Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

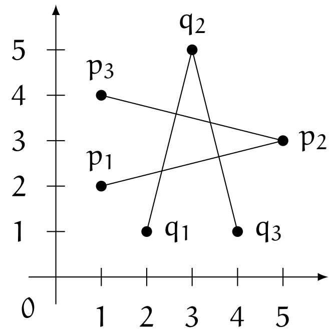
Würzburg, May 8, 2023

Prof. Dr. Alexander Wolff
M.Sc. Marie Diana Sieper

Homework Assignment #2
Algorithms for Geographic Information Systems (SS 2023)

Exercise 1 – Hausdorff and Fréchet

Let $P = (p_1, p_2, p_3)$ und $Q = (q_1, q_2, q_3)$ be two polylines with $p_1 = (1, 2)$, $p_2 = (5, 3)$, $p_3 = (1, 4)$, $q_1 = (2, 1)$, $q_2 = (3, 5)$ and $q_3 = (4, 1)$ as shown in the drawing.



a) Compute the Hausdorff distance $d_{hd}(P, Q)$ of P and Q and visualize the distance. **[2 points]**

b) Draw the free-space diagram of P and Q with respect to $\varepsilon = 2$. You don't need to draw the ellipses, but do compute their intersections with the cells and mark these intersections in the diagram.

   With the help of the free-space diagram, decide whether $d_{fréchet}(P, Q) \leq 2$. **[5 points]**

c) Prove or disprove the following statement:

   For every pair (P, Q) of polylines, it holds that $d_{fréchet}(P, Q) \geq d_{hd}(P, Q)$. **[2 points]**

d) Let Q' be the same polyline as Q, but traversed in the opposite direction, that is, $Q' = (q_3, q_2, q_1)$. Which of the following equations are correct? Justify your answer. **[2 points]**

   1. $d_{hd}(P, Q) = d_{hd}(P, Q')$
   2. $d_{hd}(Q, Q') = 0$
   3. $d_{fréchet}(P, Q) = d_{fréchet}(P, Q')$
   4. $d_{fréchet}(Q, Q') = 0$

Exercise 2 – Symmetric Hausdorff Distance

Let P and Q be two polylines, and let $P' = \{p_1, \ldots, p_n\}$ and $Q' = \{q_1, \ldots, q_n\}$ be the vertex sets of P and Q, respectively. Prove or disprove the following statement:

$$d_{hd}(P, Q) = \max\{d(p_i, Q') \mid i \in \{1, \ldots, n\}\}.$$

In other words, prove or disprove that for computing the symmetric Hausdorff distance of two polylines it suffices to consider their vertices. **[4 points]**

Exercise 3 – Free-Space Ellipses

Consider the free-space diagrams for checking whether the Fréchet distance between two polylines is at most ε, as seen in the lecture. If we are looking at just two line segments (that is, $n = m = 1$), the free space is defined as

$$F_\varepsilon \overset{\text{def}}{=} \{ (s, t) \in [0, 1]^2 \mid d(p_1(s), p_2(t)) \leq \varepsilon \}.$$

Note that this definition is over s and t, that is, in *parameter space*. Show that F_ε is an ellipse, intersected with $[0, 1]^2$. **[4 points]**

Please hand in your solutions on WueCampus until 12:00 on Thuesday, May 18.