Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

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Homework Assignment #1
Algorithms for Geogpraphic Information Systems (SS 2023)

Exercise 1 – Area preserving map projection

In the lecture, you saw how one can derive an angle conform map projection from a shere onto a cylinder. In the same manner, derive a map from a spere onto a cylinder which is area preserving. **[4 points]**

Exercise 2 – ILP-approach for state merging

The REPUBLIC OF AGIS has asked you to help them with a problem. The republic currently has a lot of states, many of which are too small to fully function on their own. Of course you agree to help!

The precise problem definition is as follows: Let $\mathcal{S}$ be a set of states, let $p\colon \mathcal{S} \to \mathbb{N}$ be the number of inhabitants living in it, and let $w_l, w_u \in \mathbb{R}_{\geq 0}$, $w_l \leq w_u$ be some bounds. Find a partition of $\mathcal{S}$ into *superstates* $S_1, \ldots, S_k$ such that, for $i \in \{1, \ldots, k\}$, it holds that $w_l \leq \sum_{s \in S_i} p(s) \leq w_u$.

a) Find an ILP formulation for this partitioning problem such that the numer of superstates is minimized. **[3 points]**

   Hint: For an introduction to (Integer) Linear Programming, take a look at the second lecture in last year's WueCampus course for Algorithmic Graph Theory (`https://wuecampus.uni-wuerzburg.de/moodle/course/view.php?id=52088`).

b) The REPUBLIC OF AGIS realized that long borders between states make logistics harder. You are now also given a function $b\colon S \times S \to \mathbb{R}_{\geq 0}$, where $b(s_1, s_2)$ expresses how long the border between regions s_1 and region s_2 is.

   Your new task is to find an ILP formulation for problem (a) such that the total length of the remaining border is minimized. **[4 points]**

c) Realizing that this can still lead to unconnected superstates, the REPUBLIC OF AGIS asks to include yet another condition.

   Find an ILP formulation for problem (b) with the additional property that every superstate is connected (meaning for every $i \in \{1, \ldots, k\}$ and for every pair of states $s, s' \in \mathcal{S}_i$, there exists a sequence $(s = s_1, s_2, \ldots s_\ell = s')$ such that $s_1, \ldots, s_\ell \in \mathcal{S}_i$, and for every $1 \leq j \leq \ell - 1$, it holds that $b(s_j, s_{j+1}) > 0$. **[5 points]**

   Hint: Fix a center state for each superstate.

Exercise 3 – Round earth

It is known since about 500 B.C. that the earth is round rather than flat. Already the old Greek had a pretty good approximation for the earth's radius, with an error of at most 15%.

How would you prove that the earth is round and compute its radius, given enough time and the technology of 500 B.C.? Describe a possible approach in detail. **[4 points]**

Please hand in your solutions on WueCampus until 12:00 on Thuesday, May 4.