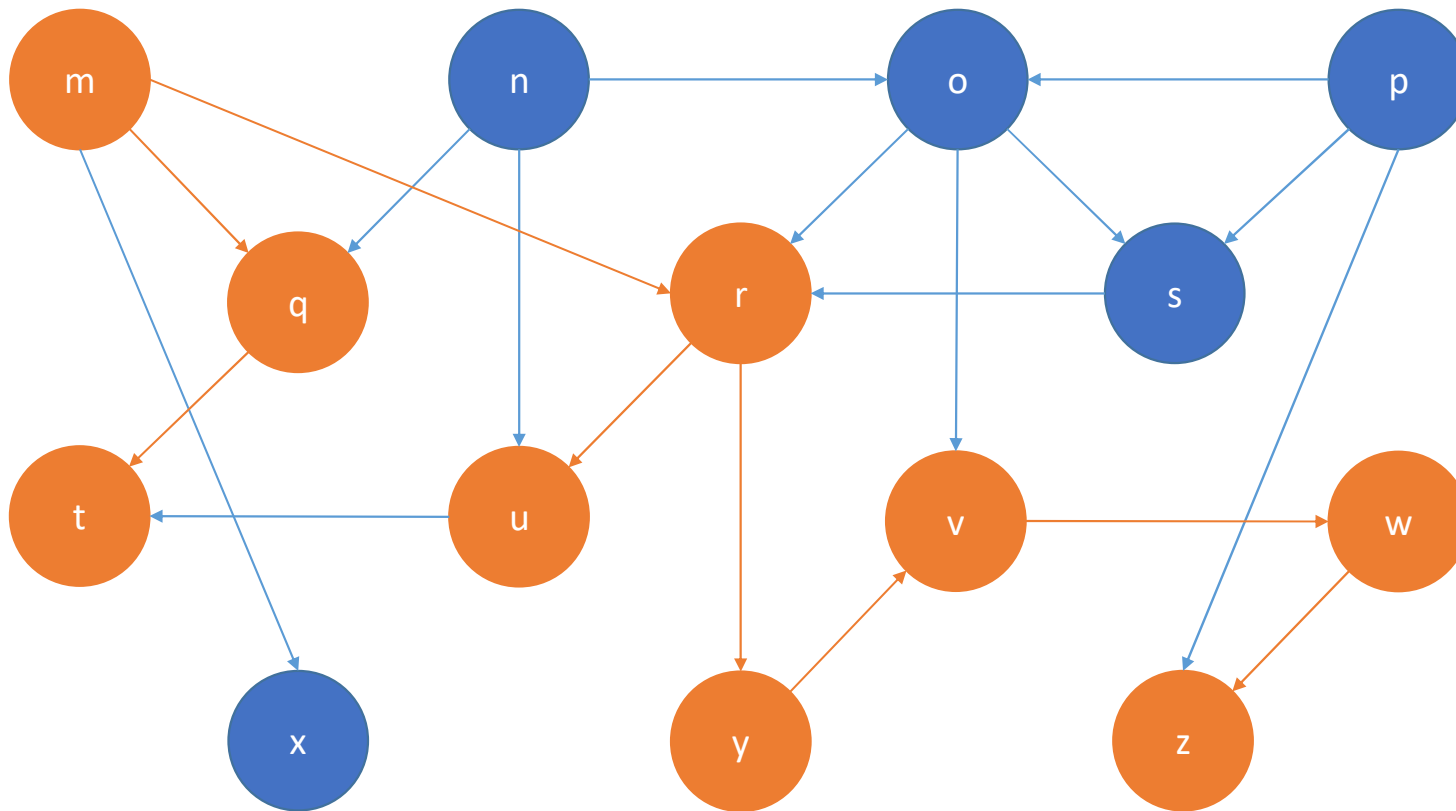
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	
w	11	14
x		
y	9	
z	12	13

# Exercise 3.1



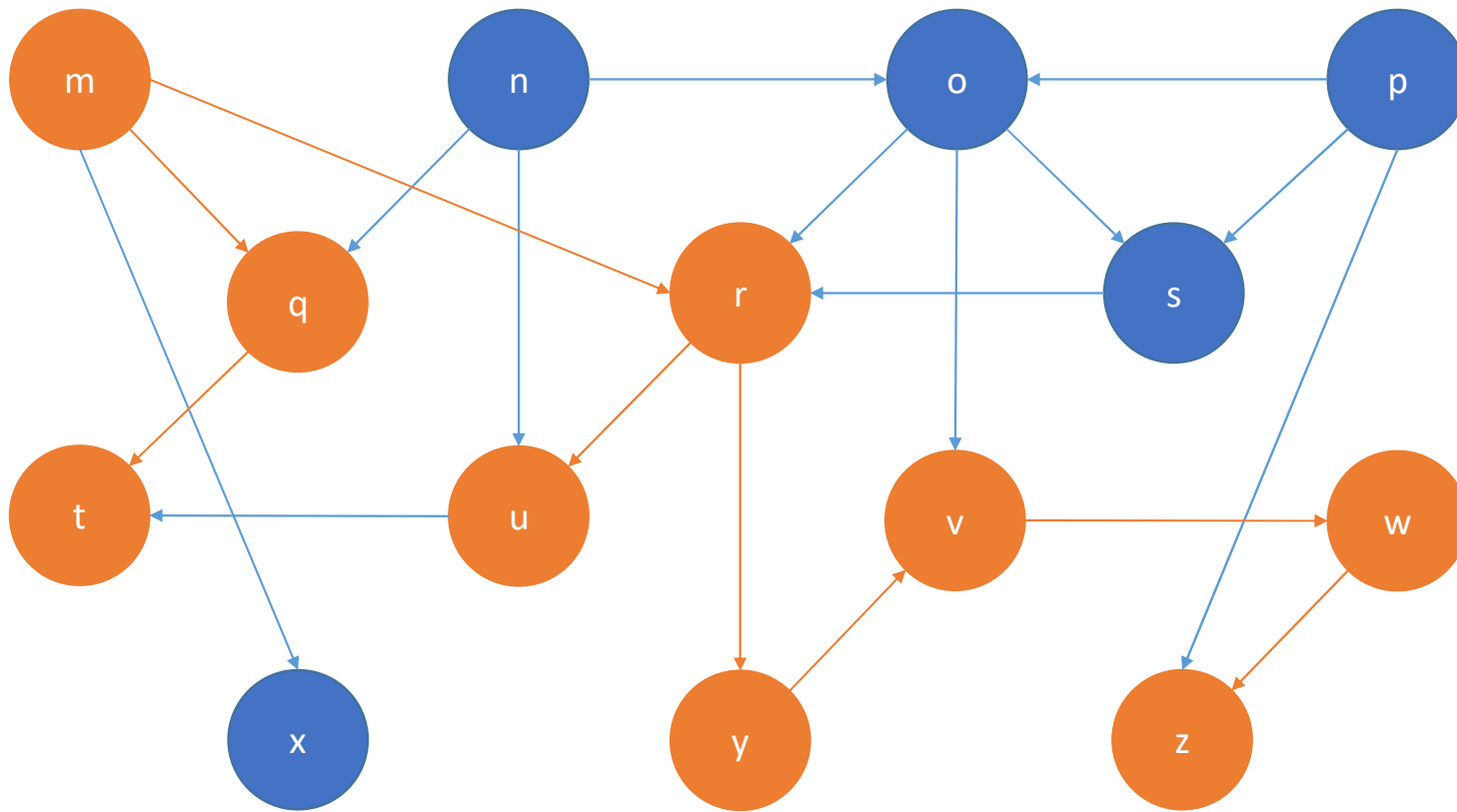
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	15
w	11	14
x		
y	9	
z	12	13

## Exercise 3.1



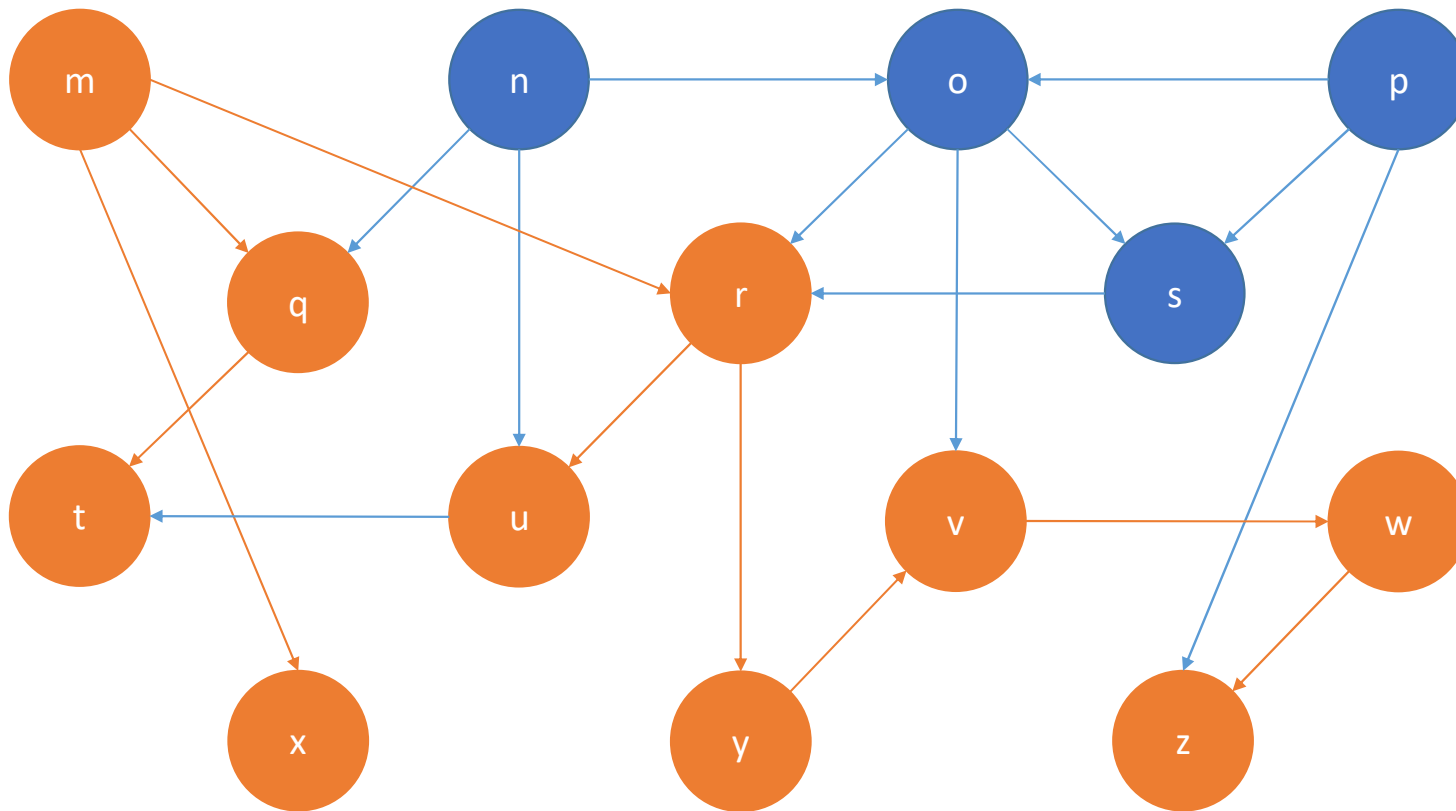
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	
s		
t	3	4
u	7	8
v	10	15
w	11	14
x		
y	9	16
z	12	13

