Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, June 1, 2023

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Homework Assignment #4(b)
Algorithms for Geographic Information Systems (SS 2023)

The first half of this exercise sheet was be published on June 1.

Exercise 3 – Map Labeling

In the lecture, you got to know an approximation algorithm for the problem of finding a maximum set of pairwise nonintersecting rectangles. The algorithm has an approximation guarantee of $O(1/\log n)$, where n is the number of rectangles.

a) Show that the analysis is tight: construct an infinite family of sets of rectangles such that the ratio of the optimum over the number of rectangles chosen by the algorithm is $\Omega(1/\log n)$.
   [2 extrapoints]

b) Prove the optimality of the greedy algorithm for the 1-dimensional problem. In other words, show that if there exists a horizontal line that intersects all rectangles, one can find an optimal solution by sweeping from left to right and always picking the rectangle that has the leftmost right border (and doesn't intersect any rectangle picked before). **[1 point]**

c) Assume that all labels have the same height. Show that the greedy algorithm from (b) does not compute an optimal solution in this case. Show that it has an approximation guarantuee of $O(1)$.
   [5 points]

d) Assume that there exist two horizontal lines such that every rectangle is intersected by at least one of these lines. Show that the optimum solution for such instances can be computed in polynomial time. **[5 extrapoints]**

Please hand in your solutions on WueCampus until 12:00 on Thuesday, June 15.