

Homework Assignment #6

Approximation Algorithms (Winter Semester 2022/23)

Exercise 1 – METRIC-k-CLUSTER

Let $G = (V, E)$ be a complete graph with edge weights $c: E \rightarrow \mathbb{Q}_{\geq 0}$ that satisfy the triangle inequality. Let k be a positive integer. We want to find a partition of V into k sets of vertices $V_1, \dots, V_k$, called *clusters*, such that the weight of the most expensive intra-cluster edge is minimized. In other words, we have to minimize

$$\max_{1 \leq i \leq k, u, v \in V_i} c(u, v).$$

- a) Devise a factor-2 approximation algorithm for this problem. [7 points]
- b) Show that under the assumption $P \neq NP$, there exists no factor- $(2 - \varepsilon)$ approximation algorithm for this problem, where $\varepsilon > 0$.

Suggestion: Use the hardness of the graph coloring problem: Given a graph $G = (V, E)$ and a parameter k , can the vertices in V be colored with at most k colors such that no two adjacent vertices share a color? [6 points]

Exercise 2 – Greedy for METRIC-k-CENTER

Consider the following greedy algorithm for METRIC-k-CENTER.

Algorithm 1: Greedy-Metric-k-Center($G = (V, E; c)$, k)

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1 Pick an arbitrary vertex  $v \in V$ 
2  $S \leftarrow \{v\}$ 
3 while  $|S| < k$  do
4    $u \leftarrow \text{Vertex with } c(u, S) = \max_{v \in V} c(v, S)$ 
5    $S \leftarrow S \cup \{u\}$ 
```

Show that this algorithm is a factor-2 approximation for METRIC-k-CENTER. [7 points]