# Exercise 3.1



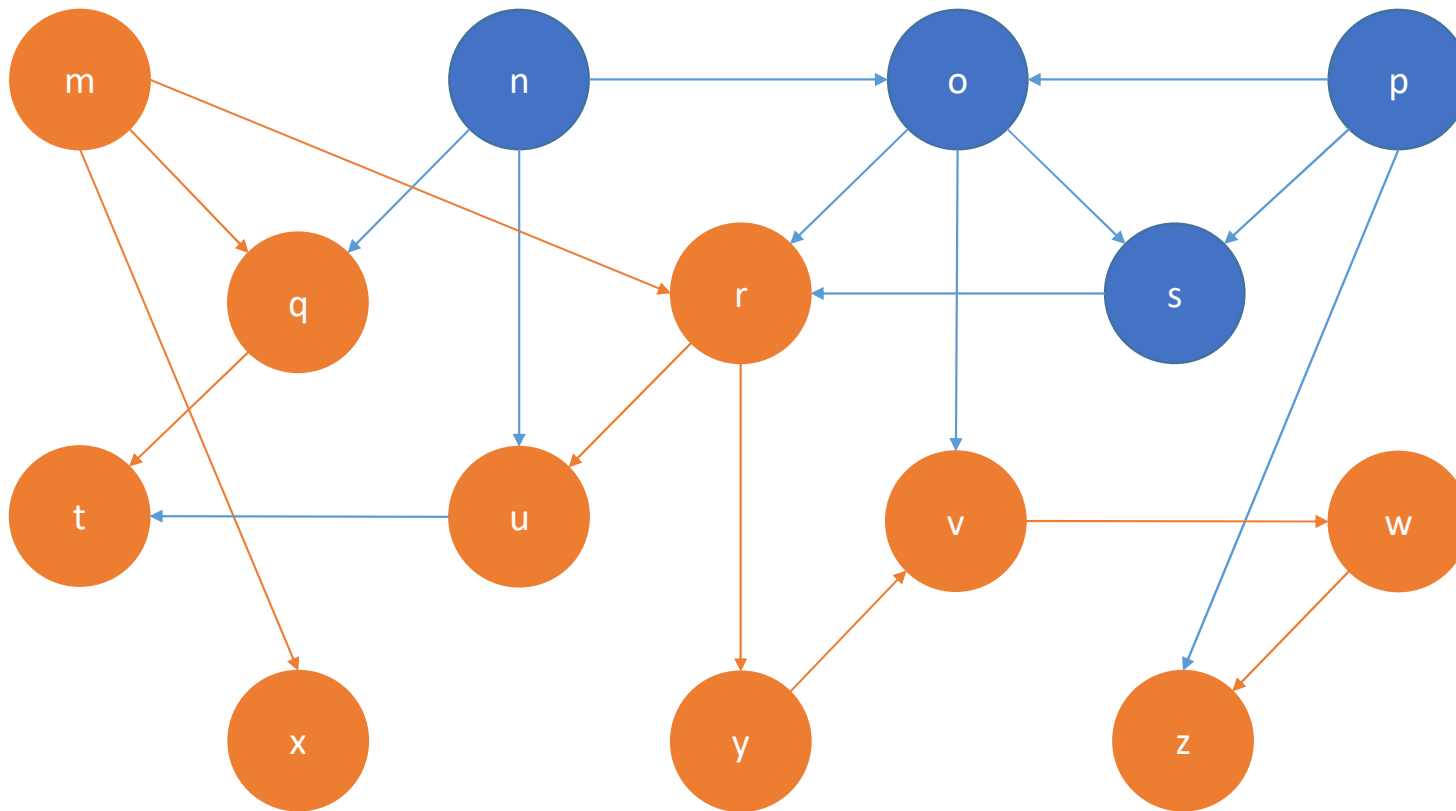
	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x		
y	9	16
z	12	13

## Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x	18	
y	9	16
z	12	13

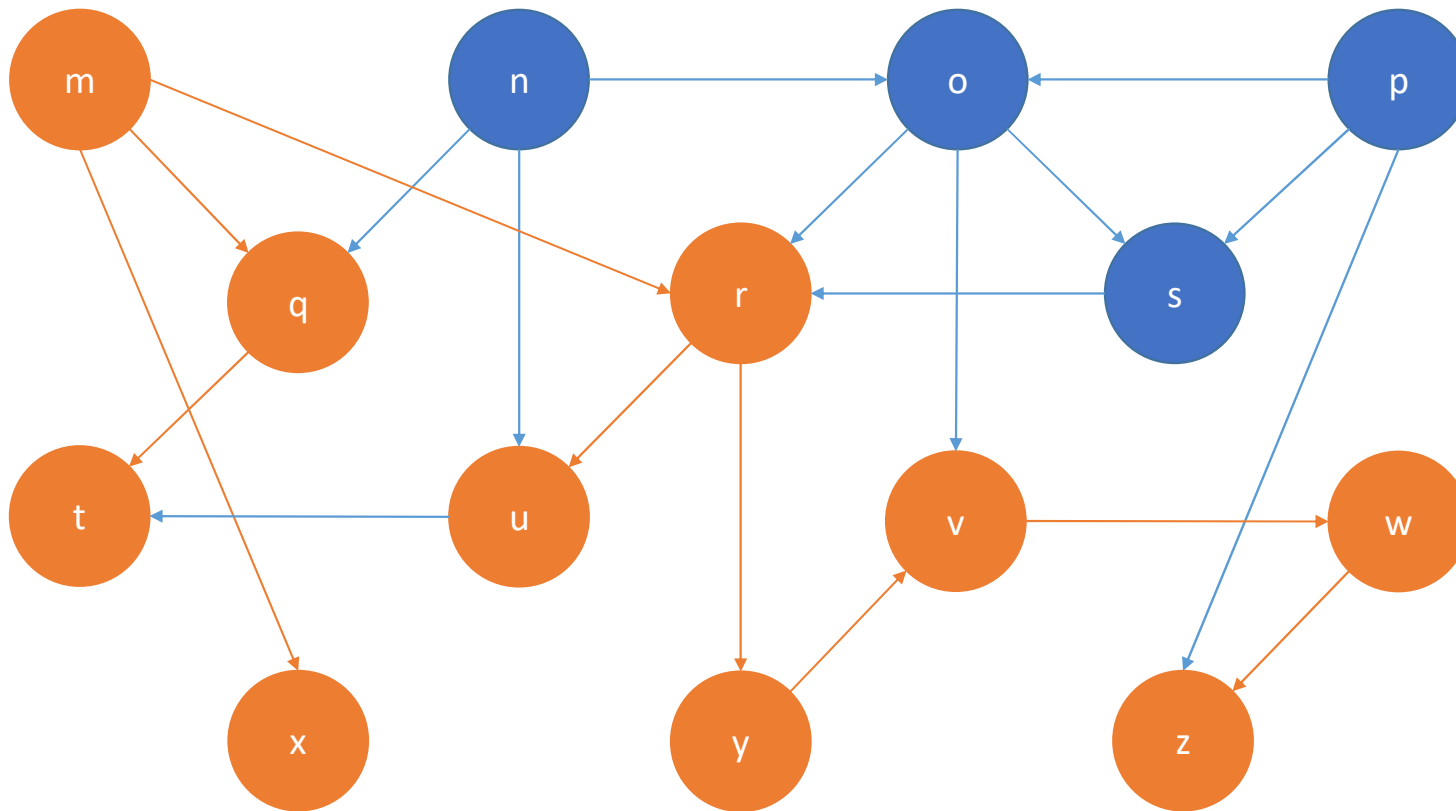
## Exercise 3.1



	vt	ft
m	1	
n		
o		
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

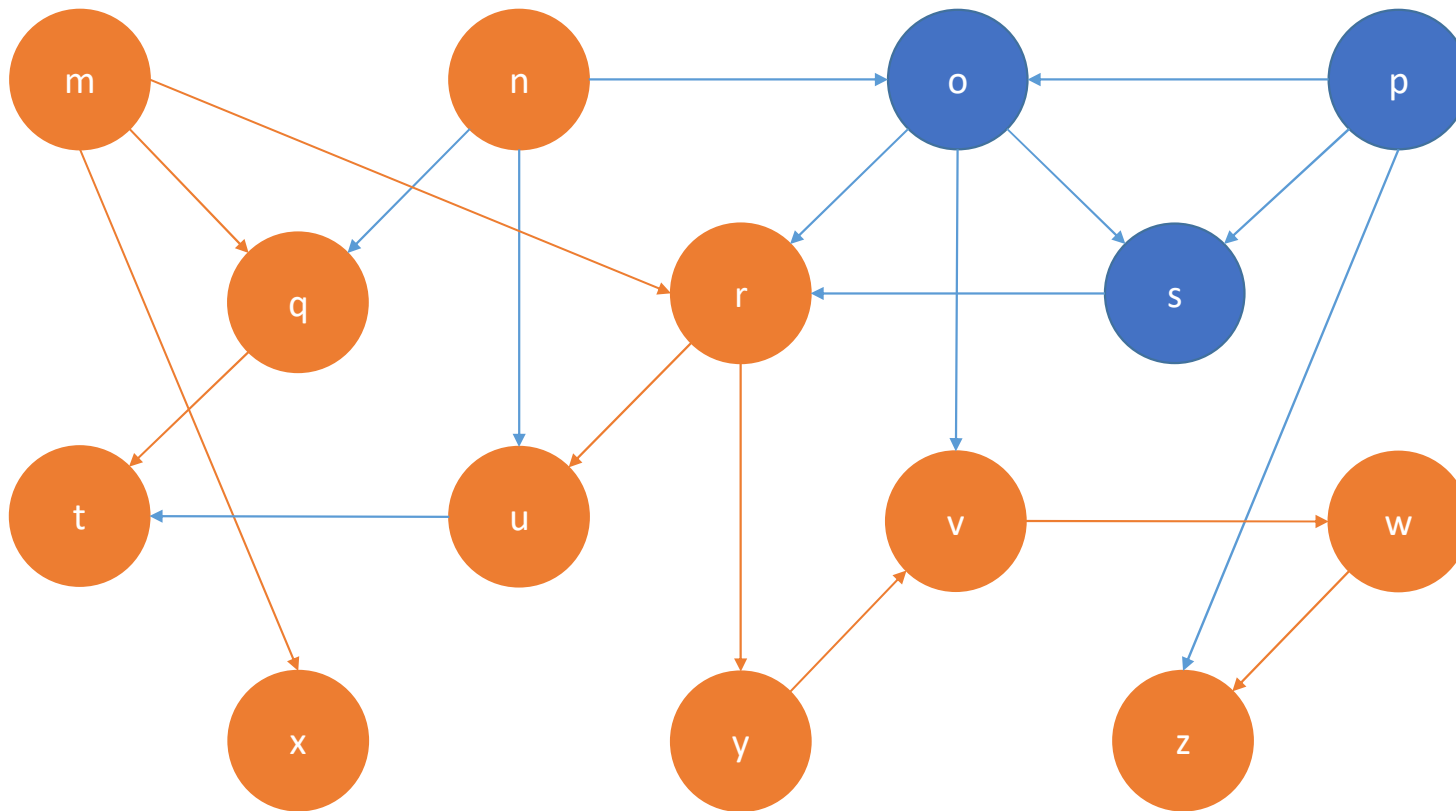


## Exercise 3.1



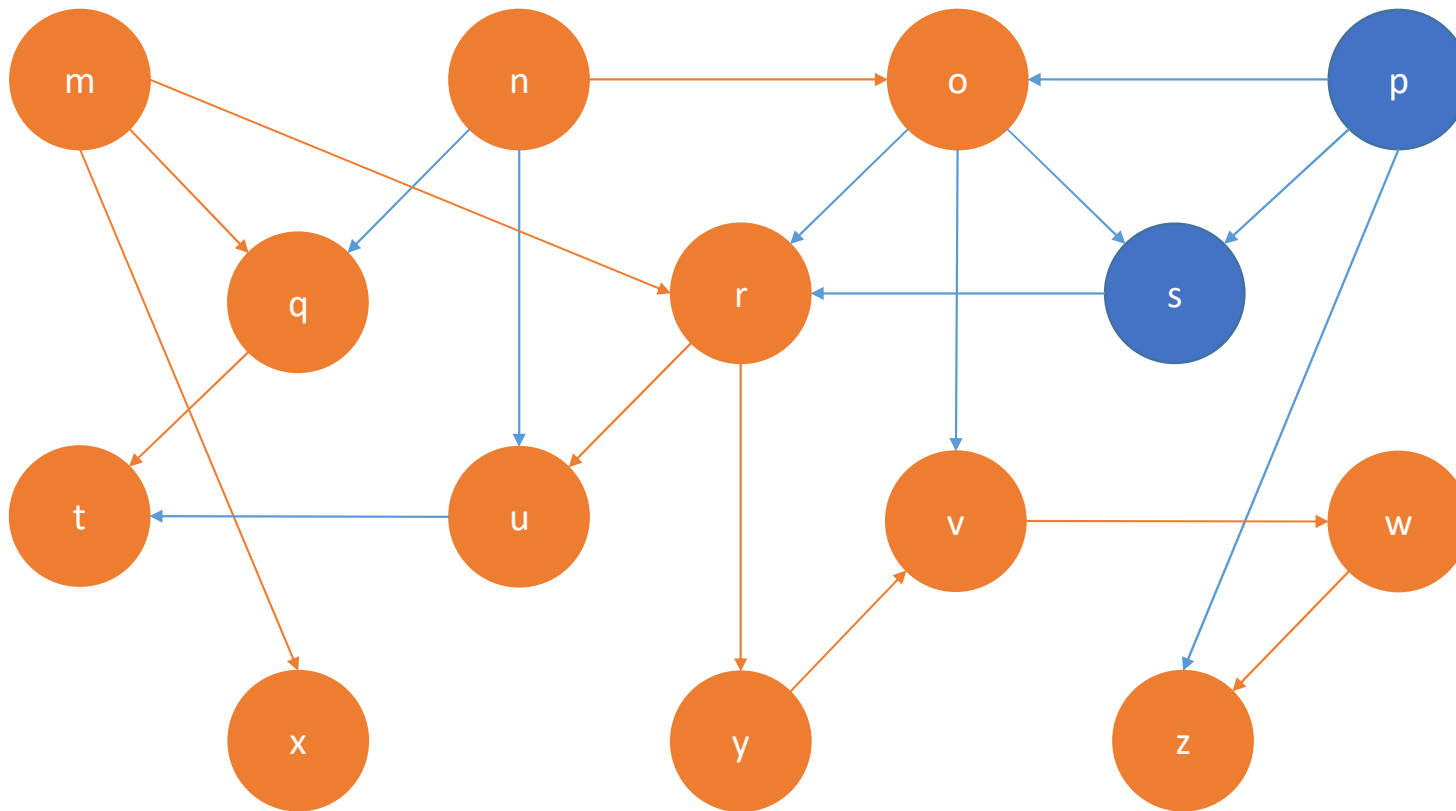
	vt	ft
m	1	20
n		
o		
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



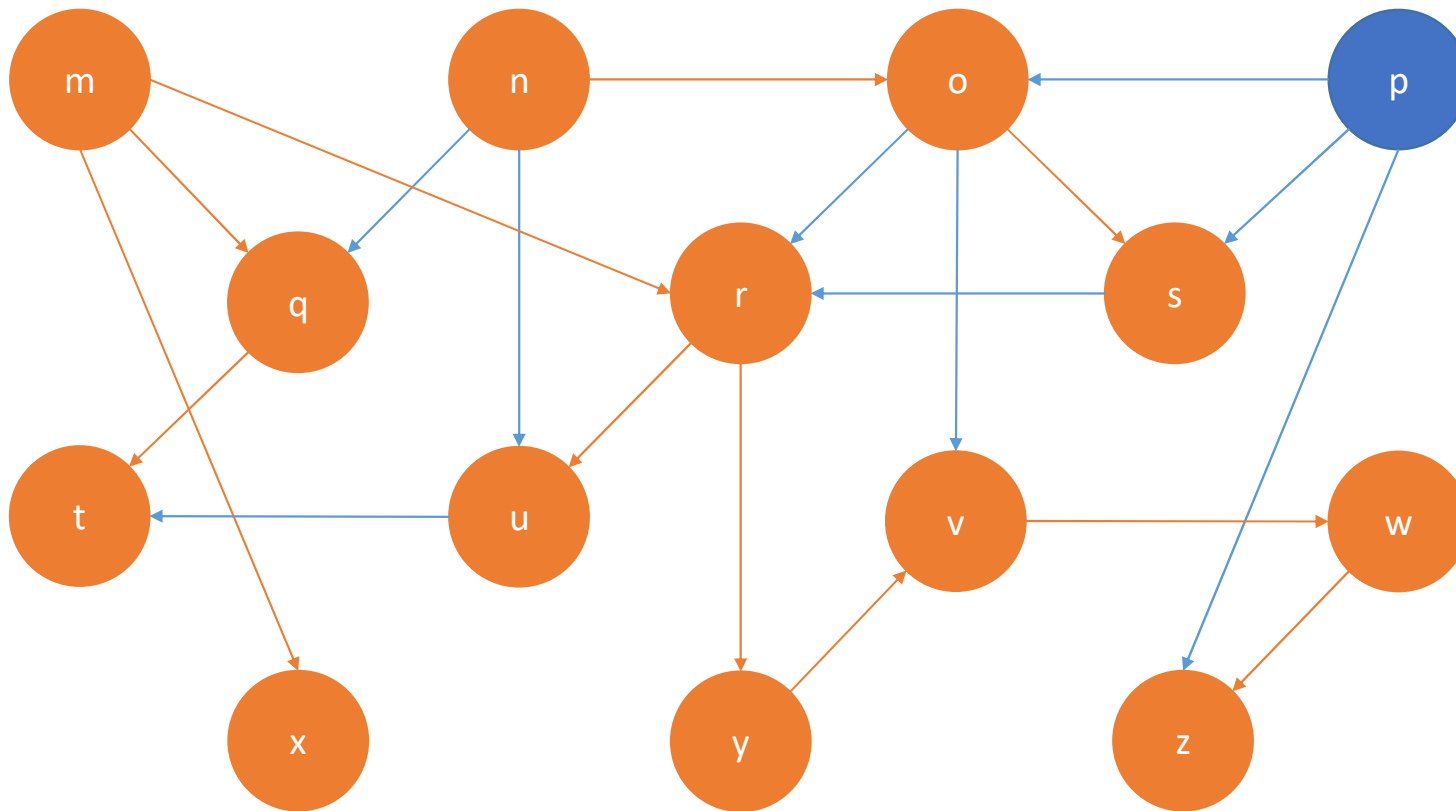
	vt	ft
m	1	20
n	21	
o		
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



