

## Exercise Sheet #4

### Graph Visualization (SS 2023)

#### Exercise 1 – Fast construction of Schnyder realizer

In the lecture we have proven that every triangulated plane graph  $G = (V, E)$  has a Schnyder labeling and a Schnyder realizer. The proof yields a recursive algorithm: contract an edge  $\{a, x\}$ , find recursively a Schnyder forest in the resulting graph and then add  $x$  consistently back. A naive implementation of this algorithm yields a runtime of  $O(n^2)$ , in particular, because we need to find the contracted edge. Explain how the algorithm can be improved to admit linear runtime.

*Hint:* Think about the candidate edges for contraction. How to update them quickly?  
**7 Points**

#### Exercise 2 – Weak barycentric representations

Let  $G = (V, E)$  be a plane triangulated graph with a weak barycentric representation  $v \in V \mapsto (v_1, v_2, v_3) \in \mathbb{R}^3$ . Let  $A, B, C \in \mathbb{R}^2$  be points in general position.

Show that the function  $f: v \in V \mapsto v_1A + v_2B + v_3C$  yields a crossing-free drawing.  
**6 Points**

#### Exercise 3 – Fast calculation of barycentric coordinates

Let  $G = (V, E)$  be an  $n$ -vertex triangulated plane graph with a Schnyder realizer  $T_1, T_2, T_3$ . As in the lecture, let  $v_i = |V(R_i(v))| - |P_{i-1}(v)|$  where  $|V(R_i(v))|$  is the number of vertices in the region  $R_i$  with respect to  $v$  (including the vertices on the boundary of  $R_i$  and  $v$  itself) and  $|P_i(v)|$  is the number of vertices on the path from  $v$  to  $a_i$  in  $T_i$ .

Show that all values  $v_i$  can be calculated for all inner vertices in a total runtime of  $O(n)$ .

*Hint 1:* Consider each  $i \in \{1, 2, 3\}$  independently. (It suffices to consider  $v_1$ .)

*Hint 2:* Gather the necessary information by traversing  $T_1, T_2$ , and  $T_3$ .  
**7 Points**

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This assignment is due at the beginning of the next lecture, that is, on May 26 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on May 24 at 16:00 and the solutions will be discussed one week after that on May 31.