

Homework Assignment #12

Approximation Algorithms (Winter Semester 2023/24)

Exercise 1 – Heuristics for STEINER FOREST

In the lecture, we considered the following two heuristics for STEINER FOREST. Show that both of them can return arbitrarily bad solutions (in terms of approximation factor). Let G be the given graph, let $c: E(G) \rightarrow \mathbb{N}$ be the given edge costs, and let R be the given set of terminal pairs.

- a) For each pair of terminals in R , compute a shortest path that connects the two terminals. Return the union of the paths. [3 points]

- b) Compute a Steiner tree for the terminal set $\bigcup R$.

Full points are given only if you show that the approximation guarantee is arbitrary bad even if G is a complete metric graph. [4 points]

Exercise 2 – Naive Primal–Dual Approach

Consider the first primal-dual algorithm `PRIMALDUALSTEINERFORESTNAIVE` from the lecture. Construct an instance for which `PRIMALDUALSTEINERFORESTNAIVE` returns a solution with approximation factor greater than 2. [5 points]

Exercise 3 – Special cases of STEINER FOREST

Given a graph G , edge costs $c: E(G) \rightarrow \mathbb{N}$, and a set R of terminal pairs, show that the algorithm `PRIMALDUALSTEINERFOREST`(G, c, R) returns an optimal solution if

- a) $R = V \times V$, [4 points]

- b) $R = \{s, t\}$ for some $s, t \in V$. [4 points]