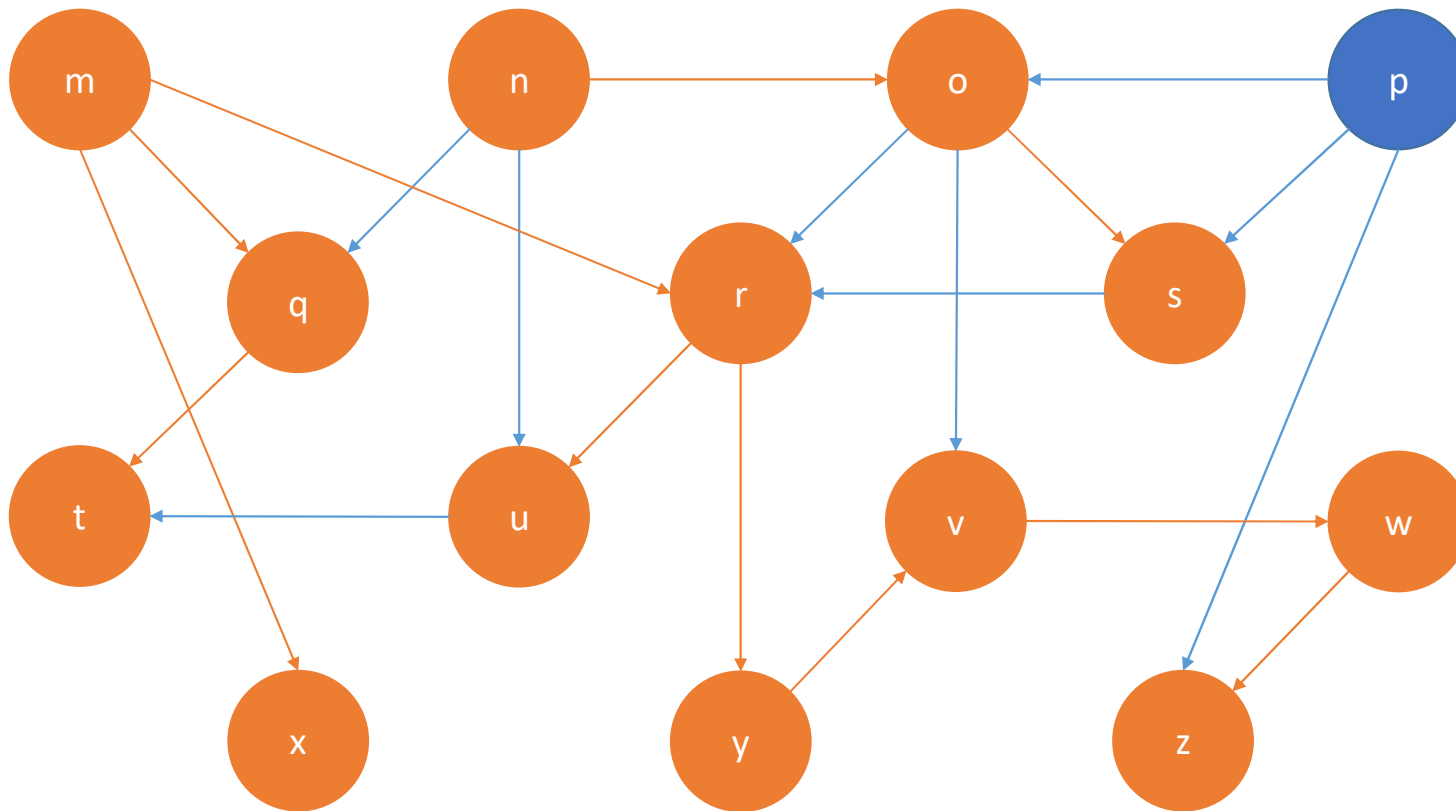
	vt	ft
m	1	20
n	21	
o	22	
p		
q	2	5
r	6	17
s		
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



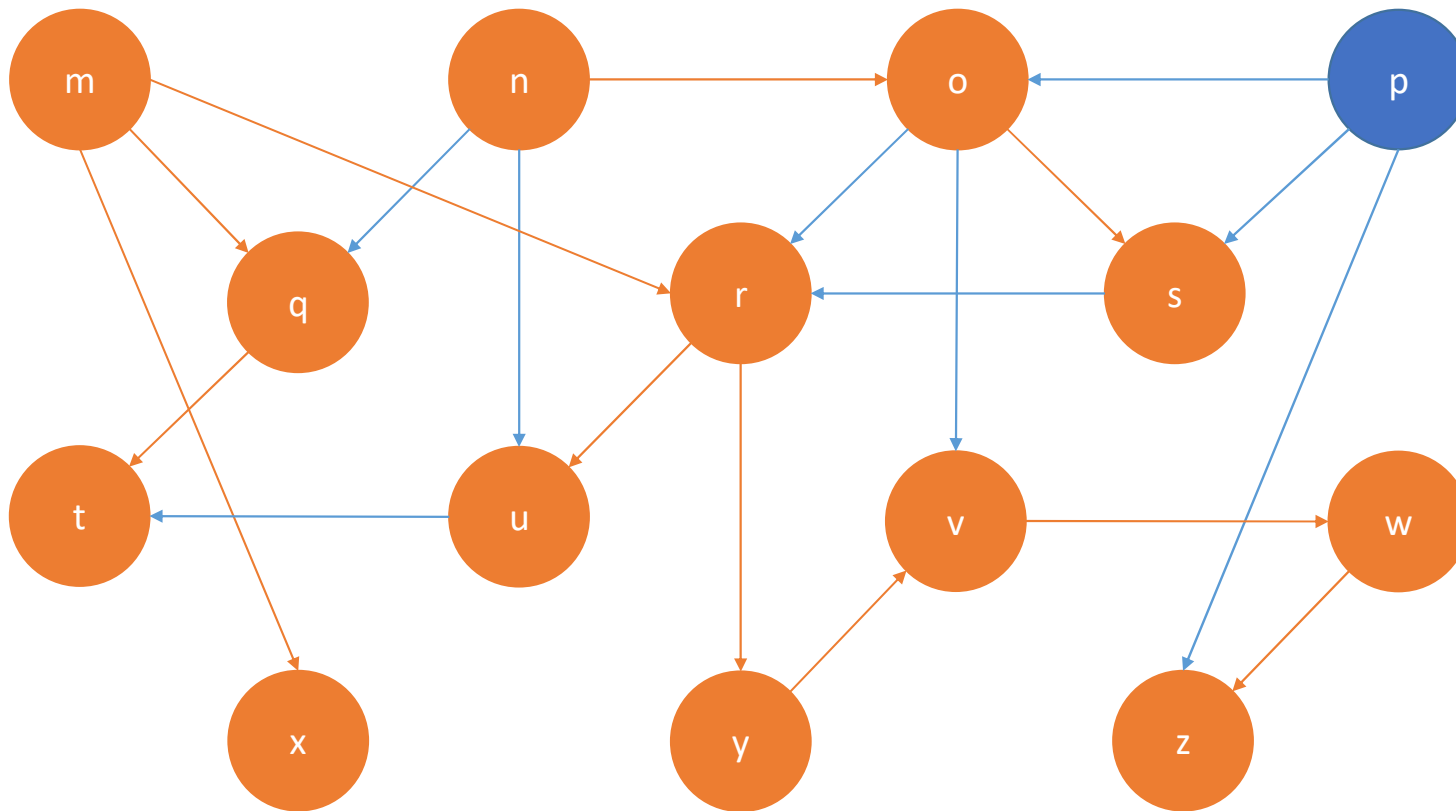
	vt	ft
m	1	20
n	21	
o	22	
p		
q	2	5
r	6	17
s	23	
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



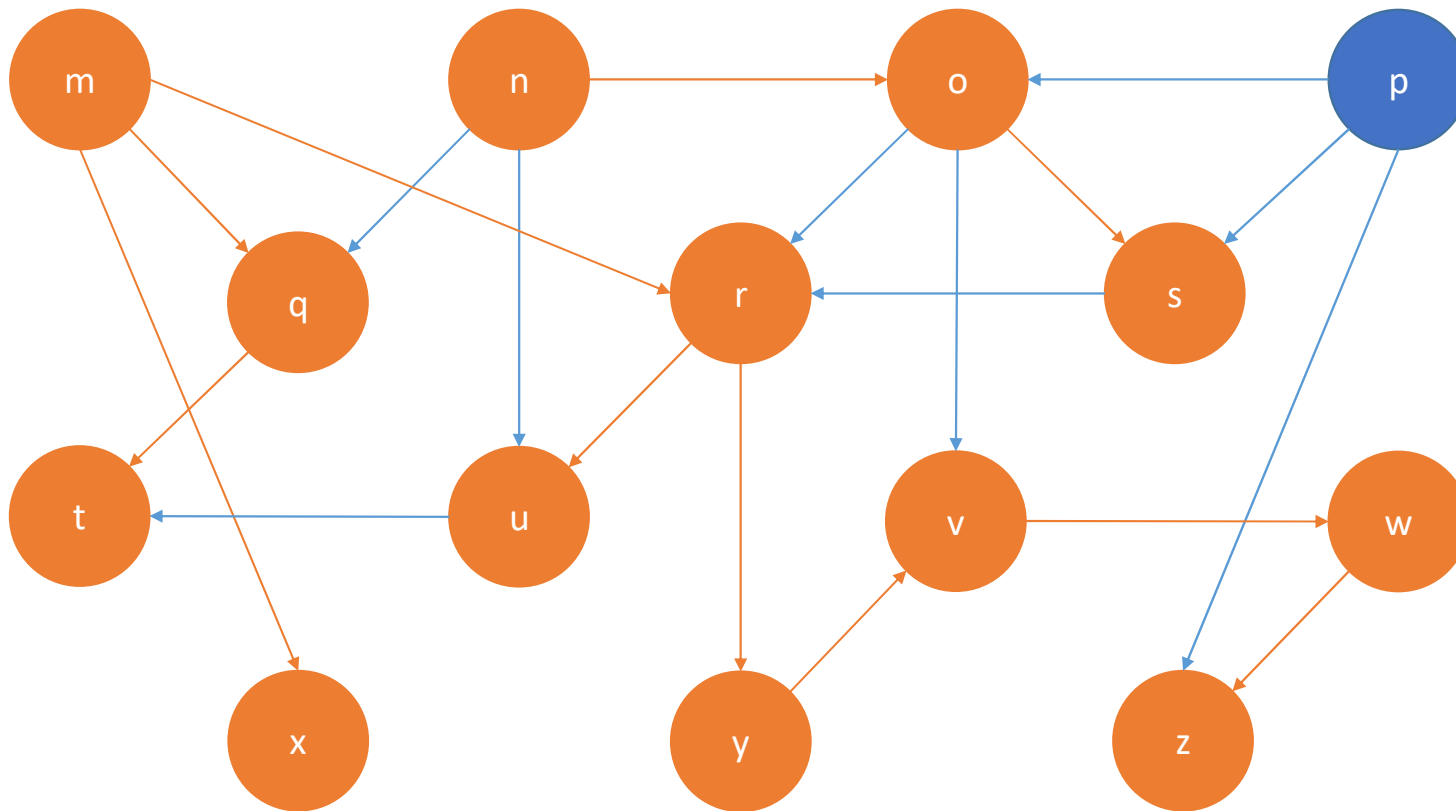
	vt	ft
m	1	20
n	21	
o	22	
p		
q	2	5
r	6	17
s	23	24
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



