Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, November 3, 2023

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Homework Assignment #3
Approximation Algorithms (Winter Semester 2023/24)

Exercise 1 – Greedy for MULTIWAY CUT

A natural greedy algorithm for calculating a multiway cut for given terminals $s_1, \ldots, s_k$ is the following: Based on G, calculate a cheapest s_i–s_j cut for all pairs s_i, s_j that are still connected and remove the cheapest of these cuts. Repeat until all terminal pairs s_i, s_j are separated.

Show that this algorithm has approximation ratio $2 - 2/k$.

Hint: Try to charge the costs of every step made by the algorithm onto some terminal-seperating cut in a way that every terminal is considered at most once.

[8 points]

Exercise 2 – FEEDBACK VERTEX SET on tournaments

A *tournament* is a directed graph $G = (V, E)$ that contains exactly one of the edges (u, v) and (v, u) for each pair of vertices $u \neq v$. The FEEDBACK VERTEX SET problem asks for a smallest set of vertices whose removal from G results in an acyclic graph.

Show that a factor-3 approximation algorithm for FEEDBACK VERTEX SET on tournaments exists.

Hint: Show that it suffices to destroy all cycles of length three. Then, develop an approximation-preserving reduction to SET COVER with frequency $h = 3$ (see Exercise 2 from Homework Assignment #2).

[5 points]

Exercise 3 – SENDER–RECEIVER

Let G be a graph with non-negative edge costs $c\colon E(G) \to \mathbb{Q}_{\geq 0}$. Let S and R be two disjoint sets of vertices, which we call *sender* and *receiver*. The *Sender–Receiver* problem asks for a minimum-cost subgraph of G where every receiver is connected to some sender by a path.

Hint: Introduce an additional vertex and connect it to G appropriately. Using an approximation-preserving reduction can simplify the solutions for questions a) and c) considerably.

a) Show that an *exact* solution can be found in polynomial time if $S \cup R = V(G)$ holds. **[4 points]**

b) *Bonus:* Show that the general version of the problem (where $S \cup R$ and $V(G)$ don't necessarily coincide) is NP-hard. **[3 extrapoints]**

c) Give a factor-2 approximation algorithm for the general version of the problem. **[3 points]**

Please hand in your solutions via WueCampus until 12:00 on Friday, November 10.