Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

Würzburg, 10 January 2024

Johannes Zink
Oksana Firman

Exercise Sheet #8

Advanced Algorithms (WS 2023/24)

Exercise 1 – Rank structure

Suppose we want to build the rank structure for a bit string of length $n = 256$.

a) Let A be the array that stores the cumulative rank of chunks and let B be the array that stores the cumulative rank of subchunks. Compute the precise lengths of A and B and how many bits each entry has; show your calculations. **3 Points**

b) Explicitly write the first 10 rows of the lookup table for the bit strings of length $\frac{1}{2}\log n$. **2 Points**

Exercise 2 – Operations of rank structure

Give pseudocode for the following methods in a rank structure. Use the methords `rank` and `select`. Catch the special case that there is no predecessor/successor. You can assume that $i \in \{1, \ldots, n\}$.

- `predecessor`(i): returns the index of the predecessor of the element indexed by i;
- `successor`(i): returns the index of the successor of the element indexed by i.

4 Points

Exercise 3 – The child operation on succinct binary trees

Consider the succinct representation of binary trees from the lecture. Let i be the index (position of its representative 1 in the bit string) of a vertex v that has two children. Prove that the indices of the children of v are then given by $2\texttt{rank}(i)$ and $2\texttt{rank}(i) + 1$. **5 Points**

Exercise 4 – Operations on LOUDS

Consider the LOUDS representation of a tree from the lecture. Let `i` be the index of a vertex v (that is, the position of its "1" in the LOUDS bit string). Give pseudocode for the following methods, which should run in constant time:

- `outDegree(i)`, which returns the outdegree of v;
- `childNum(i, j)`, which returns the index of the j-th child of v if it exists and otherwise -1;
- `isRoot(i)`, which returns whether vertex v is the (real) root; and
- `isLeaf(i)`, which returns whether vertex v is a leaf.

6 Points

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, January 17.