Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, July 6, 2023

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Homework Assignment #8
Algorithms for Geographic Information Systems (SS 2023)

This is the last AGIS exercise sheet in this semester! :-)

Exercise 1 – One-sided po-leaders

Let P be a set of points with fixed labels at one side of the enclosing rectangle R. In the lecture it was stated (but not proven) that, in a *minimum-length* po-labeling, ascending and descending leaders do not cross.

a) Prove that no ascending leader crosses a descending leader. **[2 points]**

b) Prove that no leader crosses both an ascending and a descending leader. **[2 points]**

Exercise 2 – Free segments

Let P be a set of points with fixed labels at up to four sides of the enclosing rectangle R.

a) Show that a non-uniform s-labeling of minimum length has no crossings. **[2 points]**

b) Describe an algorithm (in words) that finds an optimal non-uniform s-labeling of minimum length – even if the number of labels is greater than the number of points. **[4 points]**

Exercise 3 – Many-to-one boundary labeling

In many-to-one boundary labeling, the input consists of a point set P contained in a rectangle R, and a partition of $P = P_1 \cup \cdots \cup P_k$ into $k \geq 1$ subsets. The task is to find a placement of k labels $\ell_1, \ldots, \ell_k$ along the vertical sides of R and many-to-one leaders, that is, for every $i \in \{1, \ldots, k\}$, we want to place a horizontal line segment h_i in R incident to label ℓ_i and a set of vertical line segments connecting the points in P_i to h_i. (This can be seen as a set of po-leaders whose horizontal segments overlap.)

a) Prove or disprove: Given a point set P and a partition of $P = P_1 \cup \cdots \cup P_k$, there always exists a (not necessarily optimal) solution without crossings if the labels are free (i.e., not fixed). **[2 points]**

b) Describe an algorithm (in words) that finds in polynomial time a (not necessarily crossing-free) solution of minimum length if the labels are fixed. **[2 points]**

c) Describe an algorithm (in words) that runs in $O(|P| \cdot k)$ time and finds a crossing-minimal solution for a point set P and a partition $P_1 \cup \cdots \cup P_k$ of P if the labels are free but their order is prescribed. **[6 points]**

Please hand in your solutions on WueCampus until 12:00 on Thuesday, July 13.