Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #9

Graph Visualization (SS 2023)

Exercise 1 – Visibility representations

Let $R = [0, x] \times [0, y] \subset \mathbb{R}^2$ be an axis-parallel rectangle in the plane that contains a set S of pairwise disjoint horizontal line segments. Let the line segments $[0, x] \times \{0\}$ and $[0, x] \times \{y\}$ be contained in S.

We consider the ε-bar visibility graph $G = (S, E)$ with the set of directed edges

$$E = \{(u, v) \mid v \text{ is vertically upwards visible from } u\}.$$

a) Show that G is an st-graph. **4 Points**

b) Describe how we could derive an upward planar drawing of G from the given visibility representation. **4 Points**

Exercise 2 – Computing coordinates for a visibility representation

We want to compute an ε-bar visibility representation ψ of an st-graph $G = (V, E)$. In addition to G, we are also given minimal (vertical) distances between pairs of bars corresponding to two adjacent vertices and a minimal width for every bar. More precisely, for a given edge-weight function $h\colon E \to \mathbb{R}^+$ and edge $(u, v) \in E$, the vertical distance of the bars $\psi(u)$ and $\psi(v)$ has to be at least $h((u, v))$. The function $w\colon V \to \mathbb{R}^+$ gives the minimal width for each bar $\psi(v)$ for $v \in V$.

a) Describe a linear-time algorithm that calculates the y-coordinates for the bars in a visibility representation with the minimum maximal height. Argue why your algorithm achieves this asymptotic runtime. **6 Points**

b) To obtain a bar visibility representation, it remains to determine the x-coordinates. Describe a linear-time algorithm that calculates the x-coordinates for the bars with the minimum maximal width. **6 Points**

Hint: Consider the st-dual of G, which is the dual graph of an st-graph drawing, where the outer face has a vertex on the left and on the right side of the drawing, and all edges are directed to the right.

This assignment is due at the beginning of the next lecture, that is, on July 7 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 28 at 16:00 and the solutions will be discussed two weeks after that on July 12.