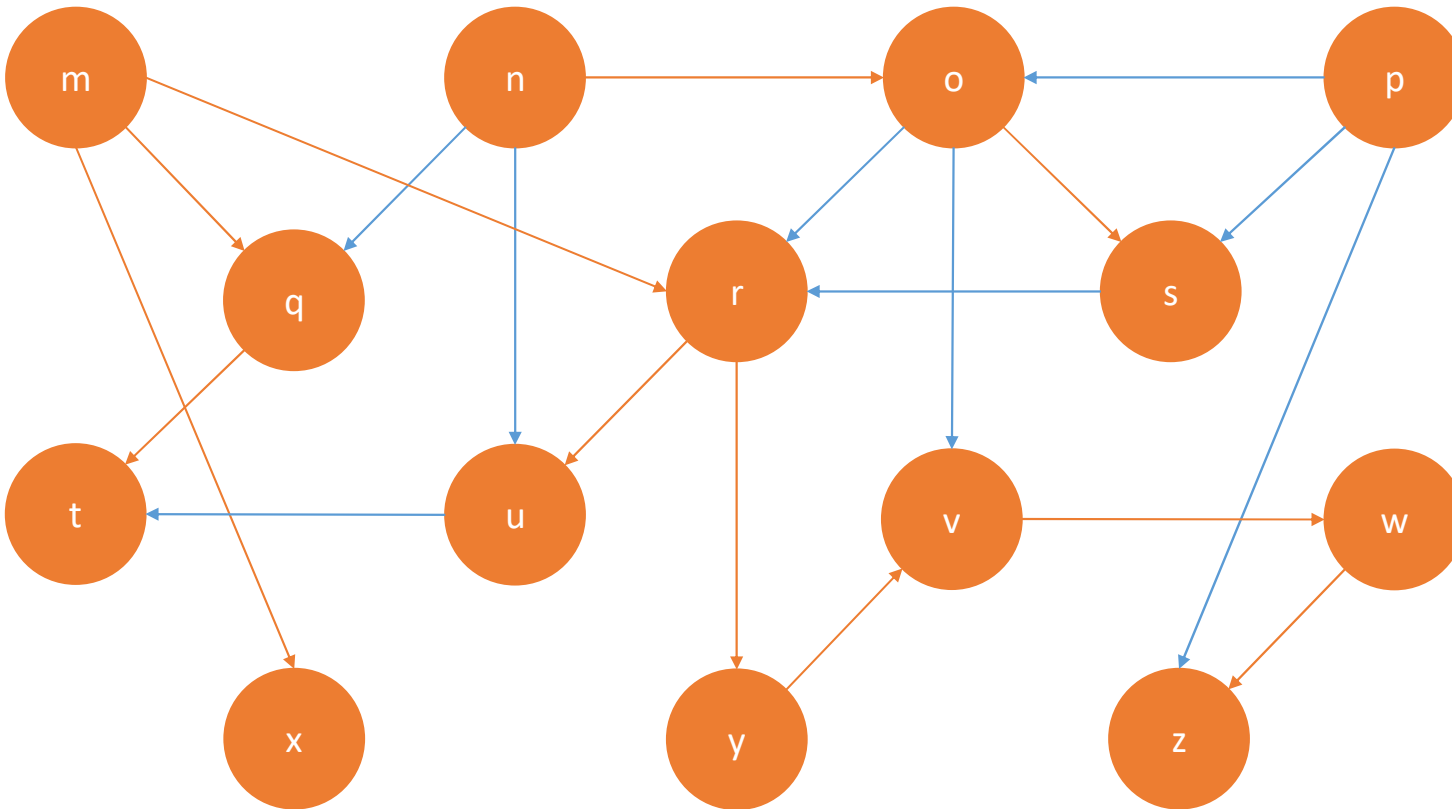
	vt	ft
m	1	20
n	21	
o	22	25
p		
q	2	5
r	6	17
s	23	24
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

## Exercise 3.1



	vt	ft
m	1	20
n	21	26
o	22	25
p		
q	2	5
r	6	17
s	23	24
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

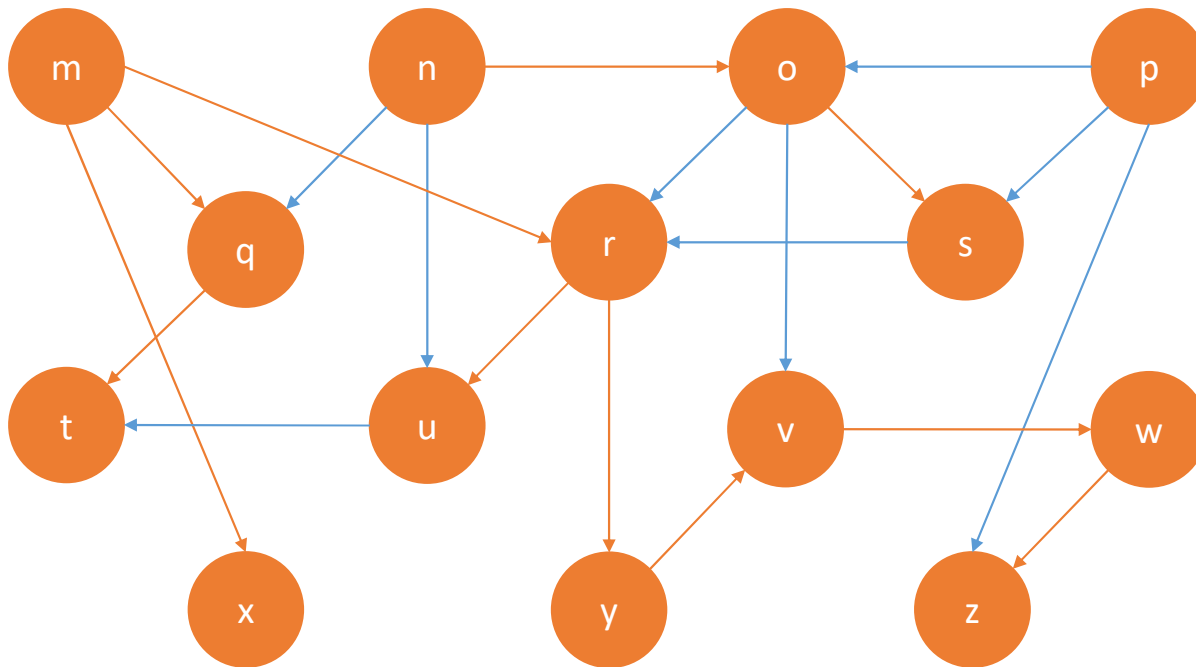
## Exercise 3.1



	vt	ft
m	1	20
n	21	26
o	22	25
p	27	
q	2	5
r	6	17
s	23	24
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13



# Exercise 3.1 - Solution



p, n, o, s, m, x, r, y, v, w, z, u, q, t

	vt	ft
m	1	20
n	21	26
o	22	25
p	27	28
q	2	5
r	6	17
s	23	24
t	3	4
u	7	8
v	10	15
w	11	14
x	18	19
y	9	16
z	12	13

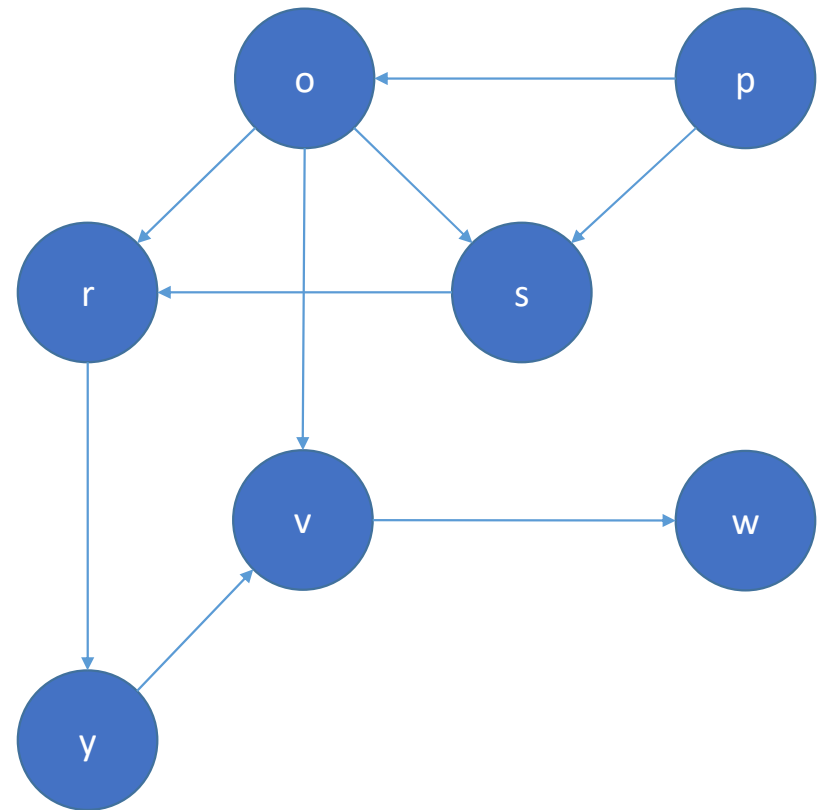
## Exercise 3.2

*Optional*

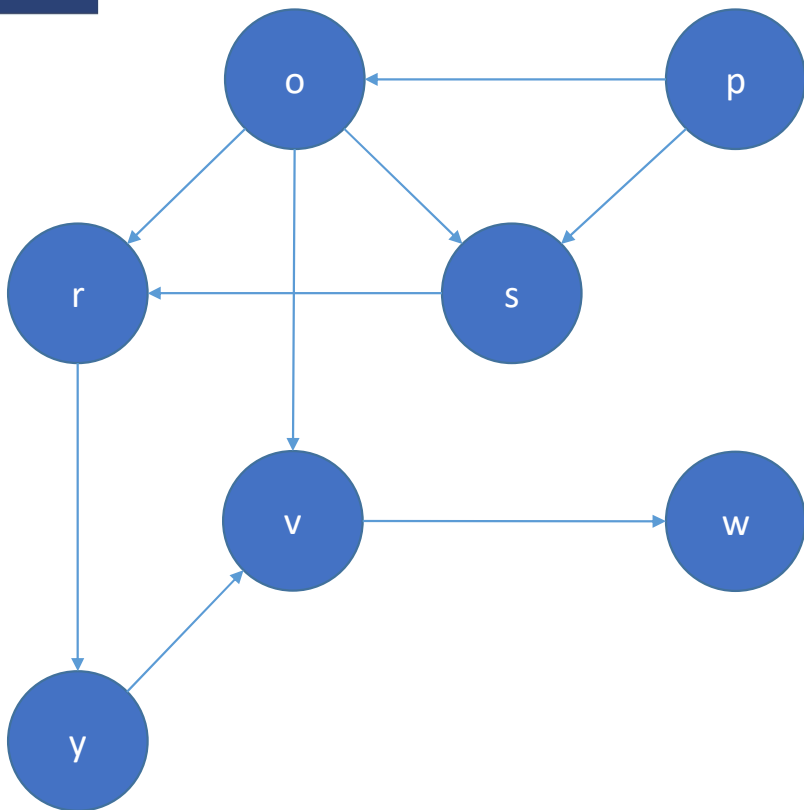
Give a linear-time algorithm (pseudocode) that, given a directed acyclic graph  $G = (V, E)$  and two vertices  $a, b \in V$ , returns the number of simple paths from  $a$  to  $b$  in  $G$ . Your algorithm needs only to count the simple paths, not list them.

