Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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# Exercise Sheet #8

# Graph Visualization (SS 2023)

### Exercise 1 – MINIMUM FEEDBACK (ARC) SET

Let $G = (V, E)$ be a directed graph. For a set of edges $E' \subseteq E$, let $E'_r := \{vu \mid uv \in E'\}$ be the set of reversed edges. A set of edges $E^\star \subseteq E$ with minimum cardinality is called

- a MINIMUM FEEDBACK ARC SET if $G_{\mathsf{FAS}} = (V, E \setminus E^\star)$ is acyclic;
- a MINIMUM FEEDBACK SET if $G_{\mathsf{FS}} = (V, (E \setminus E^\star) \cup E^\star_r)$ is acyclic.

Show that $E^\star \subseteq E$ is a MINIMUM FEEDBACK SET if and only if $E^\star$ is a MINIMUM FEEDBACK ARC SET. **6 Points**

### Exercise 2 – Optimal one-sided crossing minimization

We consider the problem of one-sided crossing minimization, i.e., we are given a bipartite graph $G = (L_1 \cup L_2, E)$ with a permutation $\pi_1$ of $L_1$ and we search for a permutation $\pi_2$ of $L_2$ that minimizes the number of crossings.

Suppose there exists a permutation $\pi_2^\star$ of $L_2$ such that no edges cross.

a) Show that in this case the *barycenter heuristic* also yields a permutation $\pi_2'$ that results in no crossings. **3 Points**

b) Show that in this case the *median heuristic* also yields a permutation $\pi_2''$ that results in no crossings. **3 Points**

*Hint:* Show how the barycenter heuristic and the median heuristic can handle the case when multiple vertices have the same barycenter or median, respectively. For the median heuristic assume that vertices with an odd number of neighbors are placed to the left of the vertices with an even number of neighbors.

### Exercise 3 – Planar Drawings

Let G be an upward-planar graph. Does the Sugiyama framework always yield an upward-planar drawing of G if we use, for the leveling, the recursive linear-time algorithm to minimize the number of layers, and a suitable method for crossing minimization. Justify your answer. **4 Points**

**Exercise 4 – Precedence-Constrained Multi-Processor Scheduling**

Give an infinite class of instances for which the $(2 - 1/W)$-approximation algorithm for the scheduling problem PRECEDENCE-CONSTRAINED MULTI-PROCESSOR SCHEDULING from the lecture yields schedules of length $(2-1/W)$ OPT. It suffices to consider a fixed value for $W$. **4 Points**

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This assignment is due at the beginning of the next lecture, that is, on June 23 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 21 at 16:00 and the solutions will be discussed one week after that on June 28.