Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #12

Advanced Algorithms (WS 2022/23)

Exercise 1 – Space consumption of skip lists

What is the space consumption of ...

a) detereministic skip lists? **2 Points**

b) randomized skip lists in expectation? **2 Points**

c) randomized skip lists in the worst case? **1 Point**

Exercise 2 – Flipping biased coins

In this exercise, we investigate how the query time and the space consumption change when skip lists are constructed with some probability $p \in (0, 1)$ for an element to appear in the next higher level. In other words, when a new element is inserted, we flip a biased coin and the probability for TAILS is p. (We assume that there is no maximum height of the skip list.)

a) What is the space consumption and the query time for a skip list for $p = 1/4$ and for $p = 3/4$? Compare them with the case from the lecture where we had $p = 1/2$. **4 Points**

b) What is the optimal p to minimize the query time? **2 Points**

Exercise 3 – Combining Bloom filters

You are given two Bloom filters F_1 and F_2 with arrays of equal lengths and with the same set of hash functions. The two Bloom filters represent two sets of numbers S_1 and S_2, respectively. However, you do not know the numbers contained in S_1 and S_2. (In the following, we don't want trivial solutions like "set every bit of the array to 1".)

a) Construct a new Bloom filter F_3 from F_1 and F_2 that answers containment checks on the set $S_1 \cup S_2$. Argue why your new Bloom filter works correctly. **2 Points**

b) Construct a new Bloom filter F_4 from F_1 and F_2 that answers containment checks on the set $S_1 \cap S_2$. Argue why your new Bloom filter works correctly. **2 Points**

c) Suppose you would construct Bloom filters F_3' and F_4' starting with an empty Bloom filter and inserting the numbers from the sets $S_1 \cup S_2$ and $S_1 \cap S_2$, respectively. Would F_3' differ from F_3? Would F_4' differ from F_4? **2 Points**

Exercise 4 – Sweeping Plane

Generalize the sweep-line algorithm for solving the CLOSEST PAIR problem in three dimensions, this is, we are given a point set of n three-dimensional points and we want to find the pair of points having the smallest Euclidean distance by using a sweeping plane similar to the sweep line in two dimensions.

Analyze the running time of your algorithm. You algorithm should have a running time in $O(n^2)$. **3 Points**

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, February 1.