## Exercise 3.2 – Idea of algorithm

- Run recursive DFS with additional constraint: return 1 if b is reached
- On the way back: store the number of paths from each node to our target node b
- Sum the paths up to root node a

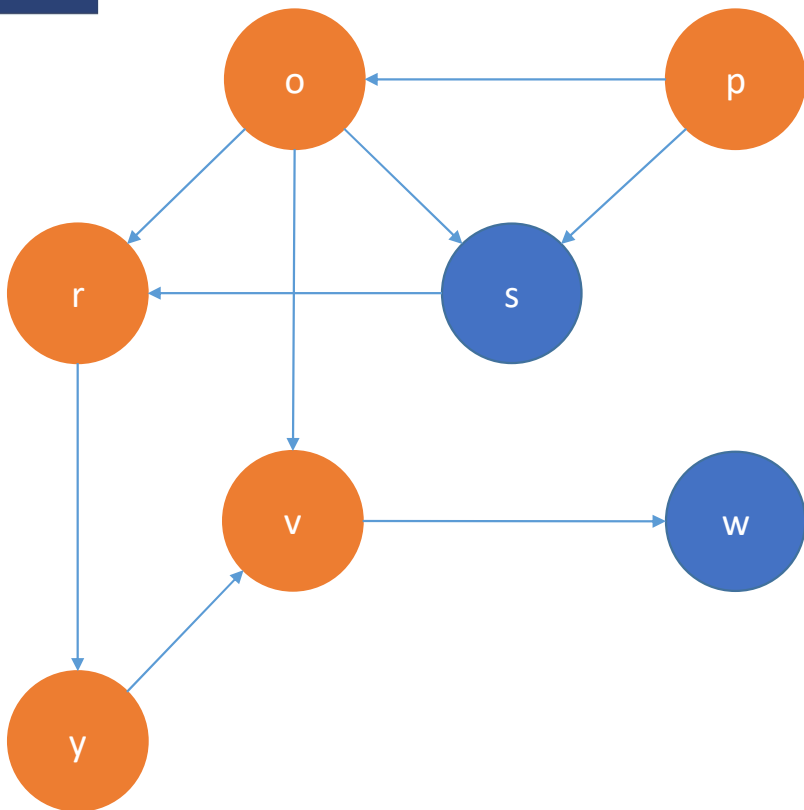


## Exercise 3.2 – Idea of algorithm



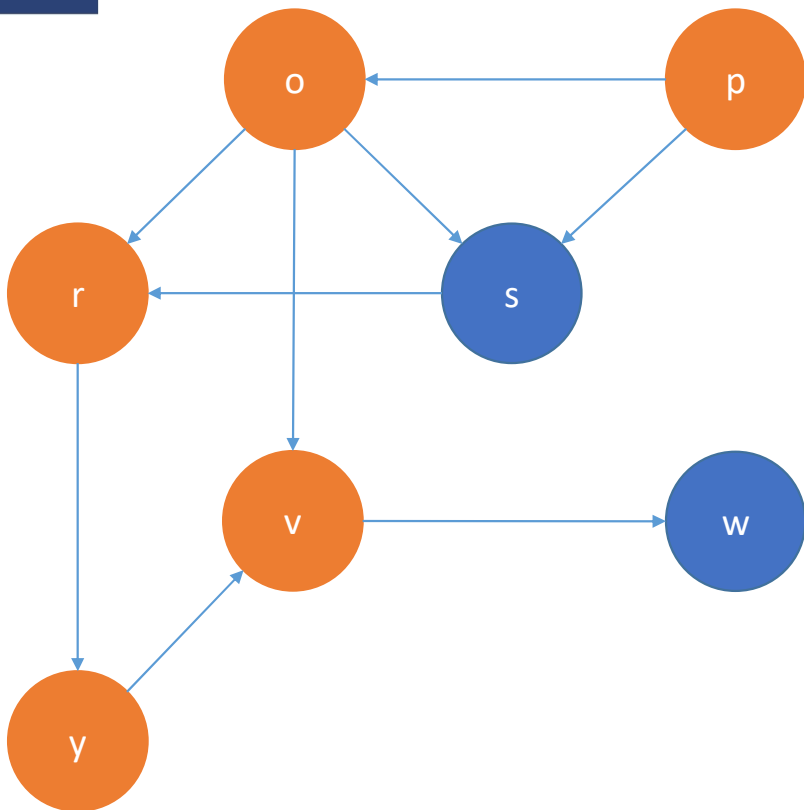
	path count
o	
p	
r	
s	
v	
w	
y	

## Exercise 3.2 – Idea of algorithm



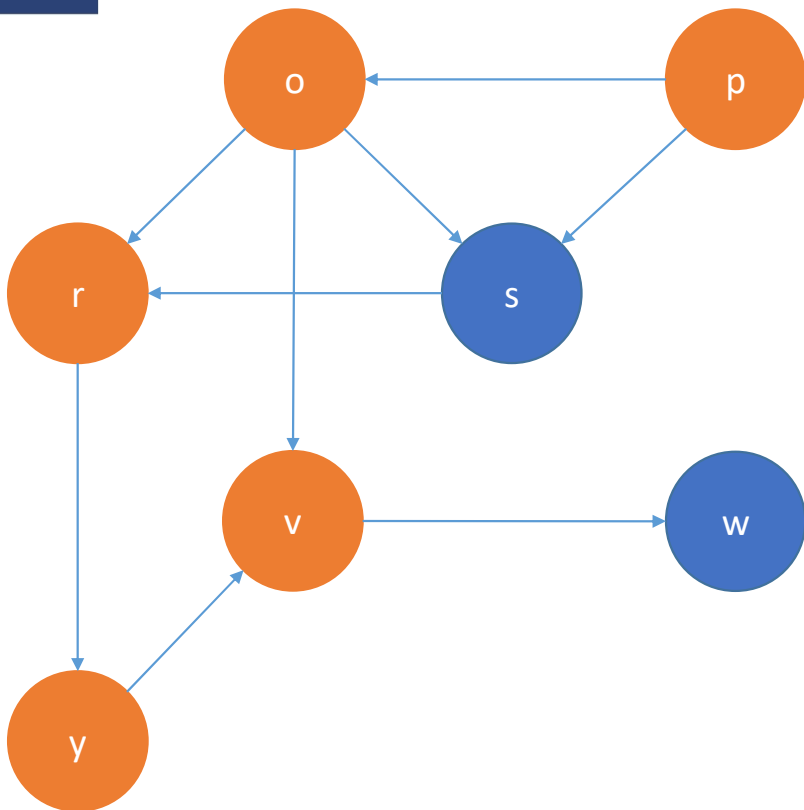
	path count
o	
p	
r	
s	
v	
w	
y	1

## Exercise 3.2 – Idea of algorithm



	path count
o	
p	
r	1
s	
v	
w	
y	1

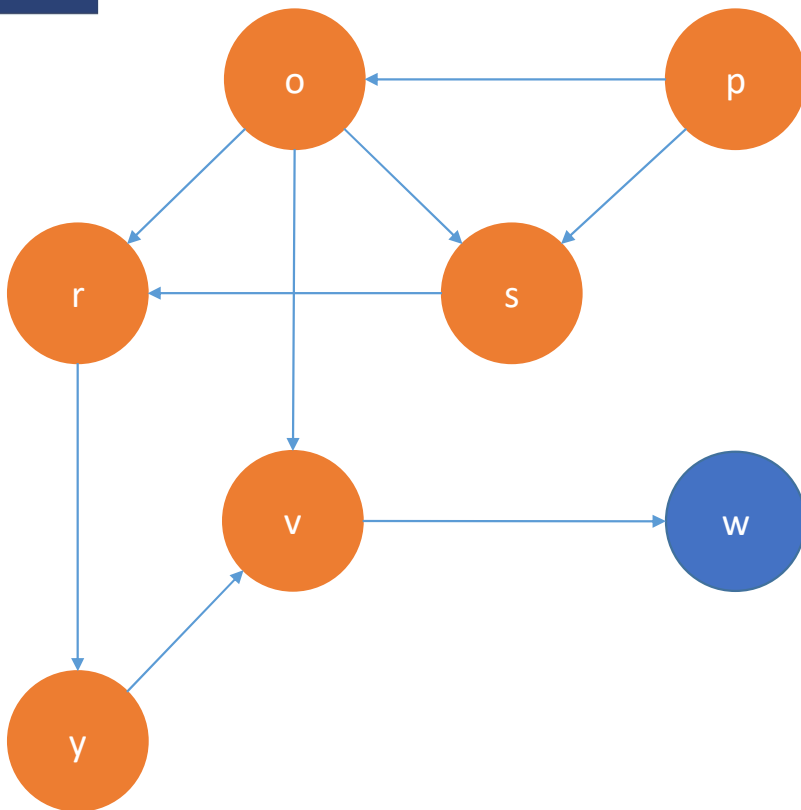
## Exercise 3.2 – Idea of algorithm



	path count
o	1
p	
r	1
s	
v	
w	
y	1

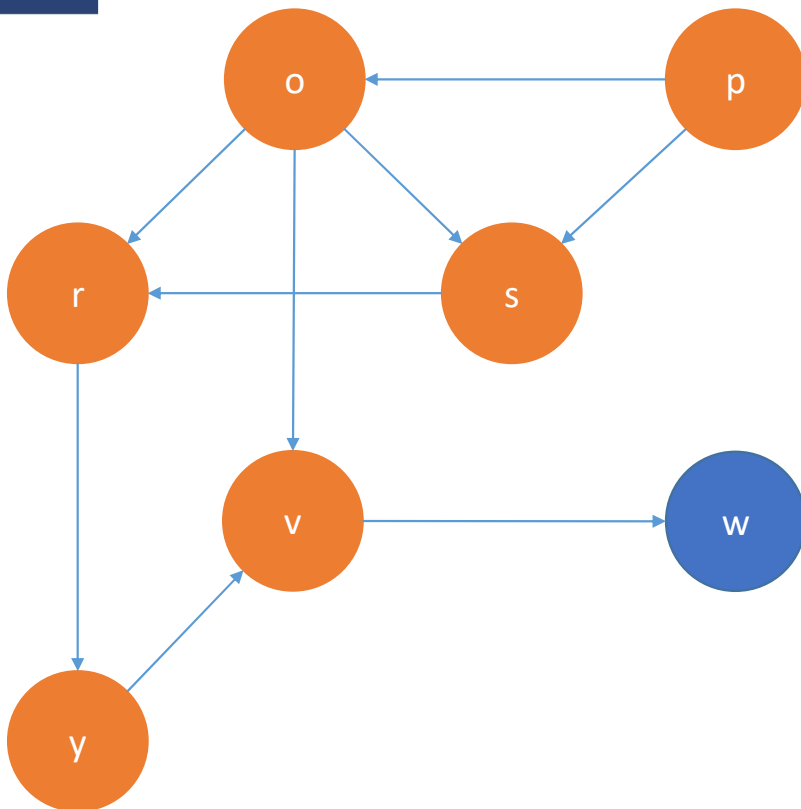


## Exercise 3.2 – Idea of algorithm



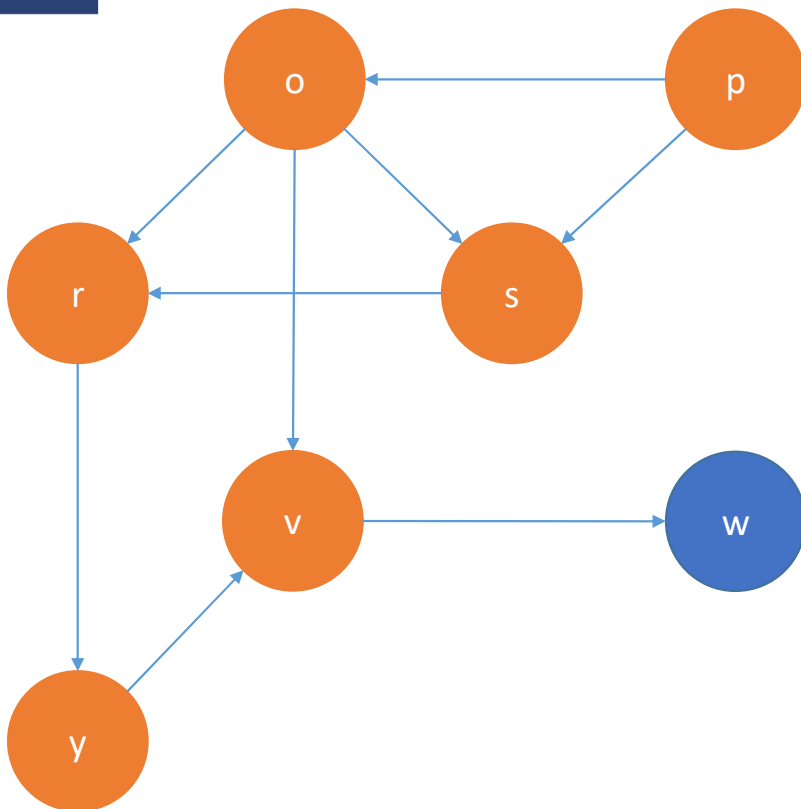
	path count
o	1
p	
r	1
s	1
v	
w	
y	1

## Exercise 3.2 – Idea of algorithm



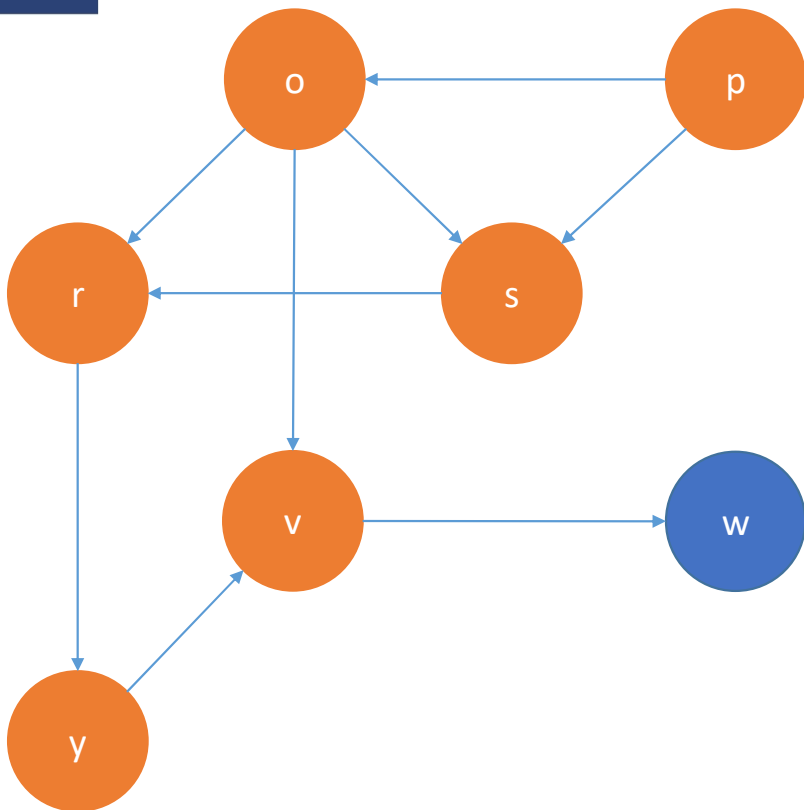
	path count
o	2
p	
r	1
s	1
v	
w	
y	1

## Exercise 3.2 – Idea of algorithm



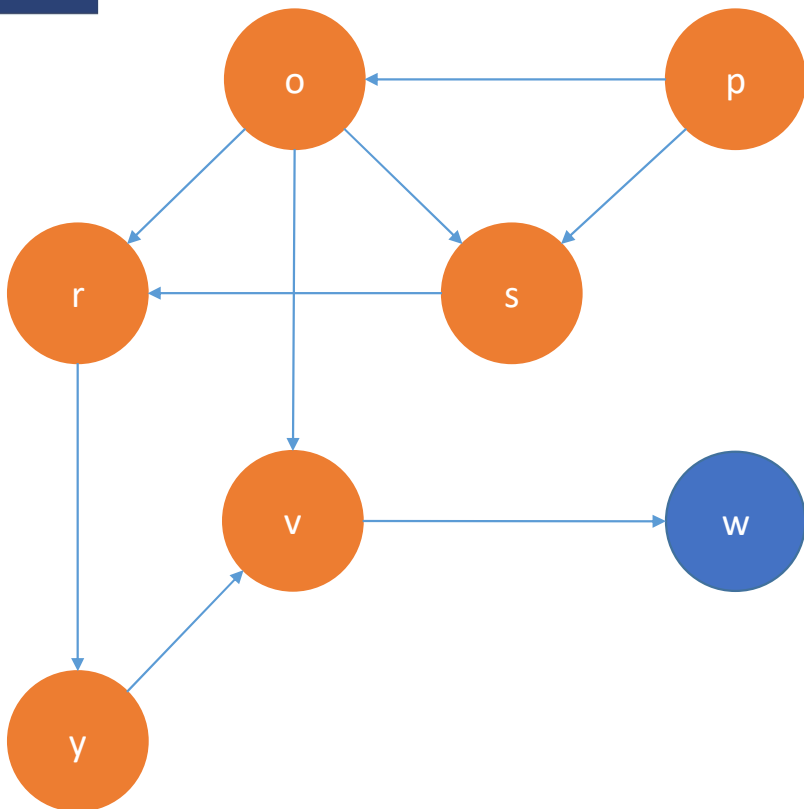
