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Tutoriumsblatt #2 Approximation Algorithms (Winter Semester 2022/23)

Exercise 1 – MAX-SAT

a) Show that the following algorithm is a 1/2-approximation for MAX-SAT. Let τ be an arbitrary truth assignment and τ' be its complement, that is, a variable is true in τ if and only if it is false in τ' . Compute the weight of clauses satisfied by τ and by τ' , then output the better assignment.

In the following, let a k-CNF-SAT instance denote a formula in conjunctive normal form, where each clause contains exactly k literals and every variable occurs at most once per clause.

- b) Show that every 3-CNF-SAT instance on at most 7 clauses is satisfiable.
- c) Recall the algorithm that just outputs a uniformly random truth assignment. It runs in polynomial time and we proved a lowerbound on the expected weight of the satisfied clauses. We then derandomised this algorithm. This makes the algorithm slower and more complicated. Consider instead the following algorithm for MAX-3-CNF-SAT: keep guessing truth assignments until you find one that satisfies 7/8 of the clauses. Show that this algorithm runs in expected polynomial time. Compare the advantages and disadvantages of these three algorithms for MAX-3-CNF-SAT.
- d) Recall the MAXCUT problem: find a set $S \subseteq V$ such that the number of edges between S and its complement is as large as possible. Solve MAXCUT using MAX-2-CNF-SAT.

We will work on these exercises in the tutorium on 07.02.2023. You do not need to hand them in and they will not be corrected.