

Exercise Sheet #7

Advanced Algorithms (WS 2023/24)

Exercise 1 – Ski rental

Consider the following randomized strategy for the ski rental problem. Instead of a coin, we throw a fair 3-sided die. If it shows 1, we buy skis on the $M/3$ -th good day. If it shows 2, we buy skis on the $2M/3$ -th good day. Otherwise we buy skis on the M -th good day.

- a) What is the (expected) competitive ration of this strategy? Is it better than the best deterministic strategies from the lecture? **4 Points**

Now we change our strategy IV from the lecture and buy skis on the $M/2$ -th good day only if we throw HEAD twice in a row and otherwise we buy skis on the M -th good day.

- b) What is the (expected) competitive ratio of this strategy? **3 Points**

Exercise 2 – Online bin packing

In the offline bin packing problem, you are given a set of items each with a size in $[0, 1]$. The task is to pack these items into bins of size 1 such that no bin is overpacked and the total number of used bins is minimized. In the online version, the items arrive in a stream-like fashion. Each item has to be packed immediately and later repacking is not allowed. We consider the *FirstFit* strategy. Here, the item is packed in the first bin where it still fits. If it fits in no bin, a new bin is opened.

- a) Prove that FirstFit is 2-competitive (up to some additive constant). **3 Points**
- b) Show with an example that it can not be better than 2-competitive (up to some additive constant). **3 Points**

Exercise 3 – Online tree coloring game

In the game of online tree coloring, a *forest warden* and a *painter* play against each other. The game starts with an empty graph G_0 . In turn i , the forest warden adds a vertex v_i to G_{i-1} and any edges from v_i to vertices in G_{i-1} . The resulting graph G_i has to be a forest, that is, a set of trees. The painter then has to assign a color to v_i such that G_i is properly colored, that is, no two adjacent vertices have the same color.

- a) Show that there is a strategy with which the forest warden can enforce an arbitrarily large number of colors. **5 Points**
- b) What is the best the forest warden can hope for, if the degree of vertex v_i in G_i may be at most k ? Explain your answer. **2 Points**

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, January 10.