	path count
o	3
p	
r	1
s	1
v	
w	
y	1

## Exercise 3.2 – Idea of algorithm



	path count
o	3
p	3
r	1
s	1
v	
w	
y	1

## Exercise 3.2 – Idea of algorithm



	path count
o	3
p	4
r	1
s	1
v	
w	
y	1

## Exercise 3.2 – Solution

---

**Algorithm 1**  $path\_count(G, a, b)$

---

```
if  $a == b$  then
  return 1
else if  $a.cp \neq NIL$  then
  return  $a.cp$ 
else
   $a.cp = 0$ 
  for  $w \in G.Adj[a]$  do
     $a.cp = a.cp + path\_count(G, w, b)$ 
  end for
  return  $a.cp$ 
end if
```

---



# Strongly Connected Component



# Exercise 4.1



## Exercise 4.1 – Recap strongly connected components (SCC)

- Given a directed graph  $G = (V, E)$
- SCC is a maximal set of vertices  $C \subseteq V$  such that for every pair of vertices  $u, v \in C$  there exists a path from  $u$  to  $v$  and a path from  $v$  to  $u$ .

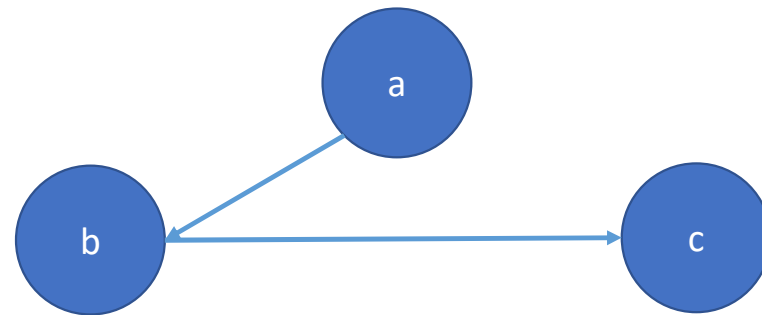
How can the number of strongly connected components of a graph change (increase, decrease or stay the same) if a new edge is added?

