Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #9

Advanced Algorithms (WS 2023/24)

Exercise 1 – Insertion & deletion for Splay Trees

Describe how insertion and deletion of an element x work for Splay Trees. Both methods should use `splay`. Give pseudocode and argue whether the running time is in $O(c(\text{Splay}(x)))$.

Hint: You may use the properties of splay trees to show the running time. **5 Points**

Exercise 2 – Working set property of Splay Trees

A BST has the *working set property* if the (amortized) cost of a query for key x is $O(\log t)$, where t is the number of keys queried more recently than x.

We want to use the Access Lemma to show that Splay Trees have the working set property. For this, we assign the weights $1, \frac{1}{4}, \frac{1}{9}, \ldots, \frac{1}{n^2}$ to the items in order by latest access (call of `Splay` on an element). That is, after the constructing of the tree is done, the root has weight 1 and the item first added has weight $\frac{1}{n^2}$.

After an access occurs, we update the weights. More precisely, suppose element x is accessed during access j and has weight $\frac{1}{k^2}$. Then set the new weight of x to 1. For every element x' with weight $\frac{1}{k'^2}$ where $k' < k$, set the new weight to $\frac{1}{(1+k')^2}$. Note that this reassignment permutes the weights $1, \frac{1}{4}, \frac{1}{9}, \ldots, \frac{1}{n^2}$.

a) Using these weight (re-)assignments, prove that Splay Trees have the working set property. **5 Points**

b) Explain why we are allowed to change the weights of nodes in this way. **2 Points**

Exercise 3 – Dynamic Program for Known Probability Distribution

Design a dynamic program that creates in polynomial time an optimal static binary search tree for a given set of n keys with known probability distribution $p_1, \ldots, p_n$, that is, the static binary search tree where the expected query time $\sum_{i=1}^{n} p_i \ell_i$ is minimized, where ℓ_i is the level of key i. What are the running time and space requirements for the dynamic program?

Hint: If you pick any key r as the root, you get two distinct subproblem: the left subtree and the right subtree of r. Which parameters do you need to describe a subproblem?

8 Points

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, January 24.