

Exercise Sheet #5

Graph Visualization (SS 2023)

Exercise 1 – Simple upward planar graphs

Prove or disprove that the following graph classes are upward planar:

- a) directed acyclic graphs whose underlying undirected graph is a simple cycle; **3 Points**
- b) directed acyclic graphs whose underlying undirected graph is a tree. **3 Points**

Exercise 2 – False friends

In the lecture we introduced three necessary conditions for a digraph G to be upward planar (planar, acyclic, bimodal). Show that these conditions are not sufficient. To do so find a directed graph with an embedding that adheres to all three conditions and prove that the graph (not only this specific embedding) is *not* upward planar. **3 Points**

Exercise 3 – Refinement of the outerface

Let $G = (V, E)$ be a directed acyclic graph with given embedding, set of faces F , and outer face f_0 . Let $\Phi: \mathcal{S} \cup \mathcal{T} \rightarrow F$ be a consistent assignment of the large angles of the sinks and sources to the incident faces. We consider the situation in which the inner faces have been refined already, i.e., there exist no large angles on the inner faces anymore.

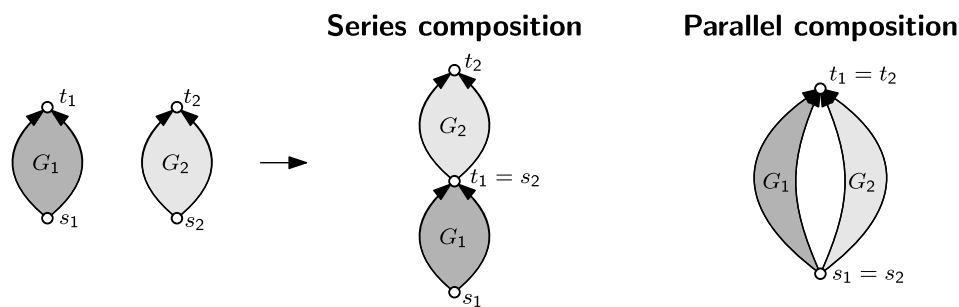
Show how you can complete the transformation into a planar st-graph by refining f_0 . **4 Points**

Hint: A possible intermediate step could be to first refine f_0 such that the sources and sinks of the graph form two disjoint intervals on the boundary of the outer face.

Exercise 4 – Upward planar drawings of series-parallel graphs

A graph G with a source vertex s and a sink vertex t is series-parallel, if

- it contains a single directed edge (s, t) , or
- it consists of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2 that are combined such that
 - $t = t_2, t_1 = s_2$, and $s = s_1$ (*series composition*), or
 - $t = t_1 = t_2$, and $s = s_1 = s_2$ (*parallel composition*)



In contrast to general acyclic graphs, all series-parallel graphs have an upward planar drawing.

Describe an algorithm that generates for a series-parallel graph with a given embedding an upward planar drawing such that the embedding is preserved. In the output drawing, edges are allowed to have up to two bends, but all bends and vertices must have integer coordinates on a grid of polynomial size.

You can assume that the series-parallel graph is given with its decomposition tree.

Hint: Try to draw each edge such that it consists of three line segments where the middle segment is vertical.

- Describe your algorithm. **4 Points**
- Argue why your algorithm generates an upward planar drawing. **2 Points**
- Estimate the area requirement of the drawings generated by your algorithm (subject to n , the number of vertices). **1 Point**

This assignment is due at the beginning of the next lecture, that is, on June 2 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on May 31 at 16:00 and the solutions will be discussed one week after that on May 7.