## Exercise 4.1 – Solution

- Increase
  - Not possible
  - Adding a new edge can never remove an existing connection between two vertices

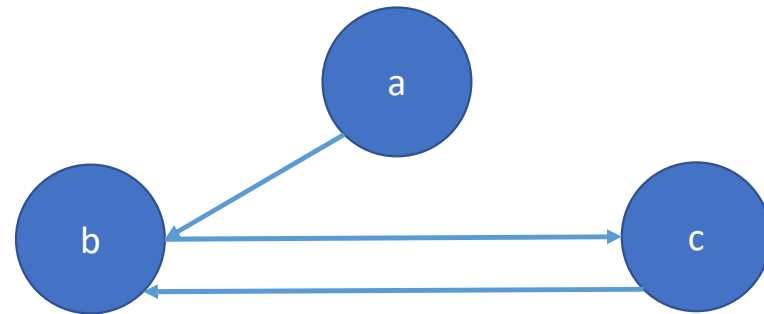
## Exercise 4.1 – Solution

- Decrease
  - Possible
  - Graph has 3 strongly connected components



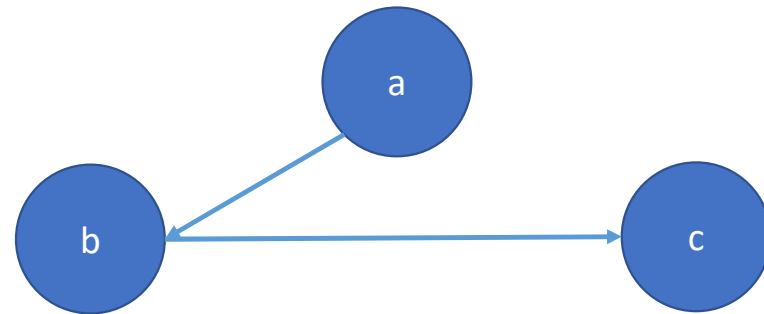
## Exercise 4.1 – Solution

- Decrease
  - Possible
  - Graph has 3 strongly connected components
  - Adding  $(c,b)$
  - Graph has 2 strongly connected components



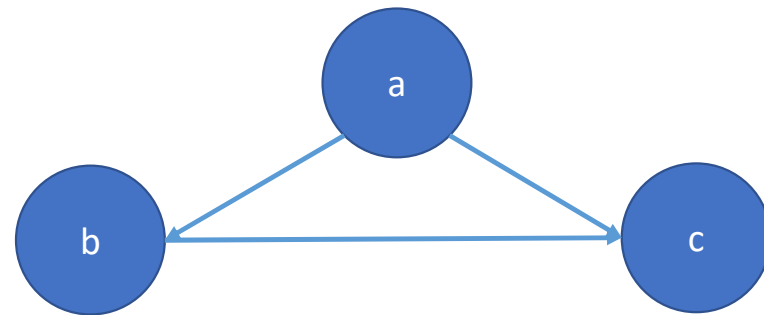
## Exercise 4.1 – Solution

- Stay the same
  - Possible
  - Graph has 3 strongly connected components



## Exercise 4.1 – Solution

- Stay the same
  - Possible
  - Graph has 3 strongly connected components
  - Adding (a,c)
  - Does not change anything





## Exercise 4.2



Teaching Assistant Benedikt rewrites Kosaraju's algorithm for strongly connected components to use the original (instead of the transpose) graph in the second depth-first search and scan the vertices in order of increasing finish times. Give a counterexample to show that the algorithm is not correct.

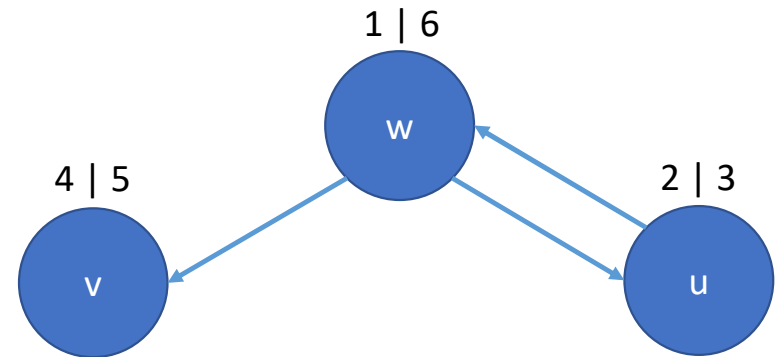
## Exercise 4.2 - Solution

Adjusted algorithm:

1. Run DFS on  $G$
2. Run DFS on  $G$  on increasing  $ft$

Result:

- Single strongly connected component  $\{u,w,v\}$
- Incorrect!

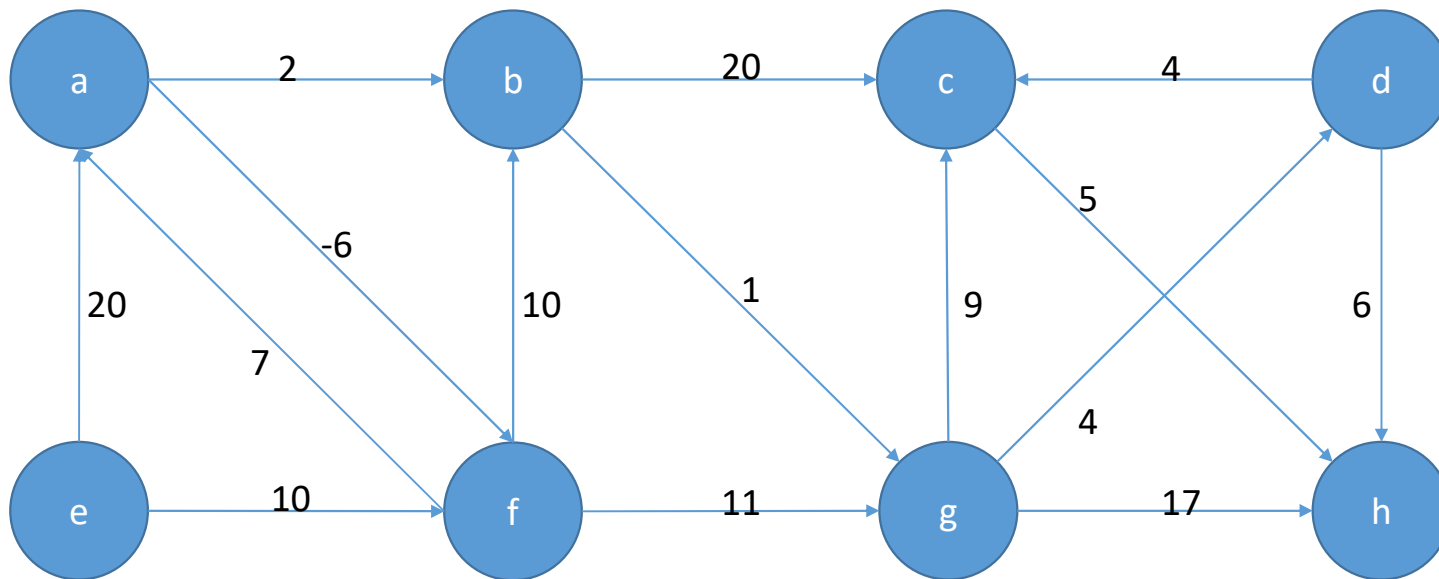




# Exercise 5.1

## Exercise 5.1

Consider the following graph:



Find the shortest path from e to h using Bellman-Ford.

## Exercise 5.1 – Recap Bellman-Ford

```
initialize(G, s)
  for each v in G.V
    v.dist = inf
    v.prev = null
  s.dist = 0
```

```
relax(u, v, w)
  if v.dist > u.dist + w(u, v)
    v.dist = u.dist + w(u, v)
    v.prec = u
```

## Exercise 5.1- Recap Bellman-Ford

```
bellman-ford(G, w, s)
  initialize(G)
  for i in 1 to |G.V| - 1
    for each edge (u, v) in G.E
      relax(u, v, w)

  for each edge (u, v) in G.E
    if v.dist > u.dist + w(u, v) # negative weight cycle
      return False

  return True
```



































## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f						
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f						
h	$\infty$	-	38	g						

## Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   $(e,f)$   $(f,a)$   $(f,b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f						
h	$\infty$	-	38	g						

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f						
h	$\infty$	-	38	g						



## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f						
h	$\infty$	-	38	g						

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g						

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g						
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	35	c				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	35	c				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

# Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   $(e,f)$   $(f,a)$   $(f,b)$   
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 $(f,g)$   $(g,c)$   $(g,d)$   $(g,h)$   
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# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
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c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				



## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
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b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
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g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

# Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   $(e,f)$   $(f,a)$   $(f,b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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d	$\infty$	-	25	g						
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d				
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
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## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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# Exercise 5.1 - Solution

*(a,b)* *(a,f)* *(b,c)* *(b,g)* *(c,h)* *(d,c)* *(d,h)* *(e,a)* *(e,f)* *(f,a)* *(f,b)*  
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
*(f,g)* *(g,c)* *(g,d)* *(g,h)*  
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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c	$\infty$	-	30	g	29	d				
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## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
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## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
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	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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h	$\infty$	-	38	g	31	d				



## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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c	$\infty$	-	30	g	29	d				
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e	0	-								
f	$\infty$	-	10	e						
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h	$\infty$	-	38	g	31	d				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
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# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
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 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d				

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   $(e,f)$   $(f,a)$   $(f,b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f,g)$   $(g,c)$   $(g,d)$   $(g,h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		



# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   $(e,f)$   $(f,a)$   $(f,b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f,g)$   $(g,c)$   $(g,d)$   $(g,h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

*(a,b)* *(a,f)* *(b,c)* *(b,g)* *(c,h)* *(d,c)* *(d,h)* *(e,a)* *(e,f)* *(f,a)* *(f,b)*  
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
*(f,g)* *(g,c)* *(g,d)* *(g,h)*  
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		



## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a,b)$   $(a,f)$   $(b,c)$   $(b,g)$   $(c,h)$   $(d,c)$   $(d,h)$   $(e,a)$   **$(e,f)$**   $(f,a)$   $(f,b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' **10** ' 7 ' 10 '  
 $(f,g)$   $(g,c)$   $(g,d)$   $(g,h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		



# Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	f						
b	$\infty$	-	20	f	19	a				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	g				
e	0	-								
f	$\infty$	-	10	e						
g	$\infty$	-	21	f	20	b				
h	$\infty$	-	38	g	31	d	30	d		

## Exercise 5.1 - Solution

$(a, b)$   $(a, f)$   $(b, c)$   $(b, g)$   $(c, h)$   $(d, c)$   $(d, h)$   $(e, a)$   $(e, f)$   $(f, a)$   $(f, b)$   
 2 ' -6 ' 20 ' 1 ' 5 ' 4 ' 6 ' 20 ' 10 ' 7 ' 10 '  
 $(f, g)$   $(g, c)$   $(g, d)$   $(g, h)$   
 11 ' 9 ' 4 ' 17

	Init		R1		R2		R3		R4	
	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev	v.dist	v.prev
a	$\infty$	-	17	<b>f</b>						
b	$\infty$	-	20	f	19	<b>a</b>				
c	$\infty$	-	30	g	29	d	28	d		
d	$\infty$	-	25	g	24	<b>g</b>				
e	0	-								
f	$\infty$	-	10	<b>e</b>						
g	$\infty$	-	21	f	20	<b>b</b>				
h	$\infty$	-	38	g	31	d	30	<b>d</b>		

Distance e to h: **30**, Shortest path e to h: **e, f, a, b, g, d, h**



# Exercise 6.1

## Exercise 6.1 – Recap Dijkstra

```
dijkstra(G, w, s)
  initialize(G)
  S = [] # empty set
  Q = G.V # set of nodes to be „finished“

  while len(Q) > 0 # while Q not empty
    u = extract_min(Q) # node with smallest u.dist
    S = S U {u}
    for each v in G.Adj[u] # total E times
      relax(u, v, w)
```

```
initialize(G, s)
  for each v in G.V
    v.dist = inf
    v.prev = null
  s.dist = 0

relax(u, v, w)
  if v.dist > u.dist + w(u, v)
    v.dist = u.dist + w(u, v)
    v.prev = u
```