Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #7

Graph Visualization (SS 2023)

Exercise 1 – Contact representations by line segments

Show that there is a planar graph which does not have a contact representation by line segments, even when the line segments can have any slopes and lengths. **5 Points**

Hint: How many edges can there be in the intersection graph of such a contact representation?

Exercise 2 – Contact representations by equilateral triangles

Show that there is a planar graph which does not have a contact representation by homothetic equilateral triangles, that is, equilateral triangles that only differ in scale and translation, but not in rotation. (E.g., all triangles have a horizontal edge and the third corner above the edge). **5 Points**

Exercise 3 – Square contact representations of maximal outerplanar graphs

Recall that an n-vertex graph G is *outerplanar* if it has a planar embedding such that all vertices are on the outer face. The graph G is a *maximal outerplanar graph* if it is internally triangulated.

Let $\pi = (v_1, v_2, \ldots, v_n)$ be the canonical order of the graph G. That is, $\{v_1, v_2, \ldots, v_n\} = V(G)$, the graph G_i induced by $\{v_1, v_2, \ldots, v_i\}$ is a maximal outerplanar graph, and the following conditions hold:

- the edge (v_1, v_2) lies on the outer face; and
- for each $i \in \{3, 4, \ldots, n\}$, v_i has exactly two neighbors on the outer face of G_i.

Show that every maximal outerplanar graph has a contact representation by squares and that, using the canonical order, it can be computed in linear time. **10 Points**

This assignment is due at the beginning of the next lecture, that is, on June 16 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on June 14 at 16:00 and the solutions will be discussed one week after that on June 21.