Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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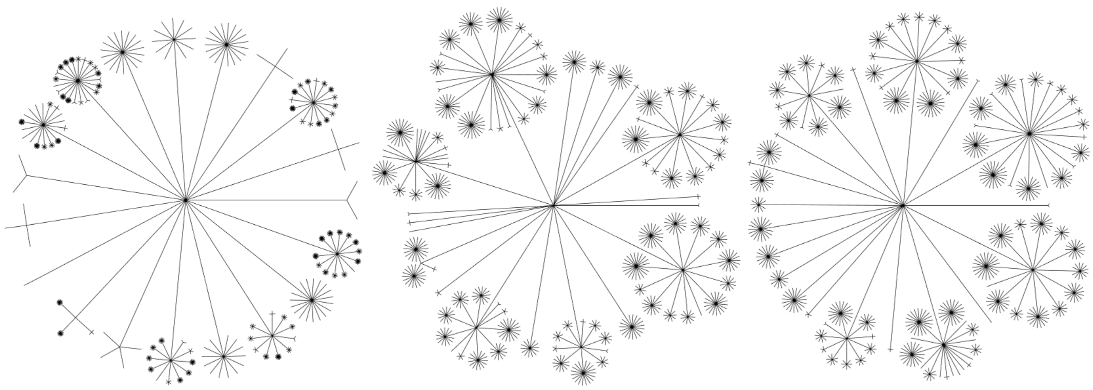
Exercise Sheet #1

Graph Visualization (SS 2023)

Exercise 1 – Drawing conventions & aesthetics of balloon layouts

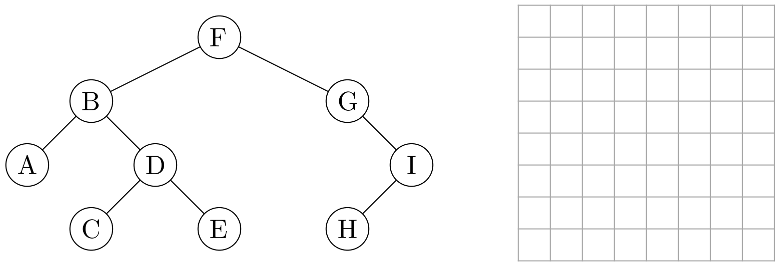
The three drawings of the same tree below are drawn with a *balloon layout*. Try to find at least two common drawing conventions and two possible drawing aesthetics to optimize for this layout style. **2 Points**



Exercise 2 – Binary trees with pre- and postorder coordinates

Let $T = (V, E)$ be a binary tree with root r. For each $v \in V$, let $x(v) :=$ preorder(v) and $y(v) :=$ postorder(v). Recall that $T(v)$ denotes the subtree rooted at v.

You may use the graph and grid below to try an example.



a) Prove that this coordinate assignment yields a planar drawing of T. **4 Points**

b) Give thight bounds on the area requirement of the generated drawing. **2 Points**

c) Prove that if you direct all edges of T such that they "point away" from r – that is, all vertices can be reached from r – then all arcs in the drawing point downwards. **2 Points**

Exercise 3 – Lower bound on the area of right-heavy HV-drawings

Prove that there are trees for which the right-heavy HV-layout algorithm from the lecture produces drawings with area $\Omega(n \log n)$. **3 Points**

Exercise 4 – Space-saving HV-drawings of complete binary trees

Let T be a *complete binary tree* of height h, that is, a binary tree where all vertices of depth $1, \dots, h-1$ have exactly 2 children and all vertices of depth h are leaves. Consider the following HV-drawing algorithm.

Algorithm 1: BalancedHVDraw(node v, depth d)

```
if v == nil then return ∅
v_l, v_r ← left / right child of v
Γ_1 ← BalancedHVDraw(v_l, d + 1)
Γ_2 ← BalancedHVDraw(v_r, d + 1)
if d odd then return horizontal combination of Γ_1 and Γ_2
if d even then return vertical combination of Γ_1 and Γ_2
```

a) Prove that BalancedHVDraw produces a drawing of T with area $O(n)$.

   Hint: use induction on the height of the tree with the following hypothesis. The area of the drawing for odd height is $2\sqrt{n+1} - 3 \times \frac{3}{2}\sqrt{n+1} - 2$ *and for even height is* $\sqrt{2(n+1)} - 2 \times \frac{3}{2}\sqrt{2(n+1)} - 3$. **5 Points**

b) Give sharp constant upper and lower bounds on the aspect ratio (i.e., the ratio between the width and the height) of the drawing generated with input T. **2 Points**

This assignment is due at the beginning of the next lecture, that is, on April 028 at 10:15 am. Please submit your solutions via WueCampus. The questions can be asked in the tutorial session on April 26 at 16:00 and the solutions will be discussed one week after that on May 03.