

Exercise Sheet #10

Advanced Algorithms (WS 2022/23)

Exercise 1 – Insertion & deletion for Splay Trees

Describe how insertion and deletion of an element x work for Splay Trees. Both methods should use splay. Give pseudocode and argue whether the running time is in $O(c(\text{Splay}(x)))$.

Hint: Think about methods to split and to join two BSTs.

5 Points

Exercise 2 – Working set property of Splay Trees

A BST has the *working set property* if the (amortized) cost of a query for key x is $O(\log t)$, where t is the number of keys queried more recently than x .

We want to use the Access Lemma to show that Splay Trees have the working set property. For this, we assign the weights $1, \frac{1}{4}, \frac{1}{9}, \dots, \frac{1}{n^2}$ to the items in order by latest access (call of Splay on an element). That is, after the constructing of the tree is done, the root has weight 1 and the item first added has weight $\frac{1}{n^2}$.

After an access occurs, we update the weights. More precisely, suppose element x is accessed during access j and has weight $\frac{1}{k^2}$. Then set the new weight of x to 1. For every element x' with weight $\frac{1}{k'^2}$ where $k' < k$, set the new weight to $\frac{1}{(1+k')^2}$. Note that this reassignment permutes the weights $1, \frac{1}{4}, \frac{1}{9}, \dots, \frac{1}{n^2}$.

a) Using these weight (re-)assignments, prove that Splay Trees have the working set property. 5 Points

b) Explain why we are allowed to change the weights of nodes in this way. 4 Points

Exercise 3 – Dynamic Program for Known Probability Distribution

Design a dynamic program that creates an optimal static binary search tree for a given set of n keys with known probability distribution p_1, \dots, p_n , that is, the static binary search tree where the expected query time $\sum_{i=1}^n p_i \ell_i$ is minimized, where ℓ_i is the level of key i . What are the running time and space requirements for the dynamic program?

Hint: If you pick any key r as the root, you get two distinct subproblem: the left and the right subtree of r . Which parameters do you need to describe a subproblem?

6 Points