Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #3

Advanced Algorithms (WS 2023/24)

Exercise 1 – Pathwidth

We consider the cactus graph G given in Figure 1.

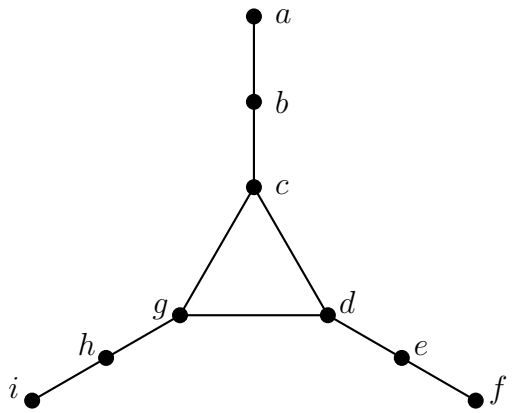

FIGURE 1: Graph G for Exercise 1.

a) Give a path decomposition of G with the minimum width. **2 Points**

b) Show that your solution of (a) is correct, i.e., that there is no path decomposition with a smaller width. **3 Points**

Exercise 2 – Computing a maximum-weight independent set

We consider the graph G with weighted vertices given in Figure 2. The weights of the vertices are specified in the figure.

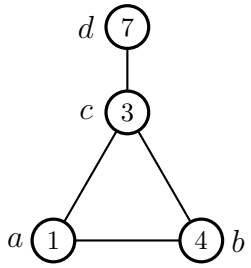

FIGURE 2: Graph G for Exercise 2.

a) Give a *nice* path decomposition of G with minimum width. What is the pathwidth of G? **3 Points**

b) Employ the algorithm from the lecture to find the weight of a maximum-weight independent set of G using the nice path decomposition from (a). Describe the intermediate steps of the computation. In particular, add all entries $D[\cdot,\cdot]$ of the dynamic program. **5 Points**

Exercise 3 – MAXCUT in graphs of bounded pathwidth

The problem MAXCUT is defined as follows. Given an undirected graph $G = (V, E)$, find a partition $(C, V \setminus C)$ of the vertex set V such that the number of edges whose endpoints are in different partitions is maximized, i. e., $\max_{C \subseteq V} |\{uv \mid uv \in E, u \in C, v \in V \setminus C\}|$.

Show that MAXCUT is FPT with respect to the pathwidth. You may assume that you are given a nice path decomposition of width k. **7 Points**

